Addendum to "Factor and Structural Equations Analyses of Within Person Trait and Ability Patterns: The Road Less Traveled"

In the following pages are the data (in the form of a Variance/Covariance matrix) from our paper, LISREL syntax for running the structural equations analysis with that data, and LISREL output including a path diagram. It is our hope that the following documentation would be helpful for users wanting to replicate the results in our paper or to learn how to apply the analysis to their own data. We have also included Appendix B from the paper that explains the commands used to constrain LISREL parameter estimates.

The Variance-Covariance Matrix for the Six Interest Variables and Four Personality Variables[a]

	Interest Variables						Personality Variables		
	REA	INV	ART	SOC	ENT	CON	SOCPO	SOCCL	T
Realistic	97.27	43.71	4.99	9.24	18.06	25.97			
Investigative	43.71	79.64	22.22	22.93	3.98	19.76			
Artistic	4.99	22.22	96.38	16.66	.63	-15.48			
Social	9.24	22.93	16.66	90.61	30.07	18.57			
Enterprising	18.06	3.98	.63	30.07	89.60	30.53			
Conventional	25.97	19.76	-15.48	18.57	30.53	87.20			
Social Potency	-6.70	5.67	16.60	20.79	33.93	-3.58	110.24		
Social Closeness	-14.96	-5.46	2.22	27.70	13.83	-4.62	31.04	98.81	
Traditionalism	2.16	-9.85	-25.54	7.72	16.01	21.09	-10.21	3.34	8
Absorption	5.89	9.96	36.98	9.01	.39	-8.78	27.06	-7.79	-

Note. REA= Realistic, INV= Investigative, ART= Artistic, SOC=Social, ENT=Enterprising, CON= Conventional, SOCPO= Social Potency, SOCCL= Social Closeness, TRAD= Traditionalism, ABSOR= Absorption.

LISREL Syntax for Structural Equations Model

Data and problem parameters
Specify the structure of the data and the type of moment matrix to be analyzed. ni: Number of input variables, no: Number of cases, ma=cm: Type of matrix to be analyzed (in this case a covariance matrix).

da ni=10 no=457 ma=cm

Specify the model for LISREL analysis. Nx: Number of *x*-variables in the model, nk: Number of *X* -variables, lx: Full matrix of fixed factor loadings, ph: Symmetric, fixed correlation matrix between latent variables.

mo nx=10 nk=5 lx=fu,fi ph=sy,fi ad=off

Variable labels
Specify labels for both observed variables and latent variables. La: Labels for observed variables, lk: Labels for latent variables.

la

REA INV ART ENT SOC CON SOCPO SOCCLOS TRAD ABSORP

lk
F1 F2 F3 F4 F5

Data for analysis
Data matrix to be analyzed. cm=sy: Symmetric covariance matrix.

cm sy

97.27									
43.71	79.64								
4.99	22.22	96.38							
18.06	3.98	.63	89.60						
9.24	22.93	16.66	30.07	90.61					
25.97	19.76	-15.48	30.53	18.57	87.20				
-6.70	5.67	16.60	33.93	20.79	-3.58	110.24			
-14.96	-5.46	2.22	13.83	27.70	-4.62	31.04	98.81		
2.16	-9.85	-25.54	16.01	7.72	21.09	-10.21	3.34	82.19	
5.89	9.96	36.98	.39	9.01	-8.78	27.06	-7.79	-9.35	98.04

Pattern matrix
Set elements of the LISREL parameter matrix fixed or free using a pattern of 1's and 0's
(1 ≡ free ; 0 ≡ fixed)

pa lx

```
1 1 1 0 0
1 1 1 0 0
1 1 1 0 0
1 1 1 0 0
1 0 1 0 0
1 1 0 0 0
0 0 0 1 0
0 0 0 1 0
0 0 0 0 1
0 0 0 0 1
```

Matrix values
Set all elements of a parameter matrix to values specified by an input matrix; these values will be the starting values for the free parameters and fixed values otherwise.

