Rational Number Project
Fraction Lessons for the Middle Grades

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Richard Lesh

LEVEL 1
ABOUT THE AUTHORS

Kathleen Cramer is a professor in the College of Education and Graduate Studies at the University of Wisconsin at River Falls. She teaches graduate and undergraduate mathematics techniques classes for students majoring in elementary education. She has taught mathematics in elementary school and in junior high school. Kathleen has a Ph.D. from the University of Minnesota in mathematics education. She has published articles and book chapters dealing with the teaching and learning of fractions and proportional reasoning. She has done numerous workshops for teachers dealing with fraction instruction.

Professor Cramer has been involved with the Rational Number Project (RNP) since 1980. She participated in the initial teaching experiments with fourth and fifth graders. She has taken the primary responsibility for revising the lessons developed from the research to form the two sets of RNP Fraction Lessons for the Middle Grades.

Merlyn Behr was for over 25 years a professor of mathematics education at Northern Illinois University in DeKalb, Illinois. He was also a faculty member at Florida State University where he received his Ph.D., and at Louisiana State University at Baton Rouge. Merlyn’s primary interest was in children’s learning of elementary- and middle-grades mathematical concepts. He contributed a great deal to our understanding of children’s cognitive processes in these areas. He was very active in the research community and served on the editorial board of the Journal for Research in Mathematics Education (JRME) and as chair of the North American chapter of the research group of the Psychology of Mathematics Education. As a co-founder of the RNP, Merlyn was instrumental in charting its course and providing much valued intellectual leadership in many aspects of RNP activity.

Merlyn died in February 1995. His wit and professional contributions are sorely missed.

Thomas Post, former high school mathematics teacher in New York State, joined the faculty of the College of Education at the University of Minnesota in 1967 after receiving his Ed.D. from Indiana University. Professor Post’s interest is closely allied with other RNP members as he is especially interested in children’s and teachers’ perceptions of middle-school mathematics. He also has an interest in interdisciplinary approaches to curriculum. He was a co-founder of the RNP and has been active in the mathematics education research community. Along with Kathleen Cramer, Merlyn Behr and Richard Lesh, he has been one of the co-authors of some 70 papers, book chapters and technical reports produced by the RNP since its inception in 1979. Tom has also served on the editorial board of the JRME and has been chair of the North American chapter of the research group Psychology of Mathematics Education.

Richard Lesh, former professor and dean at Northwestern University, received his Ph.D. from Indiana University. He spent 5 years overseeing computer software development in mathematics and science at WICAT systems in Provo, Utah. He then served as senior research scientist at ETS in Princeton, NJ where he developed innovative strategies and materials for assessing outcomes in mathematics classrooms. Professor Lesh has served as project manager of the program unit - Research on Teaching and Learning - at the National Science foundation. Currently, he is a professor of mathematics at the University of Massachusetts-Dartmouth helping to further advance our thinking about authentic assessment, principles and strategies. Dick is one of the original co-founders of the RNP and has worked on each of its six grants since 1979. He currently leads the Massachusetts site of the RNP’s Middle-Grades Teacher Enhancement Project which is the latest of the projects funded by NSF.
Rational Number Project

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Level 1

by

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Merlyn Behr was a founding member of the Rational Number Project. He developed the first draft of the fraction materials that was used in the initial research with children. His insight into children’s learning, and his thoughtful analyses of students’ thinking are an integral part of this curriculum.

These Lessons are dedicated to Merlyn Behr. After a long and illustrious career in mathematics education, he passed away in February, 1995.
The authors would like to acknowledge the important contributions made by the fourth- and fifth-grade teachers in the Rosemount-Apple Valley, MN school district who participated in the pilot study of these materials. Their suggestions for improving the lesson have been incorporated into the materials. The quality of these lessons is due in part the input these teachers provided.
# Contents

**Preface**  

I. **Teacher’s Guide**  

- Introduction  
- The RNP Curriculum  
- Theoretical Framework  
- Lesson Format  
- Manipulative Materials  
- Pilot Testing  
- Assessment  
- Special Notes on Children’s Thinking  
- Final Comments  

II. **The RNP Lessons: Level 1**  

- Detailed Scope and Sequence  
- Lesson 1: Exploring with Fraction Circles  
- Lesson 2: Modeling with Fraction Circles  
- Lesson 3: Modeling with Fraction Circles  
- Lesson 4: Comparing Paper Folding and Circles  
- Lesson 5: Connecting Models with Symbols  
- Lesson 6: Comparing Unit Fractions  
- Lesson 7: Paper Folding and Comparing Fractions  

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>vii</td>
</tr>
<tr>
<td>I. Teacher’s Guide</td>
<td>1</td>
</tr>
<tr>
<td>• Introduction</td>
<td>2</td>
</tr>
<tr>
<td>• The RNP Curriculum</td>
<td>4</td>
</tr>
<tr>
<td>• Theoretical Framework</td>
<td>5</td>
</tr>
<tr>
<td>• Lesson Format</td>
<td>6</td>
</tr>
<tr>
<td>• Manipulative Materials</td>
<td>6</td>
</tr>
<tr>
<td>• Pilot Testing</td>
<td>7</td>
</tr>
<tr>
<td>• Assessment</td>
<td>7</td>
</tr>
<tr>
<td>• Special Notes on Children’s Thinking</td>
<td>8</td>
</tr>
<tr>
<td>• Final Comments</td>
<td>8</td>
</tr>
<tr>
<td>II. The RNP Lessons: Level 1</td>
<td>11</td>
</tr>
<tr>
<td>• Detailed Scope and Sequence</td>
<td>11</td>
</tr>
<tr>
<td>• Lesson 1: Exploring with Fraction Circles</td>
<td></td>
</tr>
<tr>
<td>• Lesson 2: Modeling with Fraction Circles</td>
<td></td>
</tr>
<tr>
<td>• Lesson 3: Modeling with Fraction Circles</td>
<td></td>
</tr>
<tr>
<td>• Lesson 4: Comparing Paper Folding and Circles</td>
<td></td>
</tr>
<tr>
<td>• Lesson 5: Connecting Models with Symbols</td>
<td></td>
</tr>
<tr>
<td>• Lesson 6: Comparing Unit Fractions</td>
<td></td>
</tr>
<tr>
<td>• Lesson 7: Paper Folding and Comparing Fractions</td>
<td></td>
</tr>
</tbody>
</table>
• Lesson 8: Fraction Circles and Equivalence
• Lesson 9: Fraction Circles, Paper Folding and Equivalence
• Lesson 10: Paper Folding and Equivalence
• Lesson 11: Comparing Fractions to One-half
• Lesson 12: Introducing the Chip Model
• Lesson 13: Using the Chip Model
• Lesson 14: Using the Chip Model
• Lesson 15: Chips and Equivalence
• Lesson 16: Fraction Circles and Reconstructing the Unit
• Lesson 17: Fraction Circles and Fractions Greater than One
• Lesson 18: Fraction Circles and Names for One-half
• Lesson 19: Estimation and Fraction Addition
• Lesson 20: Fraction Circles and Fraction Addition
• Lesson 21: Estimation and Fraction Subtraction
• Lesson 22: Fraction Circles and Fraction Subtraction
• Lesson 23: Fraction Number Sense, Addition and Subtraction

III. Appendix – Fraction Circles and Assessment Ideas

• Pictures of the Fractions
• Parent and Child Activity
• Fraction Circle Masters
• Written Test
• Student Interview 1
• Student Interview 2
• Student Interview 3
• Student Interview 4
Preface

The Rational Number Project (RNP) is a cooperative research and development project funded by the National Science Foundation. Project personnel have been investigating children’s learning of fractions, ratios, decimals and proportionality since 1979. This book of fraction lessons is the product of several years of working with children in classrooms as we tried to understand how to organize instruction so students develop a deep, conceptual understanding of fractions.

- **These lessons** provide teachers with an alternative to the textbook scope and sequence for fraction instruction and are appropriate for students in grades 4 – 8.

- **These lessons** will help students develop number sense for fractions because they invest time in the development of concepts, order and equivalence ideas.

- **These lessons** provide students with daily “hands-on” experiences. Fraction circles, chips and paper folding are the manipulative models used in the Level 1 lessons.

- **These lessons** provide teachers with daily activities that involve children in large group and small group settings. All the lessons involve students using manipulative materials. Our work with children has
shown that students need extended periods of time with manipulatives to
develop meaning for these numbers.

• **These lessons** offer teachers insight into student thinking as captured from
  the RNP research with children. The “Notes to the Teacher” section shares
  examples of students’ misunderstandings, provides anecdotes of student
  thinking, and contains information on using manipulative materials.

• **These lessons** will help teachers and students attain the goals set for
  fractions by the National Council of Teachers of Mathematics in their
  *Curriculum and Evaluation Standards*.

  – Students will represent fractions using multiple concrete models
  – Students will apply fraction ideas to real-world situations.
  – Students will have opportunities to reflect on their thinking about
    fractions using their informal language as they try to make sense of the
    formal symbolic system.
  – Students will have opportunities to make connections among different
    ways to represent fractions: real world, concrete models, pictures and
    symbols.

• **These lessons** delay development of operations with fractions until
  students have developed meaning for fractions. The lessons develop
  understanding of the operations with fractions by using story problems and
  fraction circles. Estimation is emphasized throughout.

• **These lessons** have been used with over 1600 students. Teachers’
  response to them has been uniformly enthusiastic. Results of pilot testing
  show significant differences in students’ performance between students
who used these lessons as compared to students who used the textbook for fraction instruction. Students using the RNP lessons outperformed students using the textbook. Marked differences in thinking were noted in the pilot study. Students using the RNP lessons thought about fractions in a conceptual manner, while students using the textbook thought about fractions in a procedural manner.

- These lessons are for teachers who know from their own experience with children that there must be a better way to teach fractions!!! Students who use these lessons will develop concepts and will be able to operate on fractions meaningfully.
Introduction

These lessons reflect research on children’s fraction learning conducted by the National Science Foundation-sponsored Rational Number Project (RNP). Since 1978, Merlyn Behr (Northern Illinois University), Kathleen Cramer (University of Wisconsin – River Falls), Thomas Post (University of Minnesota) and Richard Lesh (University of Massachusetts – Dartmouth) have studied how elementary-aged children learn to extend their understanding of numbers to include fraction ideas.

The RNP staff conducted teaching experiments with fourth- and fifth-grade children. In a teaching experiment, researchers entered the classroom as teachers and worked with a curriculum created from a well thought-out theoretical framework. During the teaching and learning process, researchers study children’s learning as they progress through the lessons. This is done through classroom observations, student interviews and written assessments.

The curriculum created for the teaching experiments that evolved into these lessons reflected the following beliefs: (a) Children learn best through active involvement with multiple concrete models, (b) physical aids are just one component in the acquisition of concepts: verbal, pictorial, symbolic and real-world representations also are important, (c) children should have opportunities to talk together and with their teacher about mathematical ideas, and (d) curriculum must focus on the development of conceptual knowledge prior to formal work with symbols and algorithms.

The teaching experiments were conducted in two parts. The first phase of the project was ten weeks long with small groups of fourth and fifth graders at two different sites (Minnesota and Illinois). The second phase was conducted with a classroom of 30 students. A class of fourth graders participated from January of their fourth grade, through January of their fifth grade. Instruction was four days per week and covered the following topics: part-whole model for fractions, ratio and quotient models for fractions, decimals, and number lines.

Throughout both teaching experiments a subset of children were interviewed every two weeks. The interviews provided information on children’s thinking about fraction ideas. We were interested in what role manipulative materials played in their thinking as well as
what understandings and misunderstandings children have with fractions.

Our work with children helped explain why children have so much difficulty with fractions. It also informed us as to the type of experiences children need to develop a deep, conceptual understanding of fractions. Consider a few of the insights garnered from these teaching experiments:

1. Children have difficulty internalizing that the symbol for a fraction represents a single entity. When asked if 2/3 was one or two numbers, many children would say that the symbol represented two numbers. When students consider 2/3 as two numbers then it makes sense to treat them like whole numbers. For example, when students add two fractions by adding the numerators and then denominators, they are interpreting the symbols as four numbers, not two. Many errors with fractions can be traced to students’ lack of mental images for the quantity the symbol represents.

2. Ordering fractions is more complex than ordering whole numbers. Comparing 1/4 and 1/6 conflicts with children’s whole number ideas. Six is greater than four, but 1/4 is greater than 1/6. With fractions, more can mean less. The more equal parts you partition a unit into, the smaller each part becomes. In contrast, 3/5 is greater than 2/5 because 3 of the same-size parts are greater than 2 of the same-size parts. In this case, more implies more. Being able to order plays an important part in estimating fraction addition and subtraction. Ideally when a student adds, for example, 1/4 + 1/3, she should be able to reason from her mental images of the symbols that (a) the answer is greater than 1/2, but less than one and (b) 2/7 is an unreasonable answer because it is less than 1/2.

3. Understanding fraction equivalence is not as simple as it may seem. Some children have difficulty noting equivalence from pictures. Imagine a circle partitioned into fourths with one of those fourths partitioned into three equal parts. Some children we worked with were unable to agree that 3/12 equals 1/4 even though they agreed that physically the two sections were the same size. Children said that once the lines were drawn in, you could not remove them. [Therefore 3/12 ≠ 1/4]. In reality, that is just what must be done to understand fraction equivalence from a picture.

4. Difficulties children have with fraction addition and subtraction come from asking them to operate on fractions before they have a strong conceptual understanding for these new numbers. They have difficulty understanding why common denominators are needed so they revert to whole number thinking and add numerators and denominators.
The RNP Curriculum

The RNP curriculum offers an alternative scope and sequence to one suggested in fourth- or fifth-grade textbooks. The RNP philosophy is that extended periods of time invested with manipulative materials developing concepts, order, and equivalence ideas are needed before students can operate on fractions in a meaningful way. These goals are consistent with the instructional goals set forth in the National Council of Teachers of Mathematics in their *Curriculum and Evaluation Standards*. According to the *Standards*, important goals for fraction instruction include the development of number sense, the use of multiple concrete models to represent fractions, the use of context, and the ability to make connections among multiple representations. The RNP curriculum provides teachers with carefully researched lessons to meet these goals.

The RNP Level 1 materials develop the following topics: (a) part-whole model for fractions, (b) concept of unit, (c) concepts of order and equivalence and (d) addition and subtraction of fractions at the concrete level. The concrete models used are fraction circles, paper folding and chips. It de-emphasizes written procedures for ordering fractions, finding fraction equivalences, and symbolic procedures for operating on fractions. Instead it emphasizes the development of a quantitative sense of fraction.

To think quantitatively about fractions, students should know something about the relative size of fractions and be able to estimate a reasonable answer when fractions are operated on. Below, find an example of a fourth-grad student’s thought process for estimating a fraction addition problem. This student used the RNP curriculum; her thinking reflects a quantitative sense of fraction. Students using the RNP lessons develop this type of understanding for fractions.

**Problem:** John calculated the problem as follows: \( \frac{2}{3} + \frac{1}{4} = \frac{3}{7} \). Do you agree?

*Student:* I don’t agree. He did it weird. You don’t add the top numbers and bottom numbers.

*Teacher:* What would be an estimate?

*Student:* It would be...greater than \( \frac{1}{2} \) because \( \frac{2}{3} \) is greater than \( \frac{1}{2} \).

*Teacher:* Would it be greater or less than one?

*Student:* Less than one. You’d need \( \frac{1}{3} \) and \( \frac{1}{4} \) is less than \( \frac{1}{3} \).

*Teacher:* What about \( \frac{3}{7} \)?
**Student:** 3/7 is less than 1/2.

**Teacher:** How do you know?

**Student:** Because 3/7 isn’t 1/2. I just know.

**Theoretical Framework**

Children using these lessons will be using several manipulative models and will consider how these models are alike and different. They will work in small groups talking about fraction ideas as well as interacting with the teacher in large group settings. They will be drawing pictures to record their actions with fraction models. They will be solving story problems using manipulatives to model actions in the stories.

This model for teaching and learning reflects the theoretical framework suggested by Jean Piaget, Jerome Bruner, and Zoltan Dienes. Richard Lesh, a RNP director, suggested an instructional model that clearly shows how to organize instruction so children are actively involved in their learning. Consider this picture.

![Lesh Translation Model](image)

Lesh suggests that mathematical ideas can be represented in the five ways shown here. Children learn by having opportunities to explore ideas in these different ways and by making connections between different representations. This model guided the development of the RNP curriculum.

**Lesson Format**

The lessons reflect a classroom organization that values the important role teachers play in student learning as well as the need for students to work cooperatively, talking about ideas, and using manipulative models to represent rational number concepts. Each lesson
includes an overview of the mathematical idea developed. Materials needed by teachers and students are noted. The lesson begins with the development of a mathematical idea or an extension of an idea previously introduced. Teachers are provided with problems and questions to help generate the initial discussion and target the exploration. Nearly all lessons involve manipulative materials. Small group work is included in most lessons where students in small groups continue the exploration of ideas introduced in the large group. The amount of time needed for each lesson will vary from classroom to classroom. A single lesson does not necessarily reflect one day's work, thought teachers often will find that one day is sufficient to cover the material.

An important part of each lesson is the “Notes to the Teacher” section. Here insights into student thinking captured from the initial RNP teaching experiments are communicated to teachers. These notes clarify a wide variety of issues, such as why mastery at the symbolic level is not the primary objective for many of the earlier lessons. The notes also share examples of students’ misunderstandings for teacher’s reflection and anecdotes of student thinking from earlier RNP projects. These notes to the teachers also clarify methods for using manipulative materials to model fraction ideas.

**Manipulative Materials**

Fraction circles, two-sided colored counters and paper folding are the manipulative models used. Our research has shown that the fraction circles are the most important manipulative model for developing mental images of fraction symbols.

**Fraction Circles**

The master for the fraction circles are in the appendix with a page showing the different partitions and colors used for the fraction circles. The circles should be duplicated on index using colors noted on each master. Teachers who have used the fraction circles have relied on their students, parents or teacher-aids to cut out the circles and to organize them in two-pocket folders. If you choose to send home the fraction circles to be cut out with the parent’s help, you will find in the appendix a parent and child activity sheet for them to do together once the circles are cut out.

Overheads of the fraction circles are a useful teaching tool. To make them, use permanent color markers on transparencies of the fraction circles.

**Counters**

Two-sided colored counters are available from most publishers of mathematics.
Manipulative materials. A less expensive way is to purchase from a tile store, one square inch tiles (white on one side, tan on other). These cost less than 1.5 cents per tile. Thirty per student should be enough.

