Overview
Students extend their fraction concepts by reconstructing the unit when given the fraction part.

Materials
- Fraction Circles for students and teacher
- Student Page A

Teaching Actions
1. Lead a discussion around the concept of unit. Possible questions include:
   1) To show 1/3, what possible units could I use if I used fraction circles?
   2) If I used chips, what units could I use?
   3) If I used paper folding, how would I show 1/3? [use paper as a unit; partition into equal-sized parts; highlight a certain number of parts]

2. Explain that so far we have done a lot of problems in which we started with a unit and divided it into equal sized parts. Now we will reverse the process. I will give you one or more of equal-sized parts and you have to find the unit.

3. Model the idea of reconstruction the unit. Show 1 pink piece and say that this is 1 of 3 equal parts – it is 1/3 of some amount, some unit.

4. Show $\frac{1}{3}$ and ask: since this is 1 of 3 equal sized parts, how many more parts do I need to build a whole unit? What size parts do I need? All must be pink.

Comments
1. The activities in this lesson and the next, reinforce the idea that, for example, 2 halves equal 1 whole, 3 thirds equal 1 whole, and so on. It also reinforces the notion that non-unit fractions are iterations of unit fractions ($\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$).

2. Time spent on these activities continues to help children solidify mental images for fraction symbols.

3. Solution of these problems will center around the unit fraction and constructing the unit from the unit fraction.
Teaching Actions

5. Place and count

\[ \text{P} \quad \text{P} \quad \text{P} \]

1 part, 2 parts, 3 parts

so the whole unit is 3 pinks or 1 yellow.

We know that 1 pink is 1/3 of 1 yellow. We found the unit starting with 1/3 of it.

6. Ask students to take out 1 blue piece. State that this blue is 1/4 of some whole unit.

7. Ask: will the unit be bigger or smaller? How many fourths make a whole unit? Use your circles to find the unit.

8. Repeat for these pieces and values:

\[
\begin{align*}
\text{gray} &= 1/4 \quad \text{[unit is yellow]} \\
\text{red} &= 1/3 \quad \text{[unit is blue]} \\
\text{pink} &= 1/2 \quad \text{[unit is brown]} \\
\text{red} &= 1/4 \quad \text{[unit is brown]} \\
\text{gray} &= 1/2 \quad \text{[unit is blue]} \\
\end{align*}
\]

9. Present this example and explain that it is tricky:

1 gray = 1/3. Find the unit.

Ask the student to explain how to construct the unit. Then ask how we can describe the unit. Is there 1 piece to cover this amount? Since there isn’t, students can name the unit as 3 grays. You can trace the 3 grays and say this amount is the unit.

\[ \text{Gy Gy Gy} \]

10. Repeat for this example:

1 blue = 1/3. Unit = ?

Comments

4. Have students do these independently at their desks and then have them verbalize the process in a large group.

5. In each of the examples so far, the answer could be expressed as a single piece.

Ex: 1 gray = 1/4, so the unit equals 4 grays or 1 yellow.

This won’t always be the case. If 1 blue = 1/3, then the unit equals 3 blues.

\[ \frac{1}{3} \text{ unit} \]

No single piece covers 3 blues. Some children may express the unit as 3 blues; others as 1 yellow and 1 blue. Others will think their reasoning is wrong because there isn’t one piece to cover 3 blues.
Teaching Actions

11. End the class with this problem:

I know that this blue piece is half of something. How can I find the value of these pieces?

   1 red    2 pinks
   1 gray   2 yellows

12. Accept suggestions. Then model a solution by reconstructing the unit (using their new skill). Show \[ \text{\text{blue piece}} \]. Explain that if \[ \text{\text{blue piece}} \] is 1/2, then \[ \text{\text{red piece}} \] and \[ \text{\text{yellow piece}} \] make the unit. The unit is 1 yellow:

   \[ \text{\text{unit}} \]

Show 6 reds covering 1 yellow so 1 red is 1/6. Now ask students to do the other 3.
[1 gray = 1/4; 2 pinks = 2/3; 2 yellows = 2]

13. Repeat for this problem: blue = 1/3. Find the value of these pieces.

   1 red    1 yellow
   1 gray    9 reds

[1 red = 1/9; 1 yellow = 2/3; 1 gray = 1/6; 9 reds = 1]


Comments

6. To name the other pieces, student should find the unit and then compare the other pieces to that unit. Don't try to solve this abstractly:

   blue = 1/2
   red is 1/3 of blue, so 1/2 of 1/3 = 1/6.

7. This problem is challenging. Step back and let students problem solve. Ask them to explain their strategies and reasoning.
Problem Solving and Fraction Circles

I. Find the unit given the following information. Explain how you solved the problem. [You may want to draw pictures]

   a) The red piece is 1/4 of some amount. Find that amount. _______

   b) The gray piece is 1/6 of some amount. Find that amount. _______

   c) The green piece is 1/5 of some amount. Find that amount. _______

II. If the pink piece is 1/4 what value do these have? Explain your reasoning.

   a) 1 brown  
   b) 1 red  
   c) 1 white

Challenge: If the yellow piece is 2/3 what value does one gray piece have?