

Rational Number Project

Fraction Operations and Initial Decimal Ideas Lesson 1: Overview	Materials
Students review how to model fractions with fraction circles by ordering unit fractions, using 1-half as a benchmark to order two fractions, and comparing fractions close to one.	<ul style="list-style-type: none">• Fraction Circles for students and teacher• Student Pages A, B, C, D

Teaching Actions

Warm Up

Order these fractions from smallest to largest. Be prepared to explain your thinking.

$$\frac{3}{4} \quad \frac{1}{10} \quad \frac{14}{15} \quad \frac{3}{5} \quad \frac{5}{12}$$

Large Group Introduction

1. Start the lesson by asking student to share their responses to the warm up task. Record responses on the board and have students defend their answers. Encourage students to describe the mental images used to sort these fractions.
2. Questions to guide discussion:
 - Which is bigger $\frac{3}{4}$ or $\frac{3}{5}$?
 - Which fraction is just under $\frac{1}{2}$?
 - Which fractions are just greater than $\frac{1}{2}$?

Comments

Your goal is to reinforce students' informal order strategies and not to use common denominator rules.

Here is an example:

$$\frac{1}{10}, \frac{5}{12}, \frac{3}{5}, \frac{3}{4}, \frac{14}{15}$$

$\frac{1}{10}$ is farthest from the whole. Five isn't half of 12 yet. So it is under .

Then $\frac{3}{5}$. Then $\frac{3}{4}$ and $\frac{14}{15}$. Both are missing one piece but a fourth is a lot bigger than a fifteenth so it is missing a bigger piece.

Teaching Actions

- Which fractions are close to one whole?
 - Why is $\frac{1}{10}$ the smallest?
3. Spend some time ordering $\frac{3}{4}$ and $\frac{14}{15}$.
- Which fraction is closer to 1? How do you know?
 - One student suggested the two fractions are equal because the both are one piece away from the whole. What do you think?
 - Verify your conclusion by modeling each fraction with the fraction circles.
4. Test students understanding by asking them to order $\frac{4}{5}$ and $\frac{8}{9}$. Which is closer to a whole? Verify students' responses by showing both fractions with fraction circles.
5. Summarize the main ideas from this lesson introduction:
- You can judge the relative size of fractions by thinking about fraction circles.
 - Using $\frac{1}{2}$ as a benchmark is helpful.
 - Thinking about how close a fraction is to one whole is also helpful when comparing fractions like $\frac{3}{4}$ and $\frac{5}{6}$ or $\frac{4}{5}$ and $\frac{99}{100}$.

Comments

Expect some students to struggle with $\frac{3}{4}$ and $\frac{14}{15}$. Misunderstandings to expect are:

- Equal, as both are one away from the whole.
- $\frac{3}{4}$ is larger because the denominator is a lower number so it is going to have bigger pieces.

Behind this strategy is the ability to mentally represent both fractions using images of fraction circles. Here is an example of a 6th grader's use of mental images to order $\frac{8}{9}$ and $\frac{4}{5}$.

$\frac{8}{9}$ is larger because if you get the fraction circles out, $\frac{1}{5}$ is bigger than $\frac{1}{9}$.

So if you put $\frac{8}{9}$ and $\frac{4}{5}$, $\frac{8}{9}$ would be bigger because it would have smaller pieces so there is going to be a small amount left and it's going to be a bigger piece left for $\frac{4}{5}$.

Notice how this student uses his ability to order two unit fractions ($\frac{1}{5}$ and $\frac{1}{9}$) to answer this.

Small Group/Partner Work

Teaching Actions

6. Student pages A-D reinforce comparing fractions to $\frac{1}{2}$, comparing fractions to unit fractions, and comparing fractions close to 1.

7. Present this problem: What do you think of this student's reasoning?

$\frac{6}{8}$ is greater than $\frac{4}{11}$ because with $\frac{6}{8}$ you need 2 to get to the whole and with $\frac{4}{11}$ you need 7.