**Paper Folding**

Use 8.5” by 11” sheets of paper cut into strips 1” by 8.5”. Have lots on hand for students to use for lessons 7 and 10.

**Pilot Testing**

The RNP Level 1 lessons have been piloted in the Rosemount-Apple Valley, MN school district. Some 66 teachers participated in this study. Thirty-three fourth- and fifth-grade teachers used the RNP lessons while thirty-three other fourth- and fifth-grade teachers used the textbook. RNP students outperformed textbook students in all areas assessed. Particular differences were noted in students’ thinking. RNP students thought about fractions in a conceptual manner, while textbook students generally thought about fractions procedurally. RNP students were much more able to verbalize their thinking about fractions than textbook students.

Teachers from this pilot who used the RNP curriculum gave us detailed feedback on lessons. The lessons have been revised to reflect the input from these teachers.

**Assessment**

A written test is provided for use at the end of the lessons. Items reflect information directly taught in the lessons as well as extensions. Another important source of information on children’s thinking can come from talking to students. Interviews are included to be used at certain intervals in the lessons. You may want to select three students representing a range of understanding and interview them as they progress through the lessons. The interview items come from the interviews we used with children and reflect questions we feel gave us the most information on children’s thinking.

Interview questions assess understanding on content addressed in the lessons as well as questions on content to be addressed in upcoming lessons. It is interesting to see how students transfer information to new situations. Of particular interest is interview #3 which, if used, should be given before lesson 19. This interview asks students to add and subtract fractions before the lessons on addition and subtraction. We found it interesting to note how many students were able to construct their own reasonable process for adding and
subtracting fractions. A written test and four interviews can be found in the appendix.

**Special Notes on Students’ Thinking**

From our interviews with children we noted that they constructed what we now refer to as informal strategies for ordering fractions. These strategies reflect students’ use of mental images of fractions to judge the fraction’s relative size. These informal strategies do not rely on procedures usually taught: least common denominators and cross-products. We have named the four strategies noted in students’ thinking as: same numerator, same denominator, transitive and residual strategies.

When comparing 2/3 and 2/6 (fractions with the same numerator) students can conclude that 2/3 is the larger fraction because thirds are larger than sixths and two of the larger pieces must be more than two of the smaller pieces. This strategy involves understanding that an inverse relationship exists between the number of parts a unit is partitioned into and the size of the parts.

The same denominator strategy refers to fractions like 3/8 and 2/8. In this case, the same denominator implies that one is comparing parts of the unit that are the same size. Three of the same-size parts are greater than two of the same-size parts.

The student strategy that has been termed the *transitive* strategy can be modeled by comparing 3/7 and 5/9. When making this comparison, a student can conclude that 3/7 is less than 5/9 because 3/7 is less than 1/2, while 5/9 is greater than 1/2. This is the transitive strategy because students use a single outside value to compare both fractions.

When comparing 3/4 and 5/6, a student can reflect that both fractions are one “piece” away from the whole unit. Since 1/6 is less than 1/4, 5/6 must be closer to the whole and is therefore the bigger fraction. This thinking strategy has been called a *residual* strategy because students focus on the part “leftover” in judging the relative size of the fractions.

These four strategies closely parallel students’ actions with manipulatives. They are in contrast to the paper and pencil procedures, which require changing both fractions to common denominators or calculating cross-products. RNP lessons developed only these student-constructed strategies. The order questions on the interviews will assess whether students construct these strategies.

**Final Comments**

You will find at the end of each lesson a form for you to record your adaptations for
each lesson. Any curriculum will need to be “personalized” by the teacher who uses it, so it best meets the needs of his/her students. This form will act as a reminder about changes you feel are important to make the next time you teach the lesson.

Level 2 lessons, which would be appropriate for middle grade students who have used the Level 1 lessons, are available from this publisher.
The RNP Lessons
Level 1
### RATIONAL NUMBER PROJECT
**Fraction Lessons: Level 1**
**Scope and Sequence**

<table>
<thead>
<tr>
<th>LESSON</th>
<th>MANIPULATIVE</th>
<th>TOPIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fraction Circles</td>
<td>Exploration with the circles.</td>
</tr>
<tr>
<td>2</td>
<td>Fraction Circles</td>
<td>Model and verbally name: 1-half, 1-third, 1-fourth.</td>
</tr>
<tr>
<td>3</td>
<td>Fraction Circles</td>
<td>Model and verbally name unit fractions with denominators greater than 4.</td>
</tr>
<tr>
<td>4</td>
<td>Paper Folding</td>
<td>Compare paper folding to fraction circles.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Model and name (verbally and with written words) unit and non-unit fractions.</td>
</tr>
<tr>
<td>5</td>
<td>Fraction Circles</td>
<td>Model fractions and record with symbols a/b.</td>
</tr>
<tr>
<td>6</td>
<td>Fraction Circles</td>
<td>Model the concept that the greater the number of parts a unit is divided into, the smaller each part is.</td>
</tr>
<tr>
<td>7</td>
<td>Paper Folding</td>
<td>Reinforce the concept that the greater the number of parts a unit is divided into, the smaller each part is.</td>
</tr>
<tr>
<td>8</td>
<td>Fraction Circles</td>
<td>Fraction Equivalence</td>
</tr>
<tr>
<td>9</td>
<td>Fraction Circles</td>
<td>Fraction Equivalence</td>
</tr>
<tr>
<td></td>
<td>Pictures</td>
<td></td>
</tr>
<tr>
<td>LESSON</td>
<td>MANIPULATIVE</td>
<td>TOPIC</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------</td>
<td>------------------------------------------------------------</td>
</tr>
<tr>
<td>10</td>
<td>Paper Folding</td>
<td>Fraction Equivalence.</td>
</tr>
<tr>
<td>11</td>
<td>Fraction Circles</td>
<td>Order fractions by comparing to 1-half.</td>
</tr>
<tr>
<td>12</td>
<td>Chips</td>
<td>Introduce new model for fractions less than one by comparing to a familiar model.</td>
</tr>
<tr>
<td>13</td>
<td>Chips</td>
<td>Model fractions using several units for the same fraction.</td>
</tr>
<tr>
<td>14</td>
<td>Chips</td>
<td>Model fractions using chips; determine fractions that can be shown given a set of chips.</td>
</tr>
<tr>
<td>15</td>
<td>Chips</td>
<td>Fraction Equivalence.</td>
</tr>
<tr>
<td>16</td>
<td>Fraction Circles</td>
<td>Reconstruct the unit given the fraction part.</td>
</tr>
<tr>
<td>17</td>
<td>Fraction Circles</td>
<td>Model fractions greater than one sing mixed and improper fraction notation.</td>
</tr>
<tr>
<td>18</td>
<td>Fraction Circles</td>
<td>Fraction equivalence for 1-half based on a number pattern.</td>
</tr>
<tr>
<td>19</td>
<td>Fraction Circles</td>
<td>Estimate sum of two fractions within story contexts.</td>
</tr>
<tr>
<td>LESSON</td>
<td>MANIPULATIVE</td>
<td>TOPIC</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>20</td>
<td>Fraction Circles</td>
<td>Find the sum of two fractions using fraction circles.</td>
</tr>
<tr>
<td>21</td>
<td>Fraction Circles</td>
<td>Estimate and solve concretely fraction subtraction using “take-away” and “difference” contexts.</td>
</tr>
<tr>
<td>22</td>
<td>Fraction Circles</td>
<td>Estimate and solve fraction subtraction using “difference” and “how many more” contexts.</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>Summary activities to tie together students’ number sense and addition and subtraction.</td>
</tr>
</tbody>
</table>
Overview
Lesson provides guided exploration with fractions circles. Students start to become familiar with colors and relationships like 3 browns cover 1 black and 1 brown is bigger than 1 red.

Materials
• Fraction Circles for students and teacher
• Student Page A
• Transparency 1

Teaching Actions
1. Start the Lesson by asking children to sort through their fraction circles to answer these questions:
   a) How many blues cover the black circle?
   b) Which is bigger, 1 brown or 1 gray?
   c) How many pinks cover 1 yellow?
   d) How many browns cover the black?
   e) Which is bigger, 1 brown or 2 reds?
   f) How many purples cover 1 yellow?
   g) How many dark blues are there? Light blues?
2. Explain to the students that they are to continue their exploration by using the circles to complete Student Page A.

Comments
1. Students need to play with the fraction circles before developing a formal language for describing relationships among the pieces.
2. There are two different blues: a set of 4 dark blue pieces; a set of 7 light blue pieces. In the lessons the color “blue” refers to the set of 4 dark blue pieces. “Light blue” will refer to the set of 7 blues.
3. Different ways to approach Student Page A:
   • Students do page individually and then compare with a partner.
   • Students do page with a partner.
   • Do a few problems together and then students finish on their own.
Lesson 1
Teacher Page B

Teaching Actions

3. End the lesson by working through Transparency 1. The figure on the left represents the circle art you want to cover. To the right are the circle parts. Students are to determine which combination of parts will cover the shape on the left. All pieces selected do not have to be the same color.

Example:

![Image of fraction circles]

1 Yellow + 2 Blues cover 1 Black

4. Encourage students to guess first and then use their fraction circles to find the exact combination.

Comments

4. If some students finish Student Page A ahead of others, ask them to create their own problems and record them on the back of the page or put them on the board for others to solve.

5. You may want to duplicate Transparency 1 for students.

6. To encourage students to guess you might want to emphasize making “hypotheses”. Write the word hypothesis on the board. Record students’ guesses, test them out and reach a group consensus.
Exploring with the Fraction Circles

1. _____________ browns equal 1 whole circle.

2. 1 whole circle equals _______________ pinks.

3. _______________ reds equal 1 whole circle.

4. _______________ pinks equal 1 brown.

5. 1 brown equals ________________ reds.

6. 1 brown is (less than, equal to, greater than) 1 pink.

7. 1 red is (less than, equal to, greater than) 1 brown.

8. 1 yellow is (less than, equal to, greater than) 1 brown.

9. 1 yellow and 1 brown and 1 _____________ equals 1 whole circle.

10. 1 yellow equals 1 brown and 2 ________________.

11. 3 pinks and 1 ________________ equal 1 whole circle.

12. _________ grays and 1 blue and 1 yellow equals 1 whole circle.

13. 2 grays and __________ blue equals 1 yellow.

14. 1 pink equals _________ reds.

15. 4 ________________ equal 1 yellow.
Overview
Students explore relationships among circle pieces, modeling and orally naming fraction amounts for: 1-half, 1-third, and 1-fourth.

Materials
• Fraction Circles for students and teacher
• Student Page A, B

Teaching Actions
1. Ask students to take out a black circle. Model how to divide the black circle into 2 equal parts by showing that 2 yellow parts cover the whole circle.

2. Note that 1 black equals 2 yellows or 2 yellows equal 1 black. Ask: Are the 2 parts covering the whole equal?

3. Conclude by stating that when 2 equal parts equal one whole, each part (pick up 1 yellow) is called one-half. This yellow piece is one-half of the black circle.

4. Show one-half by placing 1 yellow on the black circle. “1 yellow covers half of the black circle.”

Comments
1. Flexibility of unit is stressed right from the beginning by having students find multiple representations for 1/2, 1/3, and 1/4.

The critical variable with fractions is that a unit is divided into equal parts. A single part can be given a fraction name. Its name depends on what it is being compared to.

Example:
2 blues equal 1 yellow so 1 blue is one-half of the yellow. [here yellow is the unit]

4 blues equal 1 black. 1 blue is one-fourth of the black circle. [here black is the unit]
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Continue by looking for other examples for one-half. Show one blue piece and ask: If one blue is my unit, how can we divide this piece into 2 equal parts? What color pieces will do this?</td>
<td>2. Students are naming fractions in the verbal mode only. In the next lesson students will record as: 1-fourth.</td>
</tr>
<tr>
<td><img src="Bl" alt="Bl" /> → <img src="Gy" alt="Gy" /></td>
<td></td>
</tr>
<tr>
<td>5. Use these questions: Are the 2 parts equal? 1 gray is 1 of 2 equal parts; what fraction of the blue piece is 1 gray? [1-half]</td>
<td>3. You may want to show how other units can be partitioned into halves:</td>
</tr>
<tr>
<td><img src="Gy" alt="Gy" /></td>
<td>1 pink → 2 reds</td>
</tr>
<tr>
<td>6. Show 1 yellow and ask students to consider the yellow as the unit, divide it into 2 equal parts and orally name each part.</td>
<td>1 brown → 2 pinks</td>
</tr>
<tr>
<td>7. Model, using the black circle as the unit, representations for thirds.</td>
<td>1 orange → 2 purples</td>
</tr>
<tr>
<td><img src="Black" alt="Black" /> <img src="Br" alt="Br" /> <img src="Br" alt="Br" /> <img src="Br" alt="Br" /></td>
<td></td>
</tr>
<tr>
<td>8. 3 browns cover 1 black; 1 brown is 1 of 3 equal parts; 1 brown is one-third of the black. Show as:</td>
<td></td>
</tr>
<tr>
<td><img src="Br" alt="Br" /></td>
<td></td>
</tr>
<tr>
<td>9. Find other examples for 1-third using 1 yellow and 1 brown and then 1 blue as the unit.</td>
<td></td>
</tr>
<tr>
<td>10. Model fourths using 1 black, 1 yellow, and 1 brown as units.</td>
<td></td>
</tr>
</tbody>
</table>
**Teaching Actions**

11. End the development part of the lesson with a non-example. Show how 2 blues and 1 yellow cover the black circle. Pick up 1 blue and say that this piece is 1 of 3 parts of the circle so it is one-third of the circle. Ask: Is this true? If I wanted to know what part of the black circle 1 blue is, what must I do?

   [Repeat showing 2 browns and 2 pinks covering the black circle. 1 pink does not equal 1-fourth].

12. Student pages A & B present problems similar to ones presented in large group as well as problems within realistic contexts. Assign to students in pairs, as they are to answer the questions orally.

**Comments**

4. Another question to assess understanding of the “big idea” in this lesson is to present the following scenario:

   Lianna said that 1 red piece is one-third; Rodrigo said 1 red is one-fourth. Who is correct?

   Note that 1 red is one-third of the blue; 1 red is also one-fourth of the brown. Both Lianna and Rodrigo are correct once you know what unit they are comparing the red to.

5. Another way to assess the lesson’s big idea is to put one of each of these colors: yellow, blue, pink, and red on the overhead and ask: “You have called all of these 1-half, yet they are different sizes. How is that possible?”
Directions:
The class will work together in groups or in pairs on these problems. Answers are to be given orally or by drawing a picture. On some of the problems children may want to use the fraction circles to help solve the problem.

1. The yellow piece is the unit.
   How many blues cover the yellow piece? ______________
   1 blue is ______________ of the yellow.
   (say the word)

2. The blue piece is the unit.
   How many reds cover the blue piece? ______________
   1 red is ______________ of the blue.
   (say the word)

3. The brown piece is the unit.
   How many reds cover the brown piece? ______________
   1 red is ______________ of the brown.
   (say the word)

4. What color is 1-half of the blue? ______________

5. What color is 1-third of the yellow? ______________

6. Draw a picture of a pizza. Show on your drawing the pizza cut into 2 fair shares.

   Each fair share is _____________ of the whole pizza.
   (say the word)
7. Here is a picture of a pizza with one piece removed.  

The piece is ____________ of the whole pizza.  
(say the word)

8. Here is a picture of a candy bar which someone has started to cut into pieces.  

The small piece is ____________ of the whole candy bar.  
(say the word)

Draw lines to finish cutting the candy bar into equal parts.

9. Mary’s patio is a whole circle. Draw a picture of Mary’s patio. Show on your drawing that the patio is in 3 equal-size parts. Each part is ____________ of Mary’s patio.  
(say the word)

10. John has a patio that looks like this:

11. Draw on John’s patio to show it divided into 3 equal-size parts. Each part is ____________ of John’s patio.  
(say the word)

Mary said “John’s patio is really one-half (not a whole).” What would you say to Mary?
Overview
Students model and name (orally and in written words) unit fractions with denominators greater than 4.

Materials
- Fraction Circles for students and teacher
- Student Page A

Teaching Actions
1. Show one yellow piece. Ask students to divide it into six equal parts.

2. Explain that since 6 reds cover 1 yellow, 1 red is one-sixth of the yellow.

3. Ask students to divide a black circle into 6 equal parts. What fraction piece is one-sixth of the black?

4. Make this chart to show the relationship between the number of equal parts a unit is divided into and the word name for that number of divisions.

<table>
<thead>
<tr>
<th>Number of Equal Parts Unit is divided into</th>
<th>Word Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>half</td>
</tr>
<tr>
<td>3</td>
<td>third</td>
</tr>
<tr>
<td>4</td>
<td>fourth</td>
</tr>
<tr>
<td>5</td>
<td>fifth</td>
</tr>
<tr>
<td>6</td>
<td>sixth</td>
</tr>
<tr>
<td>7</td>
<td>sevenths</td>
</tr>
</tbody>
</table>

[Continue to include 8, 9, 10, 12, 15]

Comments
1. Make a large classroom chart for students to use as reference for the rest of the fraction unit. You might include a third column showing a picture of a unit (now always a whole circle) divided into the appropriate number of equal parts.

2. You may want to have students make their own personal chart.
### Teaching Actions

For each item in the chart show at least 2 physical models. For example:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Halves" /></td>
<td><img src="image" alt="Thirds" /></td>
<td></td>
</tr>
</tbody>
</table>

Students should model each example with their fraction circles.

5. Students should help you find these different representations.
   - You may suggest the unit and ask them to divide it into a certain number of equal parts.
   - You might ask students to suggest the unit. For example, say “the next value in the chart is to model sixths. What unit can we use?”

6. Once the chart is completed, work through these problems:
   - Using the black circle as the unit, ask students to find the color that divides the unit into 4 equal parts. Hold up 1 of 4 parts, call it “one-fourth”, and record the written name as $\frac{1}{4}$.
   - Using the yellow circle as the unit, ask students to find the color that divides the unit into 4 equal parts. Hold up all 4 parts; call it “one-fourth”; record $\frac{1}{4}$.

Ask: “How are the two models for $\frac{1}{4}$ alike? Different?”

7. Repeat for sixths and twelfths using two different units.

### Comments

3. Students initially record fractions in words like: $\frac{1}{4}$-fourth; $\frac{1}{6}$-sixth. Research suggests that students make fewer reversals with the symbols (for example, writing $\frac{3}{2}$ for $\frac{2}{3}$) when they first write fractions in words.

4. You may want to do more examples.
8. To prepare students for Student Page A ask the following questions. Have students record answers using word names.

- The blue piece is the unit. What fraction name can you give 1 gray piece? 1 red piece?
- The brown piece is the unit. What fraction name can you give 1 pink? 1 white? 1 gray?

9. Assign Student Page A.

10. End the class with this game: Teacher says: “Two of the colors I am thinking of equal one yellow. What color is it? What fractional name can I give each piece?

5. You might consider assigning students Page A without this introduction. This will make the activity more of a problem solving activity.

6. Extra challenges:

- If the yellow piece is the unit, what value does the black circle have?
- If the blue piece is the unit, what value does the yellow piece have? The black one?

These questions may lead to a nice discussion. Students may question how to express the answer. If the yellow piece is the unit (or one) then the black circle is 2 units, 2 wholes or just 2.
Naming Fraction Amounts
Using Circles

Use fraction circles to find the names of the different fraction pieces.

I. The black circle is the unit. What fraction name can you give these pieces?
   
   1 yellow ___________
   1 blue ______________
   1 white ______________
   1 red ________________
   1 brown ___________________
   1 gray __________________
   1 green __________________
   1 pink ____________________

II. Now make 1 yellow unit. What fraction name can you give these pieces?
    
    1 blue __________
    1 pink ____________
    1 gray ____________
    1 red _____________
    1 gray __________________
    1 red __________________

III. Change the unit to 1 blue. What fraction name can you give these pieces?
    
    1 gray ______________
    1 red ______________

IV. Change the unit to 1 orange. What fraction name can you give these pieces?
    
    1 purple _____________
    1 green _______________
### Overview
Students use paper folding to model and name unit and non-unit fractions. Students compare the paper-folding model to fraction circles. Students record fractions in words: one-fourth, two-thirds.

### Materials
- Paper strips for folding for students
- Fraction Circles for teacher
- Student Pages A-L

### Teaching Actions
1. Prior to using paper strips to model fractions it is necessary to practice folding strips into 2, 3, 4, 6, 8, and 12 equal parts.
   
   Ask students to follow along with you as you model how to fold paper strips. Fold paper strip into two equal parts:

   ![Folded strip](image)

   2. Keep it folded. Now fold it again into two equal parts. Ask: how many equal parts do you think we have? Unfold:

   ![Unfolded strip](image)

   3. Ask students to verbalize how to fold paper strips to form four equal parts.

   4. Model folding into three equal parts. Form the letter “S” with a paper strip to get close to 3 equal parts. Press down on paper

### Comments
1. Most teachers who have used these lessons have needed 2 class periods for lesson 4.

2. Cut paper strips from 8.5” by 11” sheets of paper about 1 inch wide and 8.5” long.
### Teaching Actions

5. Model sixths. Fold paper strip into thirds and then fold into two equal parts. Have students do this and guess, before unfolding, the number of equal parts they expect.

6. Ask students if they could have gotten sixths by folding first in halves and then in thirds? Try it.

7. Ask students to think of strategies for folding 8ths and 12ths. Encourage trial and error strategy. Have them verbalize successful ways. For 12ths reinforce multiple ways.

8. Students can shade equal parts of paper strips to show fractions. Using fraction circles, show one-fourth using a black circle as the unit.

![Diagram](image)

Say: To show one-fourth of a black circle I divided it into four equal parts. Pick up one of the parts to show one-fourth.

### Comments

1. Students often will expect 5 equal parts (3+2). They are more apt to think additively than multiplicatively.

2. To get 12ths:

   - Halves ➔ halves ➔ thirds
   - Thirds ➔ halves ➔ halves
   - Halves ➔ thirds ➔ halves
Teaching Actions

9. Ask: How can you show me one-fourth on a paper strip? Have students fold into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth.

10. Discuss how the two displays for one-fourth are alike and different.


12. Look at two displays for one-third:

```
\[ \text{Br} \quad \text{Br} \quad \text{Br} \]
\[ \text{Br} \]
1-third

\[ \text{Br} \]
1-third
```

13. Shade in another third on the paper strip.

```
\[ \text{Br} \quad \text{Br} \quad \text{Br} \]
\[ \text{Br} \]
1-third
```

Ask: how many thirds are shown now? How can I show two-thirds with circles? (Pick pu two browns and say these are two-thirds of black.) State that 2-thirds is 1-third and 1-third more:

```
\[ \text{Br} \quad \text{Br} \quad \text{Br} \]
```

Comments

3. The similarity between the two displays is what’s important. A unit is divided into equal parts and one or more equal parts are highlighted in some way.

4. Non-unit fractions are introduced as sums of unit fractions: 2-fourths is 1-fourth and 1-fourth.
Teaching Actions

14. Now draw a picture of a square. Divide it into 4 equal parts and shade 3 of 4 parts. Ask students to fold paper to show the same fraction that you drew. Record fractions as 3-fourths: 1-fourth + 1-fourth + 1-fourth.

15. Return to fraction circles. Model problems as in lesson 3, this time with non-unit fractions.

Examples:

1.) The black circle = 1. What is the value of 1 blue; 3 blues- 1 brown; 2 browns; 3 reds.
2.) The yellow piece = 1. What is the value of 1 blue; 2 reds; 3 grays; 2 pinks.

16. There are several student pages in this lesson. Select the most appropriate ones for your students. Students may need some assistance to do some of the pages. See Comments for clarification.

Comments

5. Students now have seen two models for fractions. Practice pages that follow this lesson give students a chance to apply their new learning to pictures of units in different shapes.

6. Teacher Notes for Student Pages:
   **B:** Problems 6 and 7 provide some problem solving. Students reconstruct the unit given one part. For example if equals 1-half, then the whole must be two of those parts:

   If equals 1-fourth, then the whole must be four of those parts:

   **G:** Clarify with students that a picture may need to be modified to determine if 2-fourths are shaded in. For example:

   Is 2-fourths shaded?

   2-fourths can easily be seen once the picture is completed by drawing in the needed lines.

   **J:** This type of modification will be needed on some pictures to determine the fraction amount shaded.
1. Here is a picture of a candy bar.

Draw to show the candy bar divided into 5 equal-sized pieces.

2. Here is a picture of a pan of brownies.

The pan of brownies is cut into _____________ equal-sized parts.

Each piece is _____________ of the whole pan.

3. O-So-Good candy bars come in the shape of a square. After Janis ate one pieces of an O-So-Good candy bar, it looked like this:

4. The piece that Janis ate is ___________ of the whole candy bar.

5. Tom’s patio is a rectangle. Draw a picture of Tom’s patio. Show on your drawing that the patio is in 9 equal-sized parts.

Each part is _____________ of Tom’s patio.
5. Devan’s patio looks like this:

![Devan's patio drawing]

Draw on Devan’s patio to show it divided into 3 equal-sized parts.

Each part is __________ of Devan’s patio.

6. One-half of a coffee cake was left after a party was over. The half looked like this:

![Coffee cake half]

Draw a picture of the whole cake.

Explain to your classmates how you solved the problem.

7. Willis, Vang, Ellen and Marta shared part of a candy bar equally. Marta’s share looked like this:

![Candy bar share]

Draw a picture to show the whole candy bar.

Explain to your classmates how you solved the problem.
For each exercise, look at the figure and then answer the question and write the word name for each fraction.

1. ______ equal-sized parts.
   Each part is ____ _________ of the whole.

2. ______ equal-sized parts.
   Each part is ____ _________ of the whole.

3. ______ equal-sized parts.
   Each part is ____ _________ of the whole.

4. ______ equal-sized parts.
   Each part is ____ _________ of the whole.

5. ______ equal-sized parts.
   Each part is ____ _________ of the whole.
1.  
   _____ equal-sized parts.
   Each part is _____ _________ of the whole.

2.  
   _____ equal-sized parts.
   Each part is _____ _________ of the whole.

3.  
   _____ equal-sized parts.
   Each part is _____ _________ of the whole.

4.  
   _____ equal-sized parts.
   Each part is _____ _________ of the whole.

5.  
   _____ equal-sized parts.
   Each part is _____ _________ of the whole.
Directions:

You’ll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction you have shown (use words, not symbols).

1.  

2.  

3.  

4.  

5.  

6.  

7.  

8.
Directions:

You’ll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction that you have shown (use words, not symbols).

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 


Look at each picture carefully. Place an “X” beside each picture that shows 2-fourths shaded in. You may need to draw in lines to determine if 2-fourths are shaded.
2. For each diagram, fill in the blanks to tell about the diagram.

a.

Number of equal parts 

Number of equal parts shaded 

The fraction shaded is 

b.

Number of equal parts 

Number of equal parts shaded 

The fraction shaded is 1- 

c.

Number of equal parts 

Number of equal parts shaded 

The fraction shaded is 

d.

Number of equal parts 

Number of equal parts shaded 

The fraction shaded is 

3. Write words like 2-fourths, 3-fifths, and so on for the fraction shaded by each diagram.

a. Write _________________

b. Write _________________

c. Write _________________
d. Write ______________________

e. Write ______________________

f. Write ______________________

g. Write ______________________

4. Write the fraction that is shown:
   a. Write ______________________
   b. Write ______________________
   c. Write ______________________
   d. ______________________
   e. ______________________
   f. ______________________
5. Circle the figures that have equal-size parts.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

h. 

5. Circle the figures that have equal-size parts.
**Problem Solving**

**Directions:**

For each of the drawings write the color corresponding to the part marked a, b, c, and so on. Then write the word name for the fraction which the color represents. You can use fraction circles if you need them. Your teacher will help you with exercise 1.

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractions in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. yellow</td>
<td>1-half</td>
</tr>
<tr>
<td>b. blue</td>
<td>1-fourth</td>
</tr>
<tr>
<td>c.</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
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<td>c.</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
</tr>
</tbody>
</table>
## Overview
Students are introduced to fraction symbols.

## Materials
- Fraction Circles for students and teacher
- Student Pages A – E

## Teaching Actions
1. Ask students to use fraction circles to show 3-fourths. They are to show two models. For example:

   ![Fraction Circles Example](image)

   - 3 blues are 3-fourths of 1 black.
   - 3 grays are 3-fourths of 1 yellow.

2. Ask how the two models are alike.

3. Record in words fraction name: 3-fourths. Explain that there is also a symbol name for 3-fourths and it is $\frac{3}{4}$.

4. Discuss the meaning of $\frac{3}{4}$. Ask how many equal parts each unit is divided into? Point to the bottom of the fraction symbol and explain that this 4 tells us that. The 3 tells us that we are interested in $\frac{3}{4}$ of these 4 equal parts. The fraction means $1/4$ and $1/4$ and $1/4$.

## Comments
1. It’s not important for students to memorize the words: numerator and denominator.
Teaching Actions

5. Write 2/3 on the board and ask students to show that fraction with the fraction circles. Have them verbalize why their model does indeed represent 2/3.

First Divide into 3 equal parts... then

“I divided the circle into 3 equal parts to find what color is thirds. Then I only want two of them so

shows 2 of 3 equal parts. It is 1/3 and 1/3 more.”

6. Repeat for \(\frac{3}{5}, \frac{2}{6}, \frac{4}{8}, \frac{3}{3}\).

Embed examples in context:

A spinner for a game was divided into 5 equal parts. 3/5 of the spinner was blue. Show that amount with the fraction circles.

A pizza was cut into 6 equal parts. You ate 2/6 of the pizza. Show that amount with the fraction circles.

7. Student pages that follow reinforce the meaning of the symbol. Select the most appropriate (and amount of) practice your students need.

Comments

2. It's very important to help children verbalize the meaning of fraction symbols.

Have them talk through what they are doing with the fraction circles.

The action on the manipulative reinforces the meaning of the symbol.

3. You can also return to previous student pages and have students record answers in symbol form.
1. Write each fraction in words.
   a. \( \frac{2}{4} \) ________________  
   b. \( \frac{3}{7} \) ________________  
   c. \( \frac{6}{8} \) ________________  
   d. \( \frac{3}{11} \) ________________  
   e. \( \frac{7}{10} \) ________________  
   f. \( \frac{7}{15} \) ________________  
   g. \( \frac{3}{12} \) ________________  
   h. \( \frac{7}{9} \) ________________

2. Write the word name and the symbol name for each fraction described.
   a. 3 of 5 equal-size parts are shaded. ________________  
   b. 5 of 7 equal-size parts are shaded. ________________  
   c. 3 of 13 equal-size parts are shaded. ________________  
   d. 12 of 17 equal-size parts are shaded. ________________  
   e. 0 of 3 equal-size parts are shaded. ________________

3. Write the fraction symbol for each fraction word.
   a. 9-tenths ______  
   b. 7-eights ______  
   c. 6-sixths ______  
   d. 15-nineteenths ______  
   e. 13-twenty-firsts ______  
   f. 17-eighteenths ______  
   g. 0-fourths ______
4. Imagine a circle divided into 4 equal parts.
   Three $\frac{1}{4}$ parts are shaded!
   What fraction tells how much is shaded in all? _______________
   Draw a picture.

5. Imagine a rectangle divided into 5 equal parts.
   Four $\frac{1}{5}$ parts are shaded!
   What fraction tells how much is shaded in all? _______________
   Draw a picture.

6. Write the word name and the symbol name each fraction describes.
   a. A rectangle is folded into 7 equal-size parts.
      5 parts are shaded.
   b. A circle is folded into 8 equal-size parts.
      4 parts are shaded.
Directions:
Match each picture with its symbol or word name by writing the letter of the picture next to its symbol. The first one is done for you.

A. _______  $\frac{1}{6}$  F
B. _______  2-halves
C. _______  $\frac{3}{4}$
D. _______  2-thirds
E. _______  $\frac{3}{3}$
F. _______  1-fourth
G. _______  $\frac{6}{6}$
H. _______  $\frac{1}{3}$
I. _______  3-sixths
J. _______  $\frac{4}{6}$
K. _______  2-fourths
L. _______  $\frac{1}{2}$
Shade each circle to show the fractional amount.

\[
\frac{1}{4} \quad \frac{2}{2} \quad \frac{1}{6}
\]

\[
\frac{5}{12} \quad \frac{0}{2} \quad \frac{5}{6}
\]

\[
\frac{1}{3} \quad \frac{11}{12} \quad \frac{4}{4}
\]

\[
\frac{2}{12} \quad \frac{1}{2} \quad \frac{6}{8}
\]

\[
\frac{1}{6} \quad \frac{0}{3} \quad \frac{6}{12}
\]

\[
\frac{6}{6} \quad \frac{8}{8} \quad \text{You Decide}
\]
Write the name for the shaded part of each rectangle in words and then in symbols.

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<table>
<thead>
<tr>
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<tbody>
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<td>1.</td>
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<td>1/2</td>
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<td><img src="3-rectangle.png" alt="Diagram" /></td>
<td><img src="3-rectangle.png" alt="Diagram" /></td>
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<td><img src="4-rectangle.png" alt="Diagram" /></td>
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<td>5.</td>
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<td><img src="5-rectangle.png" alt="Diagram" /></td>
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<td><img src="8-rectangle.png" alt="Diagram" /></td>
<td><img src="8-rectangle.png" alt="Diagram" /></td>
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</tr>
</tbody>
</table>
### Overview
Students observe with circles that as the unit is divided into more and more equal parts, the unit parts become smaller.

### Materials
- Fraction Circles for students and teacher
- Student Pages A, B, C

### Teaching Actions
1. Start the lesson by reviewing ordering of whole numbers. For example, ask a student to select the larger of these 2 numbers, 720 or 702, and to explain his/her strategy for doing so.

2. Give another example using a context. José earns $42,175 a year. Mara earns $51,275 a year. Who earns more?

3. Introduce the idea of ordering fractions with this example. Kara entered the Pizza Factory. She saw 2 friends in 1 booth and 3 friends in another booth. Both groups have just been served a large pizza. Which group should she sit with so that she gets the most to eat?

4. Draw this diagram:

![Diagram of circles](image-url)

### Comments
1. To think quantitatively about fractions, students should know something about the relative size of fractions. Lesson 6 is the first of several lessons to help students construct informal strategies for ordering fractions. At Level 1, we want to provide the concrete experiences that students need if they are ever to reason intuitively about fraction symbols.

2. Activities in this lesson will lead students to reason, for example, that $1/4 > 1/8$ because if you divide a circle into 8 equal parts, the parts will be smaller than if you divide the same unit into 4 equal parts.
Teaching Actions

4. Ask students to show Kara’s share in booth 1 (with 2 friends) and in booth 2 (with 3 friends).

5. Which group has the most people? In which group does a person have the smallest share of pizza?

6. Conclude that 1/3 of the pizza is more than 1/4 of the pizza. [Repeat with 6 people at a table; 5 people at a table.]

7. Develop this idea of more implying less, by using Student Page A.

8. Ask students to use their fraction circles as you work together; name the black circle as the unit.

9. Ask: How many brown pieces cover the whole circle? How many orange? Which color takes more pieces to cover the whole unit? Which color has the smaller pieces?

10. Record that information in a chart.

<table>
<thead>
<tr>
<th>Color</th>
<th>How many cover 1 circle</th>
<th>Which color takes more…</th>
<th>Which color has smaller…</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Brown</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>5</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments

3. This notion of more and greater can lead to misunderstandings. Some students may want to say that 1/8>1/4 because 8>4 or because with eighths, you have more pieces than you do with fourths. Whole-number reasoning has a strong influence on how children think about fractions.

4. Children need to be reminded that to compare fractions, we look at the “size of piece,” not the “number of pieces”.

Rational Number Project - Level One
Teaching Actions

11. When completed, ask students if they see any patterns between the number of pieces to fill the whole unit and the size of the pieces.

12. As a group, write a rule similar to either of these:
   - As the number of pieces needed to fill the whole decreases, the size of each piece gets larger.
   - As the number of pieces needed to fill the whole increases, the size of each piece gets smaller.

13. Once the rule is generated use it in examples without the circular pieces.

   Ex: 20 purples = 1 whole
        80 greens = 1 whole
        Which is larger, 1 purple or 1 green?

        18 goos = 1 whole
        12 boos = 1 whole
        Which is smaller, 3 boos or 3 goos?

14. Conclude by asking: Does more always mean less with fractions?

15. Give this example: Imagine that it takes 10 maroon pieces to cover the whole circle. Which is smaller, 2 maroon pieces or 3 maroon pieces? How do you know?

16. Ask: How is this example different from all the rest we've talked about today?

17. Assign Practice Pages B and C to reinforce the day’s lesson.

Comments

4. Students tend to over generalize. This lesson leads children to order fractions with the same numerator, but different denominators (1/3 vs. 1/2, 2/5 vs. 2/10, 40/100 vs. 40/90). The same reasoning will not work for comparing fractions with the same denominator but different numerators (3/4 vs. 2/4). Since a circle is divided into the same size pieces (4ths) an ordering decision is made by looking at the numerator — “2 of same-sized pieces is greater than 2 of same-sized pieces.” Now more does mean more. No wonder students have trouble with fractions!
**Directions:** Use fraction circles to fill in the table.