ma lx

```
4.63    2.35   -3.24   ----   ----
4.63    -.44   -3.62   ----   ----
4.63   -8.73    -.11   ----   ----
4.63    1.95    4.22   ----   ----
4.63    ----    2.75   ----   ----
4.63    4.00   ----    ----   ----
----    ----   ----      5    ----
----    ----   ----      5    ----
----    ----   ----    ----     5
----    ----   ----    ----     5
```

Starting values
st: Assigns starting values for iterative estimation of fixed parameters. Setting elements lx(5,2) and lx(6,3) equal to zero will help identify the second and third factors.

```
st 0 lx 7 1 lx 7 2 lx 7 3 lx 7 5 lx 8 1 lx 8 2 lx 8 3 lx 8 5
st 0 lx 9 1 lx 9 2 lx 9 3 lx 9 4 lx 10 1 lx  10 2 lx 10 3 lx 10 4
st 0 lx (5,2)  lx (6,3)
```

Simple equality constraints
eq: Constrains other parameters to be equal to a specified parameter (the first in the list). Loadings along factor 1 are constrained to be equal.

```
eq lx (1,1) lx(2,1)
eq lx (1,1) lx(3,1)
eq lx (1,1) lx(4,1)
eq lx (1,1) lx(5,1)
eq lx (1,1) lx(6,1)
```

Complex equality constraints
co: Constrains a parameter to be equal to a function of other parameters. Orthogonality of the second and third factors can be imposed by constraining lx(4,3). Contrast constraints are imposed on the second and third factors by constraining lx(5,3) and lx(6,2). Refer to Appendix B below for a more detailed explanation of these linear and quadratic constraints.

```
co  lx (5,3) = 0 - lx(1,3) - lx(2,3) - lx(3,3) + lx(1,2)*lx(1,3)*lx(4,2)**-1 +
lx(2,2)*lx(2,3)*lx(4,2)**-1 + lx(3,2)*lx(3,3)*lx(4,2)**-1
co  lx (6,2) = 0 - lx(1,2) - lx(2,2) - lx(3,2) - lx(4,2)
co  lx (4,3) = 0 - lx(1,2)*lx(1,3)*lx(4,2)**-1 - lx(2,2)*lx(2,3)*lx(4,2)**-1 -
lx(3,2)*lx(3,3)*lx(4,2)**-1
```

Pattern matrix
Set elements of the LISREL parameter (factor correlation) matrix fixed or free using a pattern of 1's and 0's *(1 ≡ free ; 0 ≡ fixed).*

pa ph

```
0
0 0
0 0 0
0 0 1 0
0 1 0 0 0


st 1 ph 1 1 ph 2 2 ph 3 3 ph 4 4 ph 5 5
st 0 ph 2 1 ph 3 1 ph 3 2  ph 4 1  ph 4 2 ph 5 1  ph 5 3 ph 5 4

path Diagram
ou it=5000 rs se
```

Appendix B

Fitting our constraints requires an algorithm that can impose linear and quadratic constraints on SEM parameters. LISREL (Joreskog & Sorbom, 1996) can impose a variety of mathematical constraints, but the command syntax for doing so is somewhat restrictive. Here we show the LISREL commands used to help fix the rotation of our Factors 2 and 3, to impose the equality constraint on all Factor 1 loadings, to impose the orthogonality constraint Equation 4c, and to impose the contrast constraint Equation 4d.

To help identify the rotation of the second and third factors, we began by setting (st) $?_{42}$ and $?_{63}$ equal to zero using the LISREL command

```
st 0 lx (4, 2) lx (6, 3) .
```

Next we constrained all loadings along Factor 1 to be equal (eq) using the LISREL commands

```
eq lx (1, 1) lx (2, 1)
eq lx (1, 1) lx (3, 1)
eq lx (1, 1) lx (4, 1)
eq lx (1, 1) lx (5, 1)
eq lx (1, 1) lx (6, 1) .
```

These commands set all other loadings along the first factor equal to $?_{11}$.