8. Does this student's strategy work for $\frac{9}{10}$ and $\frac{98}{100}$?

9. To assess their understanding of using $\frac{1}{2}$ as a benchmark ask this question:

- Juanita said that $\frac{7}{12} > \frac{1}{2}$ because she knew that $\frac{6}{12}$ equals $\frac{1}{2}$ and $\frac{7}{12}$ is more than $\frac{6}{12}$.
- Will said he knew $\frac{7}{12} > \frac{1}{2}$ because he doubled the numerator and saw that it was greater than the denominator. That made $\frac{7}{12}$ bigger than $\frac{1}{2}$.
- I understand Juanita's strategy but I don't understand what Will meant. Can you help me? Can you use fraction circles to convince me his strategy is right?

Translations:

- Symbolic to Manipulative to Verbal;
- Symbolic to Verbal

Comments

This is a common error in student thinking. While the student gets a correct answer, her reasoning will not generalize to all fraction pairs. With fractions you have to consider the relative size of the amount away from one whole.

Possible explanations are as follows:

- Doubling the numerator gives you the size of the denominator if the fraction was equal to $\frac{1}{2}$.
- If the numerator is 7 then $\frac{7}{14} = \frac{1}{2}$. Because 12ths are bigger than 14ths, $\frac{7}{12} > \frac{7}{14}$. From this comparison $\frac{7}{12} > \frac{1}{2}$.

Additional Notes to the Teacher

Lesson 1

The RNP level 1 lessons support students' development of informal ordering strategies. Four informal ordering strategies have been identified: same numerator, same denominator, transitive, and residual. These strategies are not symbolic ones, but strategies based on students' mental representations for fractions. These mental representations are closely tied to the fraction circle model.

Same denominator: When comparing $\frac{4}{5}$ and $\frac{3}{5}$ students can conclude that $\frac{4}{5}$ is larger because when comparing parts of a whole that are the same size (in this case 5^{ths}) then 4 of those parts are bigger than 3 of them.

Same numerator: When comparing $\frac{4}{5}$ and $\frac{4}{6}$, students can conclude that $\frac{4}{5}$ is bigger because 5ths are larger than 6ths and four of the larger pieces will be bigger than 4 of the smaller pieces. Students initially come to this understanding by comparing unit fractions.

Transitive: When students use benchmark of $\frac{1}{2}$ and one they are using the transitive property. When comparing $\frac{6}{14}$ and $\frac{9}{16}$ students can conclude that $\frac{9}{16}$ is larger because $\frac{6}{14}$ is a little less than $\frac{1}{2}$ and $\frac{9}{16}$ is a little more than $\frac{1}{2}$.

Residual: When comparing fractions $\frac{2}{3}$ and $\frac{3}{4}$ students can decide on the relative size of each fraction by reflecting on the amount away from the whole. In this example, students can conclude that $\frac{3}{4}$ is larger because the amount away from a whole is less than the amount away from the whole for $\frac{2}{3}$. Notice that to use this strategy students rely on the same numerator strategy; they compare $\frac{1}{4}$ and $\frac{1}{3}$ to determine which of the original fractions have the largest amount away from one.

Students who do not have experiences with concrete models like fraction circles or students who may not have sufficient experiences with models to develop needed mental representations to judge the relative size of fractions using these informal strategies make consistent errors. On the next page we share with you examples of students' errors based on written test given to students after their RNP level 1 review lessons. We also share examples of correct thinking among this group of sixth graders. In all questions students were asked to circle the larger of the two fractions.

Misunderstandings

Students often focus on the denominator only after internalizing the relationship between the size of the denominator and the size of the fractional part. To understand what a fraction means, students need to coordinate the numerator and denominator - an idea the following students did not do.