<table>
<thead>
<tr>
<th>Color</th>
<th>How many cover 1 whole circle?</th>
<th>Which color takes MORE pieces to cover 1 whole?</th>
<th>Which color has SMALLER pieces?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Brown</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>5</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2. Orange</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Purple</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>White</td>
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<td></td>
<td></td>
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<tr>
<td>4. Gray</td>
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<tr>
<td>Green</td>
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<td>5. White</td>
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<td>Green</td>
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<tr>
<td>6. Orange</td>
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<tr>
<td>Purple</td>
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<td></td>
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<tr>
<td>7. Gray</td>
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<tr>
<td>Brown</td>
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<td></td>
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<tr>
<td>8. Brown</td>
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<td></td>
<td></td>
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<tr>
<td>Green</td>
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</tbody>
</table>
**Directions:**
Use Fraction circles to compare the two fractions. Circle the larger fraction.

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1/3</td>
<td>2/3</td>
<td>1/4</td>
<td>3/4</td>
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<tr>
<td>4/5</td>
<td>3/5</td>
<td>2/7</td>
<td>2/6</td>
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<td>6/7</td>
<td>2/7</td>
<td>4/12</td>
<td>4/15</td>
</tr>
<tr>
<td>8/12</td>
<td>11/12</td>
<td>6/7</td>
<td>3/7</td>
</tr>
<tr>
<td>2/7</td>
<td>2/9</td>
<td>9/10</td>
<td>3/10</td>
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<td>4/8</td>
<td>4/6</td>
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<tr>
<td>2/3</td>
<td>2/8</td>
<td>13/100</td>
<td>27/100</td>
</tr>
<tr>
<td>7/10</td>
<td>7/9</td>
<td>9/10</td>
<td>9/100</td>
</tr>
</tbody>
</table>

Try these without manipulatives.
Use fraction circles to solve problems.

1. Mr. Hickman made a large apple pie. His daughter ate $\frac{1}{2}$ of the pie. His son ate $\frac{1}{3}$ of the pie. Who ate less? Draw a picture to show your thinking.

2. Spinner A was divided into 6 equal parts shaded green. Spinner B was divided into 10 equal parts with 4 parts shaded green. Which spinner had the larger amount of green? Explain “in your own words” your reasoning.

3. Jessica and Kim shared a large pizza. Jessica ate $\frac{2}{6}$ of a pizza. Kim ate $\frac{3}{6}$ of the pizza. Who ate more? Draw a picture to show your thinking?

4. Mathew and Cassandra shared a bag of candy. Mathew ate $\frac{2}{3}$ Cassandra ate $\frac{2}{5}$. Who ate more? Explain your thinking.

5. Andrew spent $\frac{1}{2}$ of his allowance on candy. Ellen spent $\frac{1}{3}$ of her allowance on a movie. Is it possible that Ellen spent more than Andrew? Explain. [Use the back of the page].
Overview
This lesson reinforces the idea that as the number of parts the unit is divided into increases, the size of the parts decreases.

Materials
- 8.5” X 1” strips of paper for each student and teacher
- Student Pages A, B, C

Teaching Actions
1. Ask children to fold a strip of paper into 4 equal parts. Using the same strip of paper ask them how they can increase the number of equal parts to 8. Have them do so, but before they open up the strip of paper to show eights ask: Before you open up the strip, can you tell me if the size of the equal parts will be larger or smaller than fourths? Why?

2. Repeat for: 3rds changed to 6ths then to 12ths
   4ths changed to 12ths

3. Now ask students to fold, shape and label these fractions with paper folding:
   \[
   \frac{1}{3} \quad \frac{1}{4} \\
   \frac{2}{3} \quad \frac{2}{6} \\
   \frac{3}{4} \quad \frac{1}{4}
   \]
   [do more if needed]

Comments
1. Children need opportunities to use new ideas in order to ensure they internalize them.

Many experiences with physical models are needed to overcome the influence of children’s whole number thinking.

In this lesson students use paper folding to reexamine the relationship between size of piece and number of pieces the whole is divided into.

2. Encourage children to explain their ordering. Don’t let them refer to only one part of the fraction, as for example: 1/3 vs. 1/4 “thirds are bigger”. Thirds may be bigger, but that information is enough to order 2 fractions only if the numerators are the same. “Thirds are bigger so 1 of a larger piece is greater than 1 of a smaller pieces.” By talking like this children are coordinating numerator and denominator to approximate the size of the fraction. You want to build the notion of a fraction as a single entity!
**Teaching Actions**

4. You may want to refer back to the pizza problem from lesson 6. Model with paper folding or pictures the answer to the question in that story.

5. Put students in pairs and assign Student Page A. Student 1 will make fraction 1 with paper folding; student 2 will make fraction 2. They will then compare and circle the larger fraction.

6. Conclude the lesson by asking children to create their own context for comparing 2 fractions.

   Ex: Mary had 2/4 of large pizza; Joan had 2/4 of large pizza. Who ate more?
   Lianna ate 4/8 parts of a candy bar; Rodrigo ate 4/7 of same-sized candy bar. Who ate more?

7. Student Pages B and C offer extra practice if needed.

**Comments**

3. Students may over generalize and think bigger is always more. Check for this.

4. Some children may be able to compare without manipulatives

   1/3 vs. 1/5; 2/10 vs. 2/20

   but there is no need to push abstraction at this level.

5. Some students may try to compare fractions without the manipulatives and make errors. Encourage them to use paper folding at least to verify their guesses.

6. In grade 4 we don’t expect mastery of this idea at the symbolic level. Consider these lessons as needed foundation that students will use in level 2 to formalize these ideas with abstract notation.

7. Challenge Student Page B:

   The problem here is that the two units are not the same. 1/5<1/2 only if the two units are the same. Comparing fractions assumes equal units.
Directions:
Circle the larger fraction. Use your paper strips to determine the answers.

\[
\begin{array}{cccc}
\frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\
\frac{2}{4} & \frac{2}{6} & \frac{3}{9} & \frac{3}{3} \\
\frac{3}{4} & \frac{1}{4} & \frac{5}{6} & \frac{4}{6} \\
\frac{3}{8} & \frac{3}{4} & \frac{5}{12} & \frac{5}{8} \\
\frac{1}{3} & \frac{1}{12} & \frac{1}{100} & \frac{1}{99} \\
\frac{5}{12} & \frac{8}{12} & \frac{3}{40} & \frac{3}{50} \\
\frac{2}{6} & \frac{2}{12} & \frac{2}{10} & \frac{4}{10}
\end{array}
\]
Directions:
Circle the larger fraction. Use your paper strips to determine the answers.

\[
\begin{array}{ll}
\frac{1}{2} & \frac{1}{4} \\
\frac{1}{12} & \frac{3}{4} \\
\frac{1}{5} & \frac{1}{2} \\
\frac{2}{4} & \frac{2}{3}
\end{array}
\]

Challenge!
What's wrong with this picture?

\[
\begin{array}{ll}
\frac{1}{8} & \frac{1}{4} \\
\frac{1}{5} & \frac{1}{2}
\end{array}
\]
Directions:

A friend has been out of school for two days and missed the math lessons dealing with comparing fractions. Write your friend a letter explaining how to compare fractions like the ones you have been working with. [You may want to draw pictures].
### Overview

Students explore fraction equivalence by naming fractions equal to 1/2 with fraction circles and by finding other fraction equivalences with fraction circles.

### Materials

- Fraction Circles for students and teacher
- Large sheet of chart paper for teacher; Equivalence Chart for students
- Fraction Fill board for students and numeral cards for teacher
- Circle page divided into twelfths [optional]

### Teaching Actions

1. At the overhead, cover the whole circle with 1 yellow and ask students to find different ways to cover the remaining half of the circle. Record answers by color.

   Ex: 1 blue and 2 grays  

2. Repeat this activity, but this time specify they have to use one color to cover half the circle. Record results by color and fraction name.

   - 1 yellow = 2 blues
   - 1 yellow = 3 pinks

3. Ask what each display has in common. [They all cover 1-half of the black circle].

### Comments

1. The idea of equivalence is a prerequisite for fraction operations. To add 1/2 + 3/4, you will explain that 1/2 can be exchanged for 2/4 because 1/2=2/4.

2. Equality should first be developed from concrete models before explaining a rule that generates equal fractions \( \left( \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \right) \).

3. You are defining equivalent fractions by showing that fractions are equivalent if they cover the same amount of the circle. The digits in the fraction symbols are different, but 1 of 2 equal parts covers the same amount as 2 of 4 equal parts.
**Teaching Actions**

4. Continue to model fraction equivalences by completing the equivalence chart. Make a large classroom chart of this picture or use a transparency of student’s chart.

<table>
<thead>
<tr>
<th>Red</th>
<th>1/12</th>
<th>2/12</th>
<th>3/12</th>
<th>4/12</th>
<th>5/12</th>
<th>6/12</th>
<th>7/12</th>
<th>8/12</th>
<th>9/12</th>
<th>10/12</th>
<th>11/12</th>
<th>12/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>2/12</td>
<td>3/12</td>
<td>4/12</td>
<td>5/12</td>
<td>6/12</td>
<td>7/12</td>
<td>8/12</td>
<td>9/12</td>
<td>10/12</td>
<td>11/12</td>
<td>12/12</td>
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<td>Gray</td>
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</tbody>
</table>

5. Start by asking students to cover 2/12 of the whole circle. Now ask them if they can cover the same amount with whites (without cutting the pieces). Check other colors. Since 1 pink equals 2 reds, record on the chart, 1/6 under the 2/12 column across from “pink”.

<table>
<thead>
<tr>
<th>Red</th>
<th>1/12</th>
<th>2/12</th>
<th>3/12</th>
<th>4/12</th>
<th>5/12</th>
<th>6/12</th>
<th>7/12</th>
<th>8/12</th>
<th>9/12</th>
<th>10/12</th>
<th>11/12</th>
<th>12/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
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</tr>
<tr>
<td>Gray</td>
<td>2/8</td>
<td></td>
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</tr>
<tr>
<td>Pink</td>
<td>1/6</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Blue</td>
<td>1/4</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Brown</td>
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<td>Yellow</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

6. Repeat for 3/12, recording in fraction symbols 2/8 and 1/4 under the 3/12 column [see above].

7. Continue, completing the rest of the chart.

<table>
<thead>
<tr>
<th>Red</th>
<th>1/12</th>
<th>2/12</th>
<th>3/12</th>
<th>4/12</th>
<th>5/12</th>
<th>6/12</th>
<th>7/12</th>
<th>8/12</th>
<th>9/12</th>
<th>10/12</th>
<th>11/12</th>
<th>12/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3/9</td>
<td>6/9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9/9</td>
</tr>
<tr>
<td>Gray</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/8</td>
<td>4/8</td>
<td>6/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8/8</td>
</tr>
<tr>
<td>Pink</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>2/6</td>
<td>3/6</td>
<td>4/6</td>
<td>5/6</td>
<td>6/6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
<td>2/4</td>
<td>3/4</td>
<td>4/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/3</td>
<td>2/3</td>
<td></td>
<td></td>
<td>3/3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/2</td>
<td>2/2</td>
<td></td>
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</tr>
</tbody>
</table>

**Comments**

4. An alternative to having students put reds on the black circle is to use the circle page partitioned into 12 equal parts.

5. Teachers have found that filling in the equivalence chart to be easy for some and difficult for others. Consider completing the chart in pairs.

6. One-half equivalences are the most important ones for children to learn. Students with good “fraction number sense” use 1/2 as a reference point for estimating the size of other fractions.

7. A student will use his/her concept of 1/2 to estimate, for example, 2/4+2/3: “2/4 equals 1/2; 2/3 is greater than 1/2; so the sum is greater than 1.” This reasoning is an excellent example of thinking quantitatively. Thinking quantitatively is not rule-bound, but relies on the mental images children have for fractions. Images for 1/2 are critical.
Teaching Actions

8. Use this chart as a reference for fraction equivalencies. Students can use it as they play the following game.

9. **FRACTION FILL:**

**MATERIALS:**  Fraction numeral cards**
Fraction Fill Board

**DIRECTIONS:** Teacher randomly selects a numeral card and shows it to students. Students choose to shade that amount on one of the circles. They can only shade 1 representation for that fraction amount.

Ex: 1/4

Student can shade

![Shade options](image)

but not both

Student cannot shade the circle divided into two equal parts by adding lines to the circle to divide it into fourths.

Continue showing numeral cards. Students refer to equivalence chart to make selections. The first to shade two complete circles says “Fraction Fill.”

** Make a set of cards for 1/2; 1/3; 2/3; 1/4; 2/4; 3/4; 1/6; 2/6; 3/6; 4/6; 5/6; 1/12; 2/12; 3/12; 4/12; 5/12; 6/12; 7/12; 8/12; 9/12; 10/12; 11/12.

Comments

8. Just as students learn the basic +, -, x, ÷ facts, you want them to learn basic fraction equivalents for 1/2, 1/3, 2/3, 1/4, 2/4, 3/4. On this level, they don’t need rules, but many physical/visual examples with these fractions and their equivalents.

9. Fraction Fill is a game that can be played throughout the rest of the lesson.
# Rational Number Project - Level One

## Lesson 8

**Equivalence Chart**

<table>
<thead>
<tr>
<th>Red</th>
<th>( \frac{1}{12} )</th>
<th>( \frac{2}{12} )</th>
<th>( \frac{3}{12} )</th>
<th>( \frac{4}{12} )</th>
<th>( \frac{5}{12} )</th>
<th>( \frac{6}{12} )</th>
<th>( \frac{7}{12} )</th>
<th>( \frac{8}{12} )</th>
<th>( \frac{9}{12} )</th>
<th>( \frac{10}{12} )</th>
<th>( \frac{11}{12} )</th>
<th>( \frac{12}{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Gray</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Pink</td>
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<td>Blue</td>
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<tr>
<td>Brown</td>
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<tr>
<td>Yellow</td>
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<td></td>
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</tbody>
</table>
Fraction Fill
### Overview
Students continue to explore equivalence with pictures and fraction circles.

### Materials
- Transparencies 1& 2
- Student Pages A, B, C
- Fraction Circles for students

### Teaching Actions
1. Show transparency 1 to the class.

![Diagram](image)

2. Ask students to name section a; section b; section c. [Also ask what color fraction-circle piece matches each part]. Have them explain their reasoning.

3. Ask students if fractional parts can have more than one name. Ask students to name section a in two different ways. Record on the transparency what they say with words and/or symbols:

   Ex: 1 yellow = 1/2; [1 blue = 1/4]  
   1 yellow = 2 blues: 1/2 = 2/4

### Comments
1. Seeing equivalence from pictures is not the same as seeing it with manipulatives. Some children are better at adding and taking out lines drawn in a diagram. Don’t be surprised to see differences in how children respond to these pictures.
### Teaching Actions

3. Point to the section \((c + d + e)\). Ask: How are \(b\) and \((c + d + e)\) alike? *[cover same amount]*

4. As a group write sentences using colors and symbols that describe equivalences in the picture.

   Ex:  
   - 1 blue = 3 reds; \(1/4 = 3/12\)
   - 1 blue and 3 reds = 1 yellow; \(1/4 + 3/12 = 1/2\)
   - 6 reds = 1 yellow; \(6/12 = 1/2\)

5. Show transparency 2 to the class and talk through the naming of each part: \(a, b, c, (b + c), d, (d + e), (d + e + f + g)\) in several ways. Record symbolic sentences.

   Ex:  
   - \(a = 1/6; b = 2/6; (b+c) = 1/2;\)
   - \(c = (d+e); 2/6 = 4/12\)

6. Repeat for the second rectangle at the bottom of the page.

7. Assign in pairs Student Pages A, B, C. For problems 1, 2 and 3, children refer to their fraction circles; for the last 3 problems, children rely on diagrams. They may need to draw on the pictures. Encourage them to do so.

### Comments

2. Note: Problem 1 is already completed; this was the same as the problem on Transparency 1.
Directions:

For each of the drawings write the color corresponding to the part marked a, b, c, and so on. Then write a sentence that is true about all of the color-coded parts altogether. Use your color-coded circular parts to help you, if you need them.

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractions in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. yellow</td>
<td>1/2</td>
</tr>
<tr>
<td>b. blue</td>
<td>1/4</td>
</tr>
<tr>
<td>c. red</td>
<td>1/12</td>
</tr>
<tr>
<td>d. red</td>
<td>1/12</td>
</tr>
<tr>
<td>e. red</td>
<td>1/12</td>
</tr>
</tbody>
</table>

Sentences I can write about the parts:

a) 1 yellow and 1 blue and 3 reds equal 1 whole circle.  
   1/2 and 1/4 and 3/12 = 1 whole.
b) 1 blue and 3 reds equal 1 yellow. 1/4 and 3/12 = 12.
c) 3 reds equal 1 blue. 3/12 = 1/4.
d) 6 reds equal 1 yellow. 6/12 = 1/2.
3. Color Fractional Part of Whole Circle

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractional Part of Whole Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
</tr>
</tbody>
</table>

Sentences I can write about the parts:

4. Fractional Part of Rectangle

<table>
<thead>
<tr>
<th>Fractional Part of Rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
</tr>
<tr>
<td>b.</td>
</tr>
<tr>
<td>c.</td>
</tr>
<tr>
<td>d.</td>
</tr>
</tbody>
</table>

Sentences I can write about the parts:
5. Fractional Part of Rectangle

a. _______________________

b. _______________________

c. _______________________

d. _______________________

Sentences I can write about the parts:

6. Fractional Part of Rectangle

a. _______________________

b. _______________________

c. _______________________

d. _______________________

e. _______________________

Sentences I can write about the parts:
Sentences I can write about the parts:
Sentences I can write about the parts:

```
 a  b
 c  d  e  f  g
```

Sentences I can write about the parts:

```
 a  b  c  d  e
```
# Overview
Students explore equivalence ideas with paper folding.

## Materials
- Strips of paper (8.5” x 1”) for folding for each student.
- Student Pages A-F
- Scissors and glue

## Teaching Actions
1. Throughout this activity, teacher and students do the examples. Teacher may choose to use larger strips of paper for demonstration purposes.

2. Ask students to fold strips of paper into thirds and shade 2/3 of the paper. Write the symbol for amount shaded on that strip.

3. Now have students fold the same strip to show 6 equal parts. Before they actually open up the folded paper, ask them to guess the number of shaded parts.

4. Open up the amount and record on the paper strip.

5. Ask: Do you have more than 1 fraction written on your paper? Explain why.

6. Ask students to refold and created 12 equal parts. Unfold and note how many total equal parts; how many parts are shaded? Record that amount.

7. Ask: In what way are 2/3, 4/6, and 8/12 alike? different?

8. Replicate the above starting with 1/2; go to 2/4, 4/8.

9. Practice is provided in Student Pages A-F

## Comments
1. Students will benefit from seeing equivalent fractions with more than one manipulative.

2. Remember all this work with manipulatives is an investment that will pay off later as children learn to operate with fractions. The manipulative experiences will give them the mental images they need to operate (+, -, x, ÷) on fractions in a meaningful way.

3. Notes regarding Student Pages:
   - B and C: After cutting out rectangles and sorting into equivalent groups you may want students to paste groups onto paper and record fraction names on that paper, omitting Student Page C.
   - D and E: Students may want to just draw lines on the paper strips instead of folding.
**Lesson 10**

**Student Page A**

Name: __________________________

**Directions:**
Work through each step:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
</table>
| 1. | A. Fraction shaded: _______  
    |   | B. Make into six equal-sized parts drawing in lines on picture.  
    |   | C. Fraction shaded now: _______  |

<p>| | | |</p>
<table>
<thead>
<tr>
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<th></th>
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</tr>
</thead>
</table>
| 2. | A. Fraction shaded: _______  
    |   | B. Make into 15 equal-sized parts drawing in lines on picture.  
    |   | C. Fraction shaded now: _______  |

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
</table>
| 3. | A. Fraction shaded: _______  
    |   | B. Make into 12 equal-sized parts drawing in lines on picture.  
    |   | C. Fraction shaded now: _______  |

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 4. | A. Fraction shaded: _______  
    |   | B. Make into six equal-sized parts drawing in lines on picture.  
    |   | C. Fraction shaded now: _______  |
Directions:
Cut out the rectangles. Put the rectangles together into groups so that each rectangle in the group has the same amount shaded. After you have grouped them, fill in the table by writing the fraction name for each rectangle in the group. Fractions in the same group are called equivalent fractions.

a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

i) 

j) 

k) 

l) 

m) 

n) 

o) 

p) 

q) 

r) 

s) 

t)
Record Sheet for Student Page B

<table>
<thead>
<tr>
<th>Write equivalent fractions for each group you made.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>(\frac{5}{6})</td>
</tr>
</tbody>
</table>
PAPER FOLDING AND EQUIVALENT FRACTIONS

Cut the strips on Student page E so you can fold them to solve these problems.

1. Write the symbol for the fraction shaded on strip A: ________
   Fold to make 8 equal sized parts.
   Write the symbol for the fraction which is shown: ________

2. Write the symbol for the fraction shaded on strip B: ________
   Fold to make 12 equal sized parts.
   Write the symbol for the fraction which is shown: ________

3. Write the symbol for the fraction shaded on strip C: ________
   Fold to make 6 equal sized parts.
   Write the symbol for the fraction which is shown: ________

4. Write the symbol for the fraction shaded on strip D: ________
   Fold to make 12 equal sized parts.
   Write the symbol for the fraction which is shown: ________

5. Be careful! Think! Take folding strip E.
   Fold to make 3 equal parts.
   Shade 2 of the 3 equal parts.
   Write the fraction for the amount shaded in strip E: ________
Paper Folding and Equivalent Fractions

1) Show 3 fractions equal to 1/2. (Hint: you will need to start with 3 sheets of paper folded into 2 equal parts with one part shaded. Draw a picture to show your answers.

2) Use paper folding to find out which of these are true statements.

\[
\frac{1}{3} = \frac{2}{6} \quad \frac{1}{4} = \frac{2}{6}
\]

\[
\frac{3}{4} = \frac{9}{12} \quad \frac{4}{6} = \frac{2}{3}
\]

3) Use paper folding to find these equivalences.

\[
\frac{1}{2} = \frac{8}{12} \quad \frac{1}{3} = \frac{3}{12}
\]

\[
\frac{1}{4} = \frac{2}{12} \quad \frac{1}{3} = \frac{2}{2}
\]
**Overview**

Students use fraction circles to order 2 fractions by comparing them to one-half.

**Materials**

- Fraction Circles for students and teacher
- Student Pages A, B

**Teaching Actions**

1. Ask students to take out the black circle and to cover one-half of the circle with 1 yellow.

2. Show on overhead that 3 blues, which is 3/4 of the black, is greater than 1 yellow (1/2 of the black).

3. Record: 3 blues > 1 yellow
   \[ \frac{3}{4} > \frac{1}{2} \]

4. Ask students to find 4 other fractions greater than 1/2. Model and record their responses on the overhead.

5. Now ask them to imagine fraction pieces greater than 1 yellow or 1/2 of the circle. Have them write down at least 3 estimates for amounts greater than 1/2.

6. Ask students to share their estimates and to explain what they thought of or pictured.

   Ex: A child may say, “I can see that 3 pinks are the same as 1 yellow, so 5 pinks must be greater than 1/2.”

**Comments**

1. Students need many experiences with concrete materials to develop mental images of fractions so they can develop a quantitative notion of fraction. Comparing to 1/2 is a powerful strategy for judging the relative size of fractions and is a characteristic of having a quantitative notion of fraction.

2. Looking at specific numerical relationships between numerator and denominator to determine if fractions are greater or less than 1/2 is delayed until level 2, although some students may implicitly understand this.
### Teaching Actions

7. Have students verify each guess with their circles and record results with fraction notation.

8. Student Page A provides independent practice with circles comparing fractions to 1/2.

9. Student Page B provides more practice with ordering and equivalence ideas developed so far.

10. End class by presenting these problems for discussion. Emphasize student verbalization of their thinking as they order these fractions. They may or may not use the circles.

Which is bigger or are they equal?

Examples:

<p>| | |</p>
<table>
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<tbody>
<tr>
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<td>3</td>
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<tr>
<td>3</td>
<td>4</td>
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<td>1</td>
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<td>4</td>
<td>3</td>
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<td>3</td>
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<tr>
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<td>4</td>
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<tr>
<td>100</td>
<td>70</td>
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<p>| | |</p>
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</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
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<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

### Comments

3. Mastery at the symbolic level is not expected at Level 1, but you can use this last set of problems to evaluate to what extent students can order fractions symbolically (using, of course, mental images of fraction circles, not any Least Common Denominator rule).

4. A common error students make is to look only at the denominator to make an order decision. For example, when comparing 1/3 vs. 3/4, a student may say 1/3 is the bigger fraction since thirds are larger than fourths.

Encourage students to reflect on the numerator and denominator to determine the fraction’s relative size.
Exploring \( \frac{1}{2} \) With Fraction Circles

Use the whole circle as your unit. Make the fraction \( \frac{2}{5} \) with the fraction circles. Decide if \( \frac{2}{5} \) is greater or less than \( \frac{1}{2} \). Record your response in the box:

\[
\frac{2}{5} \quad \text{is less than} \quad \frac{1}{2}
\]

Complete the problems below. Use these choices: **is less than, is greater than** or **is equal to**.

\[
\begin{align*}
\frac{2}{3} & \quad \frac{1}{2} \\
\frac{5}{12} & \quad \frac{1}{2} \\
\frac{3}{4} & \quad \frac{1}{2} \\
\frac{2}{8} & \quad \frac{1}{2} \\
\frac{7}{12} & \quad \frac{1}{2} \\
\frac{1}{4} & \quad \frac{1}{2} \\
\frac{8}{10} & \quad \frac{1}{2} \\
\frac{1}{5} & \quad \frac{1}{2} \\
\frac{4}{6} & \quad \frac{1}{2} \\
\frac{9}{15} & \quad \frac{1}{2}
\end{align*}
\]
USING FRACTION CIRCLES TO ORDER FRACTIONS

Use fraction circles to show each fraction. Compare the fractions. Circle the largest fraction. If the fractions are equivalent, circle both.

(1) \( \frac{3}{4} \) \( \frac{1}{2} \)  
(2) \( \frac{1}{3} \) \( \frac{8}{12} \)  
(3) \( \frac{4}{6} \) \( \frac{2}{3} \)

(4) \( \frac{4}{12} \) \( \frac{2}{4} \)  
(5) \( \frac{9}{12} \) \( \frac{2}{6} \)  
(6) \( \frac{1}{3} \) \( \frac{1}{4} \)

(7) \( \frac{4}{8} \) \( \frac{1}{2} \)  
(8) \( \frac{3}{4} \) \( \frac{6}{8} \)  
(9) \( \frac{1}{8} \) \( \frac{1}{12} \)

(10) \( \frac{4}{8} \) \( \frac{5}{8} \)  
(11) \( \frac{2}{4} \) \( \frac{3}{6} \)  
(12) \( \frac{2}{6} \) \( \frac{2}{12} \)

(13) \( \frac{3}{6} \) \( \frac{5}{6} \)  
(14) \( \frac{6}{8} \) \( \frac{8}{8} \)  
(15) \( \frac{4}{8} \) \( \frac{6}{12} \)
### Overview
Students are introduced to chips as a fraction model. They learn to represent a given fraction using different sets of chips as a unit.

### Materials
- Chips or tiles with a different color on each side
- Paper strips for folding
- Display chips or overhead chips for teacher
- Student Page A

### Teaching Actions
1. You use chips while students use paper folding to show the same fraction.

2. Model the fraction 2/3 with the display chips you made [see description at end of lesson].
   
   Say: I have 6 chips (show 6 chips with the white side up). I’m going to partition them into 3 equal groups. Can you partition your paper into 3 equal parts?

   Ask: How are the displays alike and different?

| ○○ | ○○ | ○○ |     |     |

3. Turn over 3 of the 3 equal groups of chips to show the tan side of the chips. Ask children to model this action on their paper strips by shading 2 of 3 equal parts.

   Say: I made 2 of 3 equal-sized groups tan.

   Ask: What fraction of the chips are now tan?

   Ask: How are the chips and paper folding models alike? Different?

| ○○ | ○○ | ○○ |     |     |

### Comments
1. This will be a challenging lesson for students. You may want to use two class periods to cover this material.

2. To reinforce important fraction concepts students are introduced to a new model for fractions (chips) by relating this model to a previous one (paper folding). This is a translation from one physical model to another. Seeing similarities between models helps students abstract important concepts.

3. Common error: Students model 2/3 by making groups instead of three equal groups.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Summarize what you did by writing on the board.</td>
<td>4. Students often need to exaggerate the grouping of the chips to highlight the separate groups.</td>
</tr>
<tr>
<td>I started with a unit of _____ white chips. I divided the unit into _____ equal groups. I made _____ groups tan. _____ of _____ equal-sized groups are tan. What fraction of the groups is tan?</td>
<td>Ex: 2/3 Spread out the sets and perhaps even touch the two chips that form one group. At this point don’t use the array-model.</td>
</tr>
<tr>
<td>5. Ask students to verbalize what they did with paper in a similar way. Conclude that there are many different models to show fractions.</td>
<td>5. Some students say that 4/6 is covered. Informal discussion of equivalence is encouraged, but don’t rush the idea.</td>
</tr>
<tr>
<td>6. Present these two stories to students. Ask them to decide which model, paper folding or chips, would be best to show the fraction the story.</td>
<td>6. Student verbalization is important. You might ask them to write a description for showing a fraction with chips.</td>
</tr>
<tr>
<td><strong>Story 1:</strong> Janet had 15 M&amp;M’s. She shared them equally among herself and 2 friends. What fraction of candy did she get?</td>
<td></td>
</tr>
<tr>
<td><strong>Story 2:</strong> LeAnna has a Nestle Crunch bar. She plans to share it among herself and 2 friends. What fraction of the candy bar will each get?</td>
<td></td>
</tr>
<tr>
<td>7. Model other fractions using chips. Students should have their own chips. For the sake of consistency, use the white side to show the unit and use the tan side to show “amount shaded.”</td>
<td></td>
</tr>
</tbody>
</table>
8. **Say:** I have 18 chips. I want to show 4/6 using these chips as my unit.

**Ask:** How many equal-sized groups will I need? (6). Now divide the 18 chips into 6 equal groups.

![Chips divided into 6 equal groups]

**Ask:** To show 4/6, how many equal groups must I make tan? (4)

![Chips divided into 4 equal groups]

9. Repeat for several more fractions.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/3</td>
<td>15</td>
</tr>
<tr>
<td>2/3</td>
<td>21</td>
</tr>
<tr>
<td>6/7</td>
<td>7</td>
</tr>
<tr>
<td>3/4</td>
<td>20</td>
</tr>
</tbody>
</table>

10. Student Page A provides independent practice. Use if time permits.
Modeling Fractions with Chips

1) Show 3-fourths with chips. Use 20 chips in all. Draw a picture of your display.

2) Show 3-fourths with chips. Use 8 chips in all. Draw a picture of your display.

3) Show 2-sixths with chips. Use 12 chips in all. Draw a picture of your display.

4) Show 2-sixths with chips. Use 6 chips in all. Draw a picture of your display.

5) Show 4-fifths with chips. Use 20 chips in all. Draw a picture of your display.

6) On the back of this page, describe steps you would take to show 3/7 using 21 chips.
**Overview**

Students continue practicing showing fractions with chips. They determine several units that can be used to model a fraction and what units can’t be used to model fractions.

**Materials**

- Chips for students and teacher
- Student Page A

**Teaching Actions**

1. Present this picture:

```
[Image of chips]
```

Say: I want to model this fraction using chips as my unit instead of paper. What fraction is shown? If I use 12 chips as my unit, tell me the steps to show 3/4.

2. Vary the unit by asking students what they’d do if you used 4 chips as a unit and then 20 chips as the unit. Ask how these chip models are alike and how they are different.

3. Summarize by showing that to show 3/4 you used 4, 12, and 20 chips. Ask if you could have used other sets of chips as your unit.

4. Ask students to show the fraction 2/3 with chips. Allow them to choose the unit. Ask students to tell you what units they used.

**Comments**

1. Flexibility of unit is emphasized with chips, as was done with the fraction circles. Students should know that to show 2/3, a number of sets can be used: 3 chips, 6 chips, 9 chips...Regardless of the number of chips, the same action to model the fraction is used. (Partition into 3 equal groups and make 2 of 3 groups tan.)
## Teaching Actions

5. Present this chart to students. Ask them to list 3 possible units that they could use as the unit for each fraction.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Units you could use</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{4}{5} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{7} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclude the lesson by asking students to show the fraction \( \frac{3}{4} \) with 14 chips. Discuss why this cannot be done. Ask for units that cannot be used to show the fractions in the above chart. [Make another column in the table].

7. Assign Student Page A as a way to practice showing fractions with chips.

## Comments

2. The possible units are multiples of the denominator. 

\[
\begin{align*}
\frac{4}{5} & = 5, 10, 15, 20, 25, \ldots \\
\frac{5}{5} & \text{ are all possible units.}
\end{align*}
\]

3. Look closely at Student Page A. This page provides some problem solving for students.

This page will need some initial guidance. Do the first 4 or 5 rows together; students can finish the rest individually, in pairs or small groups.
Directions:
Use chips and complete the chart. The first one is done for you.

<table>
<thead>
<tr>
<th>Number of chips in units</th>
<th>Number of equal-size parts</th>
<th>Number of chips in each equal-size part</th>
<th>Number of parts tan</th>
<th>Fraction of parts tan</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>2/3</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td>2/5</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>1/3</td>
</tr>
<tr>
<td>21</td>
<td>7</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td>7</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>4/5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/4</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Overview
Students continue to model fractions with chips. They determine possible fractions that can be shown with different sets of chips.

Materials
• Chips for students
• Student Pages A, B

Teaching Actions
1. Put this chart on the board and ask children to tell you what fractions can be shown with each unit and why.

<table>
<thead>
<tr>
<th>Unit in chips</th>
<th>Fractions you can show</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 chips</td>
<td>Sixths, halves, thirds; not fourths</td>
</tr>
<tr>
<td>8 chips</td>
<td></td>
</tr>
<tr>
<td>12 chips</td>
<td></td>
</tr>
<tr>
<td>15 chips</td>
<td></td>
</tr>
<tr>
<td>20 chips</td>
<td></td>
</tr>
</tbody>
</table>

2. Ask students to name fractions that cannot be shown with the above sets.
### Teaching Actions

3. Present these problems to students. Students model with chips. You draw pictures of models as students verbally describe their models to you.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Show 1/4 with chips. Use 8 chips as a unit.</td>
</tr>
<tr>
<td>2)</td>
<td>Show 1/4 with chips. Use a unit other than 8 chips.</td>
</tr>
<tr>
<td>3)</td>
<td>Show 1/6 with chips in 2 different ways. Use 6 chips and 12 chips as your units.</td>
</tr>
<tr>
<td>4)</td>
<td>Show 3/5 with chips in two different ways. Use any two units.</td>
</tr>
</tbody>
</table>

4. Assign Student Pages A, B to reinforce chip model for fractions.

### Comments

1. To encourage communication, have students write out steps showing a fraction with chips.
1. Show each of these fractions with chips in two ways. You decide on the unit. Draw pictures of your models.

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{5} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{9} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{6} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{4}{5} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>
Draw pictures to model each story.

2. 2/3 of Mr. Vega’s math class are girls. There are 21 students in the class.

3. You can buy a box of 16 gum drops for 35 cents. If you share the box with three others, what fraction will each receive? How many gum drops will each receive?

4. William and his friend shared a small pizza evenly. How much did each eat?

5. Jessica and Jennifer shared their bag of M&M’s with LeAnna. If each got a fair share, how much of the bag did Jessica and Jennifer get together?
### Overview
Students explore fraction equivalence using chips.

### Materials
- Chips for students
- Display chips for teacher
- Student Pages A-E

### Teaching Actions
1. Tell the students this story:
   
   There are 12 pieces of hard candy in a bag. William ate 1/3 of the candy. Sonya ate 2/6 of the same-size bag of hard candy. Who ate more?

2. Have students model with chips William’s share of 12 pieces of candy.

   Ask: What fraction of the bad did William eat? How many equal-sized groups will I divide 12 into? How can I show 1 of 3 equal-sized groups? (Turn chips to tan side).

3. Repeat for Sonya’s share.

4. Have students look at each model and ask how they are alike and different. Ask again: who ate more?

### Comments
1. This will be a challenging lesson for students. You may want to use two class periods to cover this material.
Teaching Actions

5. Explain that you can name a fraction in more than one way. These two examples show that 1/3 and 2/6 are equal because the same unit is used to show both fractions, and the same number of chips are tan.

6. Show 12 chips grouped into thirds:

```
  □ □ □  □ □ □  □ □ □  1/3
  □ □ □  □ □ □  □ □ □  2/6
```

Say: This shows 1 of 3 equal-sized groups. If I reorganize the chips into different groups, then I can see a different name.

7. Show:

```
  □ □ □    □ □ □    □ □ □  1 of 3 groups
  □ □ □    □ □ □    □ □ □  2 of 6 groups
```

You may want chips in the same group to touch.

The second picture shows 2 of 6 equal-sized groups are tan or 2/6. Since the same number of chips are tan as in the previous model, 1/3 and 2/6 are equal.

Comments

2. Comparing fractions with chips depends on using the same number of chips in each unit. You cannot compare 1/3 and 2/6 if the units are different. For example:

```
  □ □ □    □ □ □    □ □ □  1/3
  □ □ □    □ □ □    □ □ □  2/6
```

Here it seems that 2/6>1/3.

3. Now reorganize groups to resemble arrays. You may still need to spread out groups to help children see different groups.

Ex:

```
  □ □ □    □ □ □    □ □ □  1/3
  □ □ □    □ □ □    □ □ □  2/6
  □ □ □    □ □ □    □ □ □  3/5
  □ □ □    □ □ □    □ □ □  6/10
```
Teaching Actions

8. Show:

![](image)

Say: I separated 12 chips into 12 groups with one chip in each group. What fraction is tan? The amount of tan chips is the same for 1/3, 2/6, and 4/12. What is true about these fractions?

9. Let’s show 3/5 with 10 chips. Can you give me another name for the fraction? How can you tell?

10. Repeat for 2/4, using 8 chips as the unit; 4/6 using 24 chips as the unit; 1/2 using 12 chips as the unit.

11. Assign Student Pages A-E.
   - You may want to do pages D and E in a large group setting.

12. End the lesson by asking for their thoughts on this story:

   Mark receives $8 a month for an allowance. Janna receives $12 a month for an allowance. Mark spent 1/4 of his allowance. Janna spent 1/6 of her allowance. Since 1/4 > 1/6, Janna spent more.

Comments

4. Emphasize the physical reorganization of the chips as a strategy for seeing different ways of grouping and naming fractional amounts.

5. Have children work together so they can discuss more than one way to name each fraction amount. Encourage children to use their chips.
Use your chips to do these problems. Name each fraction amount in more than one way if possible.

1. 8 chips is the unit. What is the value of each of these sets of counters?

   - 1 chip
   - 2 chips
   - 4 chips
   - 6 chips

2. 4 chips is the unit. What is the value of each of these sets of counters?

   - 1 chip
   - 2 chips
   - 3 chips
   - 5 chips

3. 6 chips is the unit. What is the value of each of these sets of counters?

   - 2 chip
   - 5 chips
   - 3 chips
   - 4 chips
Counters and Fractions

1. Show 1/4 with chips. Use 8 chips as your unit. Draw a picture of your model. Give another name for 1/4.

2. Show 2/3 with your chips. Use 15 chips as your unit. Draw a picture of your model. Give another name for 2/3.

3. Show 1/6 with chips in two different ways. Use 6 chips and 12 chips as your units. Draw a picture of each model.
4. Show $\frac{1}{2}$ with 12 chips as the unit. Then show $\frac{1}{2}$ with 3 other units. Draw pictures of your models and name each one in more than one way if possible.

5. Show three fractions greater than $\frac{1}{2}$ with your chips. Show three fractions less than $\frac{1}{2}$ with your chips. Draw pictures of your models and name each one.

Fractions greater than $\frac{1}{2}$

Fractions less than $\frac{1}{2}$
Directions:

In the pictures below, the (white) side of the chip and the (gray) side is the dark side. Give two fractions in symbols which tell the fraction of the chips which are light. For each exercise, complete the number sentence.

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
<th>Number Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For exercises 7-11, write the equivalent fractions which are shown in the diagram.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. __ = __

8. __ = __

9. \( \frac{2}{4} = \frac{4}{8} = 8 \)

10. __ = __

11. \( \frac{1}{4} = \frac{12}{6} = 6 \)
### Overview

Students extend their fraction concepts by reconstructing the unit when given the fraction part.

### Materials

- Fraction Circles for students and teacher
- Student Page A

### Teaching Actions

1. Lead a discussion around the concept of unit. Possible questions include:
   - 1) To show 1/3, what possible units could I use if I used fraction circles?
   - 2) If I used chips, what units could I use?
   - 3) If I used paper folding, how would I show 1/3? [use paper as a unit; partition into equal-sized parts; highlight a certain number of parts]

2. Explain that so far we have done a lot of problems in which we started with a unit and divided it into equal sized parts. Now we will reverse the process. I will give you one or more of equal-sized parts and you have to find the unit.

3. Model the idea of reconstruction the unit. Show 1 pink piece and say that this is 1 of 3 equal parts – it is 1/3 of some amount, some unit.

4. Show and ask: since this is 1 of 3 equal sized parts, how many more parts do I need to build a whole unit? What size parts do I need? All must be pink.

### Comments

1. The activities in this lesson and the next, reinforce the idea that, for example, 2 halves equal 1 whole, 3 thirds equal 1 whole, and so on. It also reinforces the notion that non-unit fractions are iterations of unit fractions \( 3/5 = 1/5 + 1/5 + 1/5 \).

2. Time spent on these activities continues to help children solidify mental images for fraction symbols.

3. Solution of these problems will center around the unit fraction and constructing the unit from the unit fraction.
Teaching Actions

5. Place and count

\[
\begin{align*}
\text{P} & \quad \text{P} & \quad \text{P} \\
\text{1 part, 2 parts, 3 parts}
\end{align*}
\]

so the whole unit is 3 pinks or 1 yellow.

We know that 1 pink is 1/3 of 1 yellow. We found the unit starting with 1/3 of it.

6. Ask students to take out 1 blue piece. State that this blue is 1/4 of some whole unit.

7. Ask: will the unit be bigger or smaller? How many fourths make a whole unit? Use your circles to find the unit.

8. Repeat for these pieces and values:

- gray = 1/4  [unit is yellow]
- red = 1/3  [unit is blue]
- pink = 1/2  [unit is brown]
- red = 1/4  [unit is brown]
- gray = 1/2  [unit is blue]

9. Present this example and explain that it is tricky:

1 gray = 1/3. Find the unit.

Ask the student to explain how to construct the unit. Then ask how we can describe the unit. Is there 1 piece to cover this amount? Since there isn’t, students can name the unit as 3 grays. You can trace the 3 grays and say this amount is the unit.

10. Repeat for this example:

1 blue = 1/3. Unit = ?

Comments

4. Have students do these independently at their desks and then have them verbalize the process in a large group.

5. In each of the examples so far, the answer could be expressed as a single piece.

Ex: 1 gray = 1/4, so the unit equals 4 grays or 1 yellow.

This won’t always be the case. If 1 blue = 1/3, then the unit equals 3 blues.

No single piece covers 3 blues. Some children may express the unit as 3 blues; others as 1 yellow and 1 blue. Others will think their reasoning is wrong because there isn’t one piece to cover 3 blues.
## Teaching Actions

11. End the class with this problem:

I know that this blue piece is half of something. How can I find the value of these pieces?

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 red</td>
<td></td>
</tr>
<tr>
<td>1 gray</td>
<td></td>
</tr>
<tr>
<td>2 pinks</td>
<td></td>
</tr>
<tr>
<td>2 yellows</td>
<td></td>
</tr>
</tbody>
</table>

12. Accept suggestions. Then model a solution by reconstructing the unit (using their new skill). Show the unit. Explain that if the blue piece is 1/2, then red and yellow make the unit. The unit is 1 yellow:

![Unit Diagram](image)

Show 6 reds covering 1 yellow so 1 red is 1/6. Now ask students to do the other 3.

[1 gray – 1/4; 2 pinks = 2/3; 2 yellows = 2]

13. Repeat for this problem: blue = 1/3. Find the value of these pieces.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 red</td>
<td></td>
</tr>
<tr>
<td>1 gray</td>
<td></td>
</tr>
<tr>
<td>1 yellow</td>
<td></td>
</tr>
<tr>
<td>9 reds</td>
<td></td>
</tr>
</tbody>
</table>

[1 red = 1/9; 1 yellow = 2/3; 1 gray = 1/6; 9 reds = 1]


## Comments

6. To name the other pieces, student should find the unit and then compare the other pieces to that unit. **Don't try to solve this abstractly:**

- blue = 1/2
- red is 1/3 of blue, so 1/2 of 1/3 = 1/6.

7. This problem is challenging. Step back and let students problem solve. Ask them to explain their strategies and reasoning.
Problem Solving and Fraction Circles

I. Find the unit given the following information. Explain how you solved the problem. [You may want to draw pictures]

   a) The red piece is 1/4 of some amount. Find that amount. ________

   b) The gray piece is 1/6 of some amount. Find that amount. ________

   c) The green piece is 1/5 of some amount. Find that amount. ________

II. If the pink piece is 1/4 what value do these have? Explain your reasoning.

   a) 1 brown  b) 1 red  c) 1 white

Challenge: If the yellow piece is 2/3 what value does one gray piece have?
### Overview
Students name fractions greater than 1 with fraction circles. Students name fractions using both mixed numbers and improper fractions.

### Materials
- Fraction Circles for students and teacher
- Transparency 1
- Student Pages A, B, C

### Teaching Actions

1. Ask students to use their fraction circles with the black circles as the unit to show 2/2, 4/4, 5/5 and 12/12. In each example, ask for another name for the amount shown (1 whole or just 1).

2. Have students show 6/8 using the whole circle as the unit. Ask if 6/8 is greater or less than 1 whole or 1?

3. Present this story and ask students to model it with their circles. Again, use whole circles as the unit.

   Last night Margo ate 3/4 of a large pizza. (Show that with circles). In the morning she ate some leftover pizza that equaled 2/4 of a pizza.

4. Ask students to try and show the extra 2/4. They realize that they don’t have enough pieces. Have them work with a partner and use 2 sets of fractions circles to model the story.

5. Continue with the story:
   How much pizza did Margo eat altogether?

### Comments
1. Modeling fractions greater than one, using fraction circles, is easier than with chips, so we can concentrate on developing the concept of changing improper fractions and vice versa with fraction circles.

2. Accept both names: 1 and 1/4 or 5/4. Do not rush any rules about changing improper fractions to mixed fractions. Our goal is for students to change from one notation to another using circles and then just with mental images of circles. No paper/pencil rules.

   You may want to label 1 1/4 a “mixed fraction;” 5/4 an “improper fraction.” Ask students why these names “make sense.”
Teaching Actions

6. Questions to lead discussion for naming amount of pizza:

1) Did Margo eat more than 1 whole pizza? How do you know?
2) Let’s count how many fourths she ate: 1/4, 2/4, 3/4, 4/4, 5/4.
3) From the picture I see that 5/4 equals 4/4 and _________.
4) What’s another name for 4/4?
5) What’s another way of describing the amount of pizza Margo ate?


This shows 5/4; point to 3/4 and 2/4 more.

This shows 5/4; 1 whole and 1/4 of another circle.

8. Ask students to use their fraction circles to show these amounts. In each case, have children name the amount in another way.

8
6
3
4
3
12
1
3
1
4
2
3

Comments

3. Students may try to name the picture of two circles with 5 parts shaded as 5/8. If they do this, ask: what is the unit?

Since the unit is one pizza, each part is named by comparing it to one pizza or one circle.
Teaching Actions

9. As students explain their models, ask if the amount is greater or less than one. Try to get them to verbalize concrete actions that show when a fraction is greater than one.

10. Have students draw pictures for each fraction to show the two ways to name the fraction.

   Ex: \( \frac{8}{6} \)

11. To name fractions greater than 2, use pictures. Show transparency 1 and ask students to name each picture. [In each case, the unit is one circle or one rectangle.]

12. Assign Student Pages A, B, C.

Comments

4. We want students to verbalize that, for example, \( \frac{8}{6} \) is greater than 1 because they need more than 1 whole circle to model it. \( \frac{4}{5} \) is less than 1 because they needed only 1 unit to model it.

5. You might want to make an overhead of Student Pages A and B to facilitate the review of answers.
Write Two Fraction Names

(1) \[\text{\texttwofraction}

(2) \[\text{\texttwofraction}

(3) \[\text{\texttwofraction}

(4) \[\text{\texttwofraction}

(5) \[\texttwofraction

(6) \[\text{\texttwofraction}

(7) \[\text{\texttwofraction}

(8) \[\text{\texttwofraction}
Write two fraction names for each picture.

<table>
<thead>
<tr>
<th>Mixed Number</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Fraction Image 1]</td>
<td>![Improper Fraction Image 1]</td>
</tr>
<tr>
<td>![Fraction Image 2]</td>
<td>![Improper Fraction Image 2]</td>
</tr>
<tr>
<td>![Fraction Image 3]</td>
<td>![Improper Fraction Image 3]</td>
</tr>
<tr>
<td>![Fraction Image 4]</td>
<td>![Improper Fraction Image 4]</td>
</tr>
<tr>
<td>![Fraction Image 5]</td>
<td>![Improper Fraction Image 5]</td>
</tr>
<tr>
<td>![Fraction Image 6]</td>
<td>![Improper Fraction Image 6]</td>
</tr>
</tbody>
</table>
Shade in the pictures to show each fraction. Write another name for each amount.

<table>
<thead>
<tr>
<th>Mixed Number</th>
<th>Shade In</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\ \frac{1}{2}$</td>
<td>![Shaded Circles]</td>
<td></td>
</tr>
<tr>
<td>$2\ \frac{1}{4}$</td>
<td>![Shaded Rectangles]</td>
<td></td>
</tr>
<tr>
<td>$2\ \frac{4}{6}$</td>
<td>![Shaded Rectangles]</td>
<td></td>
</tr>
</tbody>
</table>
| | ![Shaded Triangles] | | \[
\frac{11}{8}
\]
| | ![Shaded Triangles] | | \[
\frac{10}{3}
\]
| | | | \[
\frac{4}{3}
\]
Draw a picture for each story. Write the fraction name.

1. Brenda ate \( \frac{2}{3} \) of a candy bar for lunch. She finished it after lunch and ate \( \frac{1}{3} \) more of a second candy bar of the same type. How much candy did Brenda eat?

2. Marcia’s dad was making pancakes. He added \( \frac{2}{3} \) cup milk to the pancake mix. He decided to make a bigger batch so he poured another \( \frac{2}{3} \) cup of milk in. How much milk did he use?

(Use \( \) as a picture of a cup)

3. The dress designer needed some yellow ribbon for 3 dresses. He needs \( \frac{2}{3} \) yard for one dress, \( \frac{1}{3} \) for another, and \( \frac{2}{3} \) for the third. Draw a picture to show how many yards of ribbon he bought.

(\( \) = 1 yard)
### Overview
Students look at the numerical relationship between the numerators and denominators of fractions equal to 1/2. They use this number pattern to determine if a given fraction is less than or equal to 1/2.

### Materials
- Fraction Circles for students and teacher
- Student Page A
- Student Pages A and B from Lesson 11

### Teaching Actions
1. Ask students to take out the fraction circles and find several equivalences for 1/2 (use the black circle as unit).
   
2. Record them on chart.
   
   Fractions equal to \( \frac{1}{2} \):
   
<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

3. Tell students that you can add to the list without using circles:
   
<table>
<thead>
<tr>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>50</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>

4. Tell students to look at the numerator and denominator of each fraction equal to 1/2 and ask them if they can see any pattern or relationship between numerator and denominator that's the same for each fraction.

### Comments
1. Students with a quantitative sense of fractions use 1/2 as a reference point to estimate fraction sums and differences.

   Ex: \( \frac{3}{6} + \frac{1}{3} \)

   “3/6 equals 1/2, and 1/3 is less than 1/2, so the sum is greater than 1/2 but less than 1.”

   Notice the role of fraction equivalence for 1/2 in estimation as well as in the same numerator but different denominator strategy [Lessons 6 & 7].

2. At this point we won’t look at examples like:

   \( \frac{1}{2^2} \)

   This will be done in Level 2.
Teaching Actions

5. Help students verbalize that in each case, the denominator is double (twice) the numerator.

6. Give students these fractions with parts missing and have them make them into fractions equal to 1/2:

\[
\begin{array}{cccc}
24 & 11 & \square & 100 \\
\end{array}
\]

7. Tell students to show these fractions with their circular pieces.

\[
\begin{array}{cccc}
\frac{1}{4} & \frac{2}{6} & \frac{3}{8} & \frac{4}{10} & \frac{5}{12} \\
\end{array}
\]

Ask if they are greater or less than 1/2. Have them tell you how far away from 1/2 each amount is.

8. Without using the pieces, ask them to tell you numerators that would make each fraction greater than 1/2.

9. Present these fractions to students. Ask them if they are >1/2, <1/2, or =1/2. Use fraction circles if needed. Have them verbalize their reasoning.

\[
\begin{array}{cccc}
\frac{3}{10} & \frac{5}{12} & \frac{4}{6} & \frac{6}{10} & \frac{9}{20} & \frac{15}{18} & \frac{1}{4} \\
\end{array}
\]

10. Student Page A provides practice. You may want to use Student Pages A and B from Lesson 11 again. Now have students see if they can solve problems using number patterns for 1/2.

11. Tell students that they will be using their understanding of fractions equal to 1/2 when they learn about fraction addition and subtraction.
Comparing to 1-half

1. Margo and Jose shared a couple of large pizzas. Margo ate 5/8 of a pizza. Jose ate 6/16 of a pizza. Who ate more? Explain how you know.

2. Imagine that you shared your bag of mini doughnuts with your sister. You ate 3/5 of the bag while your sister ate 4/10 of the bag. Who ate more? Explain how you know.


4. Circle the larger fraction in each pair.

   a) \(\frac{2}{3}\) \(\frac{1}{5}\)    b) \(\frac{9}{12}\) \(\frac{6}{15}\)    c) \(\frac{5}{9}\) \(\frac{3}{7}\)

   d) \(\frac{1}{2}\) \(\frac{3}{4}\)    e) \(\frac{3}{5}\) \(\frac{4}{9}\)    f) \(\frac{11}{17}\) \(\frac{3}{9}\)

   g) \(\frac{10}{22}\) \(\frac{4}{5}\)    f) \(\frac{3}{6}\) \(\frac{2}{9}\)    i) \(\frac{8}{13}\) \(\frac{6}{16}\)
## Overview
Students are introduced to fraction addition through familiar contexts and estimating reasonable answers (by comparing sum to 1/2 and 1).

## Materials
- Fraction Circles for students and teacher
- Student Page A

## Teaching Actions
1. Present this story to the students: William ate 1/4 of a pizza for dinner. The next morning he ate a piece that equaled 11/8 of the pizza. How much of a pizza did he eat?

2. Explain that you don’t want the exact answer, but just an estimate. Ask students to imagine 1/4 of a pizza and 1/8 of a pizza. Ask: did William eat more or less than 1/2 of a pizza? Ask students to explain their responses by referring to their mental images for 1/4 and 1/8.

3. Show with fraction circles 1/4+1/8:

4. Tell students that some people would say that 1/8+1/4 is 2/12. Ask: Does that make sense? If you ate 1/4 and then 1/8 of a pizza would that be the same as 2/12? Show with circles 1/4, 1/8, and 2/12 of the black circle.

## Comments
1. The time spent developing fraction concepts, ordering and equivalence ideas will enable students to approach fraction addition and subtraction in a meaningful way. Initial experience with +, - operations will be through estimation.

2. Estimation skills depend on students’ mental images for symbols as well as the context in which the operation is embedded.

3. Students’ explanation of estimation may sound like this:
   (a) He ate less than 1/2. You need two-fourths to be 1/2, and 1/8 is less than 1/4.
   (b) 1/4 of a pizza is like the blue piece. The gray is 1/8 and it is smaller than the blue. Together they won’t make 1/2.
### Teaching Actions

5. Repeat estimation process with the following story problems. In each case have students verbalize their reasoning. Point out when students use an ordering or equivalence idea previously learned.

   (a) Maria received a chocolate chip cookie as big as a birthday cake for a present. She cut it into 6ths and shared the cookie with her friend LeAnna. Maria ate 3/6 of the cookie, Leanna ate 1/3. Together, how much did they eat?

   (b) Martin was making playdough. He added 3/4 cup of flour to the bowl. Then he added another 3/6 cup. How much flour did he use? (in this case also ask if the sum is greater or less than one).

6. Provide added practice by estimating these sums. In each case, estimate as >1/2 or <1/2, and >1 or <1.

   (a) $\frac{1}{8} + \frac{1}{4}$
   (b) $\frac{3}{6} + \frac{1}{4}$
   (c) $\frac{3}{4} + \frac{2}{4}$
   (d) $\frac{4}{6} + \frac{1}{2}$

7. Student Page A provides practice. Assign in groups so students can share strategies for estimation. Provide opportunity for students to share in a large group.

### Comments

4. Familiarity with context helps students reason about appropriateness of answers.

5. Students are asked to write out their explanations for estimating sums.
Fraction Addition and Estimation

1. Marty divided a candy bar into 12 equal parts. He ate 1-sixths of the candy bar before lunch. He ate 1-fourth of the candy bar after lunch. Did he eat more or less than 1-half of the candy bar? Did he eat the whole candy bar? Explain your reasoning.

2. Terri ate $\frac{5}{6}$ of a small pizza and $\frac{11}{12}$ of another small pizza. Did she eat more than one whole pizza? Explain your reasoning.

3. Alex used $\frac{1}{3}$ cup of flour in one recipe and $\frac{1}{4}$ cup of flour in another recipe. Together did he use more than $\frac{1}{2}$ cup of flour? Explain your reasoning.

4. Give a reasonable estimate for each sum. On the back of this sheet write out your reasoning for each problem.

\[
\begin{array}{cccc}
\frac{1}{3} + \frac{2}{6} & \frac{1}{8} + \frac{9}{10} & \frac{7}{8} + \frac{1}{6} & \frac{1}{5} + \frac{3}{12} & \frac{1}{3} + \frac{3}{4}
\end{array}
\]
### Overview
Students use fraction circles to obtain exact answers to fraction addition.

### Materials
- Fraction Circles for students and teacher
- Student Pages A, B, C

### Teaching Actions
1. Present this story from Lesson 19.
   
   William ate 1/4 of a pizza for dinner. The next morning he ate a piece that equaled 1/8 of the pizza. How much of a pizza did he eat?

2. Remind students that they already estimated the sum to be less than 1/2. Explain that to find the exact sum, they are to use their fraction circles. Model the problem and ask students to do the same with their materials.

   ![Fraction Circles Example](image)
   
   Say: This blue represents 1/4 of the pizza. This gray represents 1/8 of the pizza.

3. Say that from this picture they can see that William ate less than 1/2 of the pizza. Ask: exactly how much pizza did William eat? Is there a way to express the amount of circle covered using one fraction name? For example, is 3/4 covered? 2/12?

### Comments
1. Addition of fractions with unlike denominators is introduced right from the start. Students are led to solve the problem of naming the amount of the circle covered using one fraction name.

2. At this point you are trying to develop an implicit understanding of the symbolic procedure.
### Teaching Actions

1. Ask students to try to find a way to cover 1/4 and 1/8 of the circle using only one color.

2. Give clues if needed: (1) Think about what you know about equivalent fractions; (2) How many grays equal one blue?

3. Model the problem, verbalizing each step.

   - "I can tell how much of the whole circle is covered if I can cover the amount with pieces of the same color."
   - "I know 2 grays cover the same amount as 1 blue, so I am going to exchange 1 blue for 2 grays."


5. Repeat with the chocolate chip cookie problem from Activity 20:

   Maria received a chocolate chip cookie as big as a birthday cake for a present. She cut it into 6ths and shared the cookie with her friend LeAnna. Maria ate 3/6 of the cookie, LeAnna ate 2/6. Together, how much did they eat?

6. Ask students if they have to exchange circle pieces in this example to determine the amount of circle covered.

### Comments

1. Same denominator means, concretely, using the same color pieces to show each fraction.

2. You may choose to record your actions with circles with symbols:
   
   - $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$
   
   Please don’t record until students can easily verbalize what was done concretely.

3. Important for children to internalize that sometimes you need to find equivalent fractions and sometimes this step is not needed.
Teaching Actions

10. Repeat for these problems. Encourage children to verbalize each step with materials. Have them attend to the decision of whether or not an exchange is needed. (Don’t forget to estimate!)

   (a) $1/2 + 1/4$
   (b) $2/4 + 1/8$
   (c) $2/3 + 1/3$
   (d) $2/3 + 2/3$

11. Student Pages A and B provide added practice. Student Page C provides challenges.

12. Have groups create their own story problem for addition of fractions to share with the class. You can collect them and present them to students as practice.

Comments

6. In all exercises so far only 1 of the fractions (or none) need to be changed. In Level 2 this work with addition of fractions is extended to where both fractions need to be renamed. [Student page C does not provide problems like those for students who need a challenge].

7. You will want to extend this lesson over 2-3 days.

8. Mastery of the addition of fractions is not a major goal of the Level 1 materials. You are providing experience that will extend in the Level 2 materials.
Fraction Addition and Estimation: Finding the Exact Answer

1. Marty ate some candy. He ate 1-half of a whole Hershey bar before lunch. He ate 1-fourth of a whole Hershey bar after lunch. About how much of one candy bar did he eat? With your fraction circles, find out the exact amount of a Hershey bar that Marty ate. Draw pictures to show what you did with the circles. Estimate first!!!

Estimate: _____________________

2. Terri ate 1-half of a small pizza and 5-twelfths of another small pizza. About how much of a whole pizza did she eat? With your fraction circles, find out the exact amount. Draw pictures to show what you did with the circles. Estimate first!!!

Estimate: _____________________

3. Allie rode her bicycle 7-eights of a mile to school. Then she rode 1-fourth of a mile to her friend’s house. About how far did she ride altogether? With your fraction circles, find out the exact amount. Draw pictures to show what you did with the circles. Estimate first!!!

Estimate: _____________________
Fraction Addition Continued

4. Because of a rainstorm, the water level in a swimming pool rose \( \frac{2}{3} \) of an inch. The following day it rained again. The pool rose another \( \frac{11}{12} \) of an inch. About how high did the water level increase? With your fraction circles, find out the exact amount.
Estimate ________________

5. Alex used \( \frac{1}{4} \) cup of flour in one recipe and \( \frac{3}{8} \) cup of flour in another recipe. Together about how much flour did he use? With your fraction circles, find out the exact amount.
Estimate ________________

6. With your fraction circles, find the exact answers.

\[
\begin{align*}
\frac{1}{3} + \frac{1}{6} & \quad \frac{1}{8} + \frac{3}{4} & \quad \frac{4}{10} + \frac{1}{5} & \quad \frac{1}{6} + \frac{3}{12} & \quad \frac{1}{2} + \frac{3}{4}
\end{align*}
\]
Challenges

1. Shade parts A, B and C

   a) How much are parts A and B together?

   b) How much of the circle are Parts A and C together?

   c) How much of the circle are Parts B and C together?

2. Describe how you solved the problems.

3. Estimate a sum for each problem. Then find the exact answer using your fraction circles. Explain how these problems are different from the others you solved.

\[
\begin{align*}
\frac{1}{4} + \frac{1}{3} & \quad \frac{1}{2} + \frac{2}{5} & \quad \frac{2}{4} + \frac{5}{6} & \quad \frac{1}{2} + \frac{5}{6}
\end{align*}
\]
## Overview

Students explore fraction subtraction within take away and difference contexts. Students estimate and solve story problems with fraction circles.

## Materials

- Fraction Circles for students and teacher
- Student Page A, B and C

## Teaching Actions

1. Consider the development of fraction subtraction as a problem solving activity. Let students explore, in groups, using the circle pieces and creating their own solution strategies.

2. Introduce this take away problem. Ask students to act out the problem in their groups. Explain that you will be soliciting, at random, someone to explain their group’s work to the whole class.

   Alice noticed that there was \( \frac{3}{4} \) of a pizza left after the party. She ate a slice of pizza that was the size of \( \frac{1}{8} \) of a whole pizza. How much pizza was left after Alice ate a slice?

3. Before group work, ask for estimates. Will there be more or less than \( \frac{1}{2} \) pizza left? Try picturing the \( \frac{3}{4} \) pizza in your mind. Does this help in your estimate? Explain your thinking. (Record estimates on the board).

## Comments

1. Developing subtraction of fractions from contexts is more difficult for fraction addition. There is more than one context for subtraction: take away, finding the difference, how many more.

2. You should expect students to solve each story problem type at the concrete level differently. Students will match their actions with the fraction circles with the action from the story problem.

At Level 1 you should be only interested in concrete solutions to subtraction problems and estimation of reasonable answers.
### Teaching Actions

4. Have students try to act out the problem. As you monitor their work, consider these hints:

   a) Can you show 3/4 of a pizza with the circles?
   b) Is 1/8 > or < than 1/4?
   c) What is the action in the story?
   d) Can you take away 1/8 if the model is shown with 3 blues?
   e) Can you use your ideas of equivalence to help you?

5. Randomly select a student to explain how his/her group attempted to act out the problem. Deemphasize an exact answer, but help students verbalize their group’s reasoning and discussion.

6. Repeat the same story problem with these new numbers:

   - 1/2, 1/8
   - 1, 3/4
   - 2/3, 1/6

7. End the lesson with a story problem that represents a difference model. Ask students to act out this story problem:

   Joe & Renata each receive the same allowance. Joe spent 2/3 of his allowance on records. Renata spent 1/6 of her allowance repairing her bicycle. How much more did Joe spend than Renata?

   Ask: Is it easier to compare amounts when they are shown with the same colored pieces?

### Comments

3. In the take-away model, students may build the first fraction and take away the second.

   ![Take away diagram]

4. In the difference model, students may construct 2 separate models for the 2 fractions in the story and compare models.

   ![Difference model diagrams]

   For example:

   - Joe’s amount
   - Renata’s amount

   Change to pink:

   ![Change to pink diagram]

   Joe spent 3/6 more.

5. Some may still use the take-away model.
Teaching Actions

8. Estimate answer. Will the difference be greater or less than 1/2? Why?

9. As you monitor their group work consider these hints:
   a) Is the action in this story different than in the pizza problem?
   b) Can you show Joe’s amount? Renata’s amount?
   c) Can you use your ideas of equivalence?

10. Have students share solution strategies. Talk about differences in actions in the two problem types.

11. Repeat the same story with these numbers:

   \[
   \begin{align*}
   &\frac{1}{2} \quad \frac{1}{4} \\
   &\frac{2}{3} \quad \frac{2}{3} \\
   &\frac{6}{8} \quad \frac{1}{2}
   \end{align*}
   \]

12. Student Pages A and B provide practice with story problems. Spend time in class having students explain their strategies for solving these problems.

   Student Page C provides some challenges

Comments

6. Again, you will want to use this lesson plan over 2 – 3 days. You may want to introduce the take-away model one day & the difference model the next day.

7. What you want is for students to be able to act out the problems with fraction circles, talking about what they are doing.
Fraction Operations: Finding the Exact Answer

1. Joe lives $\frac{4}{10}$ a mile from school. Mary lives $\frac{1}{5}$ of a mile away. How much farther from school does Joe live than Mary? Draw pictures to show what you did with the circles.
Estimate: _____________

2. Because of a rainstorm the water level in a swimming pool rose by $\frac{9}{12}$”. The following day it dropped by $\frac{4}{6}$”. What was the total change in water level? Draw pictures to show what you did with the circles.
Estimate: _____________

3. Velicia spent $\frac{1}{3}$ of her allowance on a CD and $\frac{4}{6}$ of her allowance on movie. What fraction of her allowance did she have left? Draw pictures to show what you did with the circles.
Estimate: _____________
Fraction Operations Continued

1. A clerk sold three pieces of ribbon. The red piece was $\frac{1}{3}$ of a yard long. The blue piece was $\frac{1}{6}$ of a yard long. The green piece was $\frac{10}{12}$ of a yard long.

   a) How much longer was the green ribbon than the red ribbon?

   b) How much longer was the green ribbon than the blue ribbon?

   c) Are the red ribbon and blue together greater than, less than, or equal in length to the green ribbon?

   d) If the red and blue together are greater than the green, how much greater are they? If shorter, how much shorter are they?

5. With your fraction circles, find the exact answers

\[
\frac{4}{5} - \frac{3}{10} \quad \frac{1}{4} + \frac{5}{8} \quad \frac{7}{8} - \frac{1}{3} \quad \frac{1}{2} + \frac{3}{8} \quad \frac{3}{5} - \frac{2}{5}
\]
Challenges

1. Shade parts A, B and C

   a) How much larger is part A than part B?
   b) How much larger is part A than part C?
   c) How much larger is part C than part B?
   d) How much are parts A and C together?
   e) How much greater are parts A and C together than part B?
   f) Are parts C and B together greater than A? If so, how much greater? If not, how much smaller?

2. Describe how you solved the problems.

3. Circle the larger fraction in each pair. Then find out how much larger that fraction is.

   \[
   \begin{array}{ccc}
   \frac{1}{4} & \frac{1}{3} \\
   \frac{1}{2} & \frac{2}{5} \\
   \frac{2}{4} & \frac{5}{6} \\
   \frac{1}{2} & \frac{5}{6}
   \end{array}
   \]
# Overview
Students explore fraction subtraction within difference and how many more contexts. Students estimate and solve story problems with fraction circles.

# Materials
- Fraction Circles for students and teacher

# Teaching Actions

1. Continue with the difference story problems. Students act out the problem in a group with fraction circles.

   Joe and Renata each receive the same allowance. Joe spent 1/2 on records. Renata spent 1/8 on repairing her bicycle. How much more did Joe spend?

2. Ask for estimates. Is it reasonable to expect the answer to be less than 1/2? Greater or less than 1/4?

3. As you monitor their group work consider these hints:
   a) How can you show Joe and Renata’s amounts with the fraction circles?
   b) Is 1/8 greater or less than 1/2?
   c) How many 8ths equal 1/2?

4. Have student share solutions again, emphasizing the group’s thinking rather than the exact answer.

# Comments
1. You may want to spend more than one day on this lesson.
### Teaching Actions

5. Present these two problems. Ask students to act out both problems and explain how they are different.

   a) José ate 3/4 of a pizza. Mara ate 5/8. Who ate more and how much more?

   b) A movie costs Josie 1/2 of her allowance. If she only had 3/4 of her allowance left before the move, how much of her allowance did she have left after the movie?

6. Present this “how many more” problem and ask students to act it out.

Marcia agreed to mow about 1/2 of the lawn before dinner. At 3:00 she finished about 3/8 of the lawn. How much of the job does she have left?

7. As you monitor group work, consider these questions:

   a) Can you show how much she completed by 3:00?

   b) Even if you can reason the answer in your head, can you show how to do it with circles?

   c) If she has completed 3/8, how much more to 1/2?

8. Share responses and actions on circles. Discuss how this problem is different from previous ones.

9. Talk about how all 3 problem-types are subtraction. Pick any 3 examples and substitute whole numbers in for fractions. Ask students to solve with whole numbers. Note that in each case they wrote a subtraction problem. Now write the fraction problems symbolically. Explain that they first solve these problems with their fraction circles. They will learn how to solve them symbolically in the next level.

### Comments

1. The movie problem is a “take away” problem. The pizza problem models subtraction as “finding a difference”.

2. Expect students to act out this story differently from the others. Students may start with 3/8 (amount completed).

   ![Fraction Circle](image)

   They will try to fill in the space left to make 1/2 (1/8 more).

3. Even though actions in story problems are different, they are all subtraction problems. Symbolically, all will use “-” (minus sign). It’s important for students to see that these are subtraction contexts.
Teaching Actions

10. In groups ask students to create a story problem for each subtraction category and one addition problem. Ask them to solve the problems using their fraction circles, recording steps symbolically if you think they are ready to do so.

Then have students exchange their set of problems with another group.

Spend the time in large group for sharing these stories and the strategies for solving them.

11. Provide symbolic problems for students to act out with the fraction circles. They can use any of the three ways shown. Many might choose the take-away model.

Examples:

\[
\begin{align*}
2/3 - 1/3 \\
4/5 - 1/10 \\
1/2 - 3/8 \\
7/12 - 1/2 \\
6/9 - 1/3
\end{align*}
\]

Comments

4. You decide whether students are ready to record their work symbolically.

Their symbol records may not look like “standard algorithm,” but reflect each step they act out with the fraction symbols.

For example, for the lawn problem, the students might record as follows:

\[
\begin{align*}
\frac{3}{8} \\
\frac{1}{2} = \frac{4}{8} \\
\frac{4}{8} - \frac{3}{8} = \frac{1}{8}
\end{align*}
\]
# Overview

This lesson helps to highlight for students how much they know about fractions: meaning for symbols as well as how to add and subtract them in a meaningful way.

## Materials

- Fraction Circles for students
- Student Page A

## Teaching Actions

1. Show students this problem and ask them to write a paragraph explaining how they could estimate a reasonable answer.

   Marge ran 3/4 mile and stopped to catch her breath. She then ran another 1/8 mile. Did Marge run more or less than one mile? Did Marge run 4/12 mile?