Imposing the orthogonality and contrast constraints of Equations 4c and 4d within the syntax grammar of LISREL required rewriting those constraints.

The orthogonality constraint was imposed by placing a restriction on the parameter $?_{53}$. We began by solving the orthogonality Equation 4c, $S_{v}?_{v2}?_{v3} = 0$, for $?_{53}$ and utilizing the fact that $?_{42}$ and $?_{63}$ were set to zero above. This yields the following result:

$$?_{53} = -(S_{v?5}?_{v2}?_{v3})/?_{52} = -(S_{v=3}?_{v2}?_{v3})\,?_{52}^{-1} = -?_{12}?_{13}\,?_{52}^{-1} -?_{22}?_{23}?_{52}^{-1} -?_{32}?_{33}?_{52}^{-1} \qquad \text{(B1)}$$

Within the rather restrictive syntax of LISREL that does not allow parentheses except to designate subscripting, the constraint (co) in Equation B1 becomes

$$\text{co lx}(5, 3) = 0 - \text{lx}(1, 2)*\text{lx}(1, 3)*\text{lx}(5, 2)^{-1} - \text{lx}(2, 2)*\text{lx}(2, 3)*\text{lx}(5, 2)^{-1}$$

$- lx(3, 2)*lx(3, 3)*lx(5, 2)^{-1}$.

Next, we imposed the contrast constraint Equation 4d on Factor 2 with a restriction on $?_{62}$. The basic constraint equation, $S_v?_{v2} = 0$, was solved for $?_{62}$ which by utilizing the fact that $?_{42}$ was set to 0 above, yields

$$?_{62} = -S_{v\neq 6}?_{v2} = -S_{v?4,6}?_{v2} = -?_{21} - ?_{22} - ?_{32} - ?_{52} \qquad \text{(B2)}$$

Within the syntax of LISREL, Equation B2 translates into the constraint command co lx(6,2) = 0 – lx(2,1) – lx(2,2) – lx(3,2) – lx(5,2).

Last, we imposed the contrast constraint in Equation 4d on Factor 3 with a restriction on $?_{43}$. We began by solving the contrast constraint Equation, $S_v?_{v3} = 0$, for $?_{43}$ and then utilizing the fact that $?_{63}$ was set to 0 above and utilizing the restriction on $?_{53}$ in Equation B1 to arrive at the result in Equation B3. In an iteration, LISREL estimates parameters one at a time, and a parameter estimated later in the iteration cannot be specified on the right side of a constraint on a parameter estimated earlier in the iteration. Since $?_{53}$ was estimated later than $?_{43}$, $?_{53}$ cannot be used on the right side of the constraint on $?_{43}$. This forced us to substitute the expression on the right of Equation B1 for $?_{53}$ in the constraint on $?_{43}$.

$$?_{43} = -S_{v\neq 4}?_{v3} = -S_{v?4,6}?_{v3} = -?_{13} - ?_{23} - ?_{33} - ?_{53}$$

$$= -?_{13} - ?_{23} - ?_{33} + ?_{12}?_{13}\,?_{52}^{\ -1} + ?_{22}?_{23}?_{52}^{\ -1} + ?_{32}?_{33}?_{52}^{\ -1} \ . \qquad \text{(B3)}$$

In the syntax of LISREL, Equation B2 translates to the following constraint command:

co lx(4, 3) = 0 – lx(1, 3) – lx(2, 3) – lx(3, 3) + $lx(1, 2)*lx(1, 3)*lx(5, 2)^{-1}$

$+ lx(2, 2)*lx(2, 3)*lx(5, 2)^{-1} + lx(3, 2)*lx(3, 3)*lx(5, 2)^{-1}$.

In summary, we partly determined the rotation by setting $?_{42}$ and $?_{63}$ to 0, constrained all of the loadings along Factor 1 to be equal using LISREL equality statements, imposed orthogonality on the second and third factors with a constraint on $?_{53}$, and imposed contrast constraints on the second and third factors with constraints on $?_{62}$ and $?_{43}$.