$\frac{4}{9}$	$\frac{4}{15}$	This is bigger because you need less to make a whole than the other fraction. R
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$\frac{3}{4}$	$\frac{2}{3}$	Smaller denominator
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$\frac{3}{4}$	$\frac{2}{3}$	I got my answer by knowing that thirds are bigger than fourths
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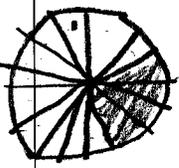
$\frac{6}{14}$	$\frac{9}{16}$	in Fractions 4 is bigger than 6
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An underlying assumption when ordering fractions is that the units for both fractions must be the same. Not realizing the unit needs to be the same is a common error as shown in this student's picture.

$\frac{3}{4}$	$\frac{6}{8}$	
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Whole number thinking also dominates students thinking when they first start working with fractions. This is shown in different ways. Without models students might say that $\frac{6}{8} > \frac{3}{4}$ because $6 > 3$ and $8 > 4$. But even after students use concrete models, their whole number thinking may still dominate. In the following examples, note that some students determine the larger fraction by deciding which fraction had the larger number of pieces. In other cases, students look at the difference between numerator and denominator to identify the larger fraction. In both instances, students have yet to focus on the relative size the fractional part being examined. These students need more time with concrete models to overcome their whole number thinking.

$\frac{3}{4}$	$\frac{6}{8}$	 has more pieces.
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$\frac{4}{9}$	$\frac{4}{15}$	 has more pieces.
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$\frac{3}{4} = \frac{2}{3}$	They are both equal because there both one fraction from a whole.
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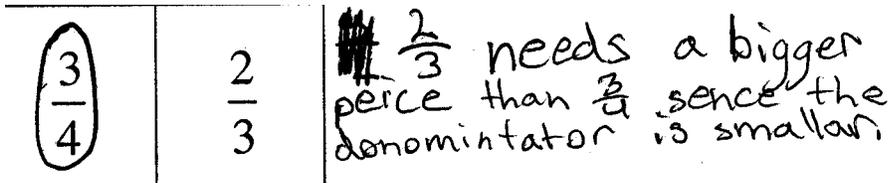
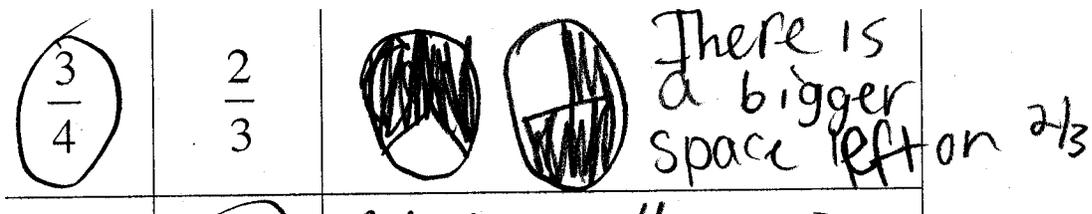
$\frac{3}{4}$	$\frac{6}{8}$	it can't need more to be half
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Understandings

But with enough experiences with concrete models, students do overcome these misunderstandings. Below find student examples for the transitive and residual strategies:

$\frac{6}{14}$	$\frac{9}{16}$	$\frac{6}{14}$ is a little less than $\frac{1}{2}$ and $\frac{9}{16}$ is a little more.
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$\frac{6}{14}$	$\frac{9}{16}$	$\frac{9}{16}$ is bigger because it's over a half and $\frac{6}{14}$ is less than a half.
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When you teach lesson 3 you will notice students using these informal ordering strategies along with other benchmarks to estimate fraction and subtraction problems effectively.

Order these fractions from smallest to largest. Be prepared to explain your thinking.

$$\frac{3}{4} \quad \frac{1}{10} \quad \frac{14}{15} \quad \frac{3}{5} \quad \frac{5}{12}$$

Fraction Estimation

Picture these fractions in your mind. Is the fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Put a $>$ or $<$ sign in each box to show your answer. When in doubt use fraction circles or draw pictures to help you decide if the fraction is more or less than 1-half.