   Have students share their paragraph with others in their group and then select a sample for large group sharing.

2. Have students act out the problem with fraction circles.

3. Repeat for this subtraction story.

   Allie ran 2/3 of a mile. Mark ran 5/12 of a mile. Who ran the farthest? How much further?

   (Don’t be surprised if students can mentally find the exact answer).

## Comments

1. The estimation problems here allow you to assess to what extent students’ mental images of fractions help them reason about adding and subtracting fractions.

   First problem looks at students’ understanding of fractions >1/2 as well as comparing fractions with the same numerator (1/4 vs. 1/8).

   Possible student response: “It has to be greater than 1/2 but less than 1. To make one whole you’d need to add 1/4 more. But 1/8<1/4 so it’s not 1 mile.”

   “4/12 doesn’t make sense since 4/12<1/2.”
Teaching Actions

4. In groups have students estimate a reasonable answer to each problem on Student Page A. They can show their estimates by marking the interval on the numberline with an “x”. Encourage students to talk about their strategies for estimation.

\[ \frac{2}{3} + \frac{1}{20} \]

“2-thirds is greater than 1/2. 1-twentieth is a very small fraction (much smaller than 1/3). Therefore the sum has to be greater than 1/2 but less than one.”

5. End class by sharing thinking strategies. Congratulate children for learning so much about fractions. Point out to them examples of their quantitative thinking that shows just how much they know about these new numbers.

Comments

2. Students are not to find the exact location on the numberline. The numberline is being used only as a way to record a reasonable range for the problem.

You may want to use a multiple choice format:

a) between 0 and 1/2
b) between 1/2 and 1

3. The problems may be challenging, but use them to assess the strength of students’ mental representations for fraction symbols.
ESTIMATING FRACTION SUMS AND DIFFERENCES

1) $\frac{2}{3} + \frac{1}{20}$

2) $\frac{2}{3} - \frac{1}{10}$

3) $\frac{9}{10} - \frac{8}{9}$

4) $\frac{5}{6} + \frac{1}{15}$

5) $\frac{1}{3} + \frac{1}{100}$

6) $\frac{4}{5} - \frac{3}{4}$
Appendix
Pictures of the Fraction Circles

- **Black**: 2 Pieces
- **Yellow**: 3 pieces
- **Brown**: 4 pieces
- **Blue**: 5 pieces
- **Orange**: 6 pieces
- **Pink**: 7 pieces
- **Light Blue**: 8 pieces
- **Gray**: 9 pieces
- **White**: 10 pieces
- **Purple**: 12 pieces
- **Red**:15 pieces
Explorations With the Fraction Circles

1. How many different colors are in your folder? _____________

2. Which color has the most pieces? _____________

3. Which color has the fewest pieces? _____________

4. How many light blue pieces are there? _____________

5. How many dark blue pieces are there? _____________

6. How many dark blue pieces does it take to cover a yellow piece? _____________

7. Can you find two different pieces to cover a yellow piece? Which colors did you use? __________________________

8. Cover the yellow piece using smaller pieces of one color.
   What color did you use? _____________
   How many did you use? _____________

9. Is a pink piece larger, equal to or smaller than two red pieces? __________________________

10. Cover a brown piece by using several smaller pieces.
    Which colors did you use? _____________
    How many did you use? _____________
YELLOW
BLUE
ORANGE
LIGHT BLUE
Fraction Assessment

The purpose of this test is to find out what you know about fractions. You will be shown 3 problems on the overhead. Estimate the answer by recording in each box the whole number the answer is closest to.

1. 

2. 

3. 

Rational Number Project - Level One
Continue to work on the rest of the test on your own.
Show all the work you do to solve each problem.

4. Bert’s father cuts a cake into 8 pieces. He is going to take 3-fourths of the cake to the party. How many pieces of cake will he take with him?

   A. Draw a picture below to solve the problem.

   B. The number of pieces of cake taken to the party is:

5. \[ \text{is the unit. You have two pieces this size. What fraction is this?} \]

6. Draw in the box a set of circles 3-fourths as many circles as you see here:

7. \[ \text{What fraction of the circle is part c?} \]
8. How many fifths are shaded? 

9. Circle \( \frac{2}{3} \)

10. ________ is 3-fourths of some length. 

   Draw the whole length below. 

   Explain why it is the whole.

For problems 11-13, give two fraction names for the shaded amount.

11. 

12. 

13. 

Rational Number Project - Level One
For problems 14-18, circle the larger fraction. If equal, circle both. Explain your reasoning for each.

14.

\[
\frac{3}{4} \quad \frac{2}{3}
\]

15.

\[
\frac{1}{2} \quad \frac{5}{8}
\]

16.

\[
\frac{3}{12} \quad \frac{7}{12}
\]

17.

\[
\frac{4}{9} \quad \frac{4}{11}
\]

18.

\[
\frac{8}{14} \quad \frac{4}{9}
\]
You may use your fraction circles on the last five problems. Draw pictures to show what you did with the circles.

19. Liana ate $\frac{3}{8}$ of a small pizza. The next day she ate $\frac{1}{4}$ of a small pizza. How much of a pizza did she eat altogether?

20. Ann and Josie received the same allowance. Josie spent $4$-ninths of hers on CDs. Ann spent $1$-third of her allowance repairing her bicycle. Josie spent how much more of her allowance than Ann?
21. Show Pictures Here

\[
\frac{3}{4} + \frac{2}{3}
\]

22.

\[
\frac{1}{2} - \frac{3}{8}
\]

23.

\[
\frac{1}{3} - \frac{2}{6}
\]
1) \[ \frac{7}{8} + \frac{12}{13} = \]

2) \[ \frac{3}{8} + \frac{5}{12} = \]

3) \[ \frac{8}{9} - \frac{7}{8} = \]
Interview #1:
[Used after Lesson 10 or 11]

I am going to ask you some questions about fractions. I am very interested in how you come up with the answers, so it is important for you to tell me what you are thinking about. The interview will not be graded, so you do not have to worry about wrong answers. Are you ready?

Concept Questions

1. 
   (A) Use your fraction circles to show a model for the fraction 3/5.

   (B) Now I want you to explain how you know that this models 3/5.

   (C.) Now can you show me another way to model 3/5 using the same fraction circles? Explain how the two models are alike and different.
Interview #1 continued

2. Display 15 tiles without counting or telling the child how many there are. [This is an extension. Tiles have not been introduced as a model yet].

   (A) Say: You can arrange the tiles any way you want. I want you to show me the fraction 3/5 with these tiles.

   (B) Explain what you were thinking in order to solve this problem.

   (C) Show me a model for 3/5 using a different number of tiles. How are the two tile-models alike? How are they different?

   (D) How is this model for 3/5 like your fraction circle models? How are they different?

Order Questions

[same numerator]

3. Say: I’m going to show you word names for two fractions. I’ll ask you to tell me whether they are equal or one is less.

   Ready? 1-fifth and 1-sixth. Are they equal or is one less? Which is less? Explain your reasoning. [ex: Do you imagine or picture something in your mind to help you tell which is less?]
Interview #1 continued

[Same Denominator]
4. Say: I’m going to show you word names for two new fractions. I’ll ask you to tell me whether they are equal or if one is less.

   Ready? 3-ninths and 4-ninths. Are they equal or is one less? Which is less? Explain your reasoning. [ex: Do you imagine or picture something in your mind to help you tell which is less?]

[Transitive]
5. Read this story to the student:

   Jon and Lara each ordered a small pizza at Domino’s. Jon’s pizza was cut into 8 equal-sized parts. Lara’s was cut into 6 equal-sized parts. Jon ate 5 pieces; Lara ate 2 pieces. Did they eat the same amount, or did one eat less?

   Explain your reasoning. [ex: Do you imagine or picture something in your mind to help you tell which is less?]
Interview #1 continued

[residual]

6. Read this story to the student:

Mark and William both had bags of M&M peanut candies. The bags held the same number of candies. Mark ate 2-thirds of his bag. William ate 3-fourths of his bag. Did they eat the same amount or did one eat less?

Explain your reasoning. [ex: Do you imagine or picture something in your mind to help you tell which is less?]

Concept of Unit Questions

7. Show and read the statement:

Tim used 6 cubes to build 1/5 of an orange tower. How tall will the whole tower be when it's finished?

(A) Provide unifix cubes and ask students to show you how to use these cubes to solve the problem. Ask students to talk aloud as they solve the problem.

(B) [If correct repeat changing the data: 8 cubes; 2/3]
Interview #2
Use after Lesson 15 or 16

I am going to ask you some questions about fractions. I am very interested in how you come up with the answers, so it is important for you to tell me what you are thinking about. The interview will not be graded, so you do not have to worry about wrong answers. Are you ready?

Concept Questions

1. Display 15 tiles without counting or telling the child how many there are.

   (A) Say: You can arrange the tiles any way you want. I want you to show me the fraction 2/3 with these tiles.

   (B) Explain what you were thinking in order to solve this problem.

   (C) Show me a model for 2/3 using a different number of tiles. How are the two tile-models alike? How are they different?

2. Read this story to the student:

   Mary and Jose both have some money to spend. Mary spends 1/4 of hers and Jose spends 1/4 of his. Is it possible that Mary and Jose spent the same amount of money? Tell me what you are thinking.
Interview #2 continued

Order Questions

[Same numerator]
3. Say: Here are two fractions.

Show: $\frac{2}{5}$ $\frac{2}{3}$

Say: Are they equal or is one less? Which one is less? Tell me how you know. Did you picture anything in your mind as you thought about these fractions?

[Same numerator]
4. Say: Here are two fractions.

Show: $\frac{1}{17}$ $\frac{1}{19}$

Say: Are they equal or is one less? Which one is less? Tell me how you know. Did you picture anything in your mind as you thought about these fractions?

[Same denominator]
5. Say: Here are two fractions.

Show: $\frac{14}{26}$ $\frac{18}{26}$

Say: Are they equal or is one less? Which one is less? Tell me how you know. Did you picture anything in your mind as you thought about these fractions?
Interview #2 continued

[Residual]
6. Read this story to the student:

Alice and Janis both receive the same allowance. Alice spent \( \frac{4}{5} \) of hers on a movie. Janis spent \( \frac{9}{10} \) of hers on a new CD. Did they spend the same amount or did one spend less? [if less ask: who spent less?]

Say: Tell me how you know. Did you picture anything in your mind as you thought about these fractions?

[Transitive]
7. Read this story to the student:

Mark and Jenny walk home from school. Mark walks \( \frac{3}{7} \) of a mile. Jenny walks \( \frac{6}{9} \) of a mile. Do they walk the same amount or does one walk less?

Say: Tell me how you know. Did you picture anything in your mind as you thought about these fractions?
Interview #2 continued

Concept of Unit Questions

8. Show the red fraction piece.

Say: This is 1/6 of my unit. With your fraction circles, show me the unit. Talk aloud as you solve the problem explaining each step. Record answer.

[If correct change data: pink is 2/3; find the unit].

9. Read this story to the student:

Ten children went to a party in a group. This group of 10 children was 2/5 of all the children who were invited. How many children were invited?

Provide tiles and ask the student to show you how to use these tiles to solve the problem. Ask students to talk aloud as they solve the problem. Record answer.
Interview #3
Use before Lesson 19

I am going to ask you some questions about fractions. I am very interested in how you come up with the answers, so it is important for you to tell me what you are thinking about. The interview will not be graded, so you do not have to worry about wrong answers. In fact, you have not worked on the type of problems I will ask about. I am interested in how you try to solve them before you learn from your teacher how to do them.

1. Read this story to the student:

_Sally ate \(\frac{2}{3}\) of a pizza for dinner. The next morning she ate another \(\frac{1}{6}\) of a pizza. How much of a pizza did she eat altogether?_

(A) Say: Without working out the exact answer, give me an estimate that is reasonable. (If needed, provide clues: Is the answer \(>\frac{1}{2}\) or \(<\frac{1}{2}\)? Is the answer \(>1\) or \(<1\)?

(B) Say: Tell me what you were thinking to reach this estimate.

(C) Say: Using fraction circles, act out how you would find the exact answer. Talk aloud as you solve the problem.
Interview #3 continued

(D) If the student was successful, ask student to record each step with the fraction circles with symbols.

2. Read this story to the student:

A movie costs Jane $\frac{1}{6}$ of her allowance. If she only had $\frac{11}{12}$ of her allowance before the movie, what fraction of her allowance did she have after the movie?

(A) Say: Without working out the exact answer, give me an estimate that is reasonable. (If needed, provide clues: Is the answer $>\frac{1}{2}$ or $<\frac{1}{2}$? Is the answer $>1$ or $<1$?).

(B) Say: Tell me what you were thinking to reach this estimate.

(C) Say: Using fraction circles, act out how you would find the exact answer. Talk aloud as you solve the problem.

(D) If the student was successful, ask student to record each step with the fraction circles with symbols.
Interview #3 continued

3. Read this story to the student:

Josie and Al each receive the same allowance. Al spent 6/10 of his allowance on a book. Josie spent 4/5 of her allowance to fix her skateboard. How much more did Josie spend?

(A) Say: Without working out the exact answer, give me an estimate that is reasonable. (If needed, provide clues: Is the answer >1/2 or <1/2? Is the answer >1 or <1?).

(B) Say: Tell me what you were thinking to reach this estimate.

(C) Say: Using fraction circles, act out how you would find the exact answer. Talk aloud as you solve the problem.

(D) If the student was successful, ask student to record each step with the fraction circles with symbols.
Interview #3 continued

4. (A) Say: Tell me about where the answer to this addition problem would be on this numberline.

\[
\frac{3}{4} + \frac{1}{3}
\]

(B) Say: Tell me how you know.

5. (A) Say: Tell me about where the answer to this addition problem would be on this numberline.

\[
\frac{7}{8} - \frac{1}{2}
\]

(B) Say: Tell me how you know.

6. Say: Jon calculated the problem as follows: \( \frac{2}{3} + \frac{1}{4} = \frac{3}{7} \)

Ask: Do you agree? Tell me all you can.
Interview #4: End of Unit

I am going to ask you some questions about fractions. I am very interested in how you come up with the answers, so it is important for you to tell me what you are thinking about. The interview will not be graded, so you do not have to worry about wrong answers. Are you ready?

Concept Questions

1. Display a bag of chips without counting or telling the child how many there are.

   (A) Say: I want you to show me $\frac{2}{3}$ using the chips as your model. Use as many chips as you want.

   (B) Explain what you were thinking in order to solve this problem.

   (C) Can you do this in another way? How are your two alike and different?

2. Read this to the student:

   Martin ate $1 \frac{2}{3}$ pizza.

   (A) Name the amount Martin ate in another way.
Interview #4 continued

(B) Draw a picture to verify your answer. Explain what the picture shows.

Order Questions

[Same numerator]
3. Say: I am going to write 3 fractions

Write: $\frac{1}{5}$  $\frac{1}{3}$  $\frac{1}{4}$

(A) Say: I want you to write them in order from smallest to largest.

(B) Say: Tell me how you know.

[Same numerator]
4. Say: I'm going to write two fractions.

Write: $\frac{4}{35}$  $\frac{4}{29}$

(A) Say: Are they equal or is one less? Which one is less?

(B) Say: Tell me how you know.
Interview #4 continued

[Transitive]
5. Say: I’m going to write two fractions.

Write: 8/12   3/7

(A) Say: Are they equal or is one less? Which one is less?

(B) Say: Tell me how you know.

[Same Denominator]
6. Say: I’m going to write two fractions.

Write: 27/64   19/64

(A) Say: Are they equal or is one less? Which one is less?

(B) Say: Tell me how you know.
Interview #4 continued

[Residual]
7. Say: I’m going to write two fractions.
   Write: 4/5  9/10
   (A) Say: Are they equal or is one less? Which one is less?
   (B) Say: Tell me how you know.

[Equivalence Questions]
8. Say: I’m going to write two fractions.
   Write: 3/4  9/12
   (A) Say: Are these fractions equal or is one less? Which one is less?
   (B) Say: Tell me how you know.
Interview #4 continued

[Equivalence Questions]
9. Say: I’m going to write two fractions.

   Write: 6/9  4/6

   (A) Say: Are they equal or is one less? Which one is less?

   (B) Say: Tell me how you know.

Concept of Unit Questions

10. Show and read the statement:

   Tina was building towers with cubes. Tina finished building 1 1/3 towers. She used 12 cubes for these towers. How many cubes is one tower?

   Ask student to solve the problem. Provide unifix cubes. Ask students to talk aloud as they solve the problem.

11. (A) Say: This brown piece is 4/6 of some unit. What is the unit? Use the fraction circles to show me. Talk aloud as you do this.

   (B) If correct ask this: This yellow piece is 1 and 1/2 of some unit. What is the unit? Use the fraction circles to show me. Talk aloud as you do this.
Marty was making two types of cookies. He used 1/4 cup of flour for one recipe and 2/3 cup of flour for the other. How much flour did he use altogether?

(A) Say: Without working out the exact answer, give me an estimate that is reasonable. (If needed, provide clues: Is the answer >1/2 or <1/2? Is the answer >1 or <1?).

(B) Say: Tell me what you were thinking to reach this estimate.

(C) Say: Using fraction circles, act out how you would find the exact answer. Talk aloud as you solve the problem.

(D) If the student was successful, ask student to record each step with the fraction circles with symbols.
13. Read this story to the student:

Martin and Jane each ordered small pizzas at Dominos. Jane ate $\frac{5}{8}$ of her small pizza. Martin ate $\frac{3}{4}$ of his small pizza. Who ate more? How much more?

(A) Say: Without working out the exact answer, give me an estimate that is reasonable. (If needed, provide clues: is the answer $>\frac{1}{2}$ or $<\frac{1}{2}$? Is the answer $>1$ or $<1$?)?

(B) Say: Tell me what you were thinking to reach this estimate?

(C) Say: Using fraction circles, (or paper folding) act out how you would find the exact answer. Talk aloud as you solve the problem.

(D) If the student was successful, ask the student to record each step with the fraction circles with symbols.

14. (A) Say: Tell me about where the answer to this problem would be on this numberline.

2/3 + 1/6

(B) Say: Tell me how you know.
Interview #4 continued

(C) Say: Using paper and pencil, how can you figure out the exact answer? Explain what you are doing.

15. (A) Say: Tell me about where the answer to this problem would be on this numberline.

\[
\frac{8}{9} - \frac{1}{3}
\]

(B) Say: Tell me how you know.

(C) Say: Using paper and pencil, how can you figure out the exact answer? Explain what you are doing.