LISREL Output From The Structural Equation Analysis

LISREL Estimates (Maximum Likelihood)

LAMBDA-X

	F1	F2	F3	F4	F5
REA	4.53 (0.21) 21.72	1.00 (0.41) 2.46	-4.57 (0.48) -9.60	- -	- -
INV	4.53 (0.21) 21.72	-1.45 (0.39) -3.73	-3.16 (0.42) -7.51	- -	- -
ART	4.53 (0.21) 21.72	-6.85 (0.51) -13.41	1.56 (0.39) 3.96	- -	- -
SOC	4.53 (0.21) 21.72	- -	2.76 (0.46) 6.02	- -	- -
ENT	4.53 (0.21) 21.72	3.11 (0.43) 7.26	3.42 (0.43) 7.88	- -	- -
CON	4.53 (0.21) 21.72	4.19 (0.41) 10.28	- -	- -	- -
SOCPO	- -	- -	- -	6.08 (0.71) 8.52	- -
SOCCLOS	- -	- -	- -	5.52 (0.66) 8.38	- -
TRAD	- -	- -	- -	- -	4.54 (1.55) 2.92
ABSORP	- -	- -	- -	- -	-1.63 (0.68) -2.40

PHI

	F1	F2	F3	F4	F5
F1	1.00				
F2	- -	1.00			
F3	- -	- -	1.00		
F4	- -	- -	0.71 (0.08) 8.56	1.00	
F5	- -	0.93 (0.29) 3.23	- -	- -	1.00

THETA-DELTA

	REA	INV	ART	SOC	ENT	CON
REA	49.27 (5.67) 8.69					
INV	- -	42.32 (4.23) 10.01				
ART	- -	- -	35.76 (7.81) 4.58			
SOC	- -	- -	- -	61.79 (5.06) 12.21		
ENT	- -	- -	- -	- -	46.32 (5.17) 8.95	
CON	- -	- -	- -	- -	- -	50.18 (4.80) 10.45
SOCPO	- -	- -	- -	- -	- -	- -
SOCCLOS	- -	- -	- -	- -	- -	- -
TRAD	- -	- -	- -	- -	- -	- -
ABSORP	- -	- -	25.34 (6.45) 3.93	11.19 (3.82) 2.93	- -	- -

THETA-DELTA

	SOCPO	SOCCLOS	TRAD	ABSORP
SOCPO	75.61 (8.58) 8.81			
SOCCLOS	- -	68.29 (7.15) 9.55		
TRAD	- -	- -	61.62 (14.22) 4.33	
ABSORP	30.05 (4.51) 6.66	- -	- -	96.00 (6.53) 14.71

Squared Multiple Correlations for X - Variables

REA	INV	ART	SOC	ENT	CON
0.46	0.44	0.66	0.31	0.47	0.43

Squared Multiple Correlations for X - Variables

SOCPO	SOCCLOS	TRAD	ABSORP
0.33	0.31	0.25	0.03

Goodness of Fit Statistics

Degrees of Freedom = 28
Minimum Fit Function Chi-Square = 158.81 (P = 0.0)
Normal Theory Weighted Least Squares Chi-Square = 156.41 (P = 0.0)
Estimated Non-centrality Parameter (NCP) = 128.41
90 Percent Confidence Interval for NCP = (92.85 ; 171.49)

Minimum Fit Function Value = 0.35
Population Discrepancy Function Value (F0) = 0.28
90 Percent Confidence Interval for F0 = (0.20 ; 0.38)
Root Mean Square Error of Approximation (RMSEA) = 0.10
90 Percent Confidence Interval for RMSEA = (0.085 ; 0.12)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.00

Expected Cross-Validation Index (ECVI) = 0.47
90 Percent Confidence Interval for ECVI = (0.39 ; 0.56)
ECVI for Saturated Model = 0.24
ECVI for Independence Model = 1.79

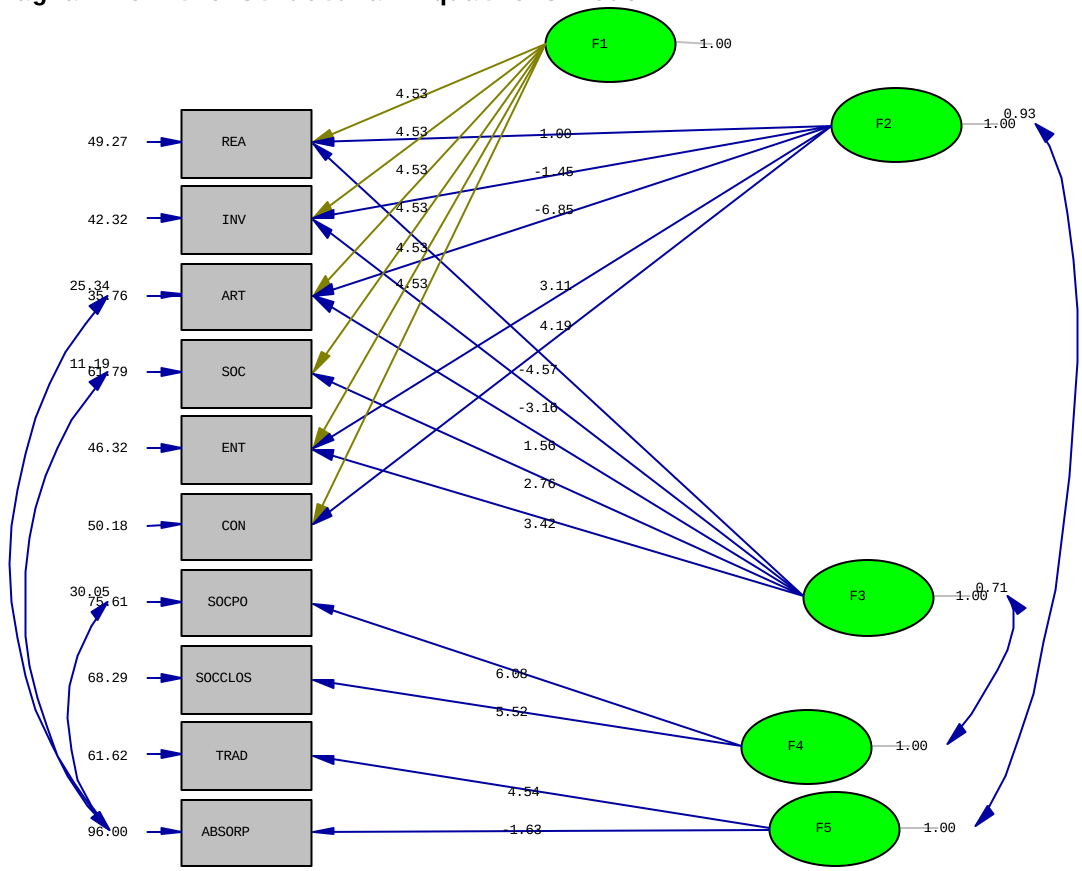
Chi-Square for Independence Model with 45 Degrees of Freedom = 798.27
Independence AIC = 818.27
Model AIC = 216.41
Saturated AIC = 110.00
Independence CAIC = 869.52
Model CAIC = 370.15
Saturated CAIC = 391.86

Normed Fit Index (NFI) = 0.80
Non-Normed Fit Index (NNFI) = 0.72
Parsimony Normed Fit Index (PNFI) = 0.50
Comparative Fit Index (CFI) = 0.83
Incremental Fit Index (IFI) = 0.83
Relative Fit Index (RFI) = 0.68

Critical N (CN) = 139.62

Root Mean Square Residual (RMR) = 6.89
Standardized RMR = 0.074
Goodness of Fit Index (GFI) = 0.94
Adjusted Goodness of Fit Index (AGFI) = 0.87
Parsimony Goodness of Fit Index (PGFI) = 0.48

LISREL's Path Diagram for the Structural Equations Model



Chi-Square=156.41, df=28, P-value=0.00000, RMSEA=0.100