$\frac{2}{3} \square \frac{1}{2}$	$\frac{1}{4} \square \frac{1}{2}$	$\frac{15}{18} \square \frac{1}{2}$
$\frac{3}{10} \square \frac{1}{2}$	$\frac{3}{9} \square \frac{1}{2}$	$\frac{6}{10} \square \frac{1}{2}$
$\frac{6}{11} \square \frac{1}{2}$	$\frac{5}{12} \square \frac{1}{2}$	$\frac{4}{6} \square \frac{1}{2}$
$\frac{9}{12} \square \frac{1}{2}$	$\frac{3}{7} \square \frac{1}{2}$	$\frac{9}{20} \square \frac{1}{2}$

Complete these fractions so they are all close to $\frac{2}{3}$, but just a bit bigger.

$\frac{\quad}{10}$ $\frac{\quad}{12}$ $\frac{\quad}{11}$ $\frac{\quad}{20}$ $\frac{\quad}{13}$ $\frac{4}{\quad}$ $\frac{7}{\quad}$

Fraction Estimation

Work with a partner to order the fractions in each set from smallest to largest. Explain your thinking to each other.

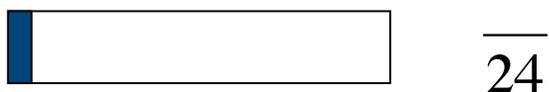
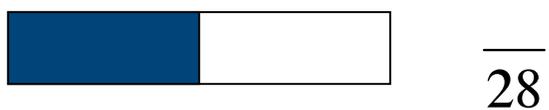
$\frac{9}{16}$ $\frac{8}{9}$ $\frac{1}{14}$	$\frac{19}{20}$ $\frac{2}{22}$ $\frac{6}{14}$
$\frac{99}{100}$ $\frac{6}{11}$ $\frac{3}{100}$	$\frac{6}{7}$ $\frac{2}{88}$ $\frac{6}{10}$
$\frac{10}{13}$ $\frac{2}{47}$ $\frac{4}{7}$	$\frac{3}{50}$ $\frac{13}{15}$ $\frac{5}{11}$

Which fraction is the smallest? $\frac{4}{7}$ or $\frac{12}{33}$; $\frac{2}{3}$ or $\frac{1}{10}$; $\frac{6}{14}$ or $\frac{10}{18}$

Which fraction is the largest? $\frac{9}{10}$ or $\frac{11}{12}$; $\frac{2}{3}$ or $\frac{3}{4}$; $\frac{99}{100}$ or $\frac{4}{5}$

On the back of this paper, describe your strategies for ordering these last two fraction pairs.

Estimate the amount shaded in each example:



Shade to show about $\frac{4}{7}$



Shade to show about $\frac{18}{40}$



<p>Order from smallest to largest. Explain your reasoning:</p> $\frac{1}{10} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{5} \quad \frac{1}{100}$	<p>Picture each fraction. What fraction away from one whole is each one?</p> <p>$\frac{10}{11}$ is ___ away from one whole. $\frac{6}{7}$ is ___ away from one whole.</p> <p>Which fraction is larger? $\frac{10}{11}$ or $\frac{6}{7}$</p>
<p>Margo and Joshua both had a candy bar (same size). Margo ate about $\frac{2}{3}$ of her candy bar. Joshua ate about $\frac{5}{12}$ of his candy bar. Who ate less? How do you know?</p>	<p>Ruby ran $1\frac{5}{6}$ miles. Robert ran $1\frac{3}{4}$ miles. How much further would Ruby need to run to run 2 miles?</p> <p>How much further would Robert need to run to run 2 miles?</p> <p>Who ran the furthest? Ruby or Robert?</p>
<p>If you live $\frac{2}{8}$ of a mile from school And you friend lives $\frac{2}{5}$ of a mile, who lives the nearest to the school? Explain your thinking.</p>	<p>Order from smallest to largest. Explain your reasoning:</p> $\frac{3}{4} \quad \frac{1}{12} \quad \frac{4}{9} \quad \frac{11}{22} \quad \frac{98}{100}$

Post Lesson Reflection

Lesson _____

1) Number of class periods allocated to this lesson: _____

2) Student Pages used: _____

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: