Rational Number Project

Fraction Operations & Initial Decimal Ideas

2009

Kathleen Cramer
Terry Wyberg
Seth Leavitt

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ABOUT THE AUTHORS

Kathleen Cramer is an Associate Professor in Mathematics Education at the University of Minnesota. She teaches mathematics for elementary teachers courses, elementary math methods courses, and other courses for graduate students in mathematics education. Her research interests include teaching and learning of rational numbers.

Terry Wyberg is a Lecturer in Mathematics Education at the University of Minnesota. He teaches mathematics courses for pre-service and in-service teachers at the elementary and secondary levels. His research interests include teaching and learning of rational numbers and teacher content knowledge.

Seth Leavitt teaches middle school mathematics for the Minneapolis Public Schools. He has also taught elementary school in Minneapolis. He is interested in revolutionizing public school education in general and mathematics in particular.

GRADUATE STUDENTS

Debbie Monson is a graduate student at the University of Minnesota and has taught high school mathematics in the Saint Paul Public Schools for 14 years.

Stephanie Whitney is a graduate student at the University of Minnesota and has taught high school and middle school mathematics in Bloomington Public Schools for 8 years.

RNP Website: http://www.cehd.umn.edu/ci/rationalnumberproject/
Fraction Operations & Initial Decimal Ideas
Curriculum Module

Companion Module to -
RNP: Initial Fraction Ideas*

By

Kathleen Cramer
University of Minnesota

Terry Wyberg
University of Minnesota

Seth Leavitt
Minneapolis Public Schools

With Assistance from Graduate Students:
Debbie Monson
Stephanie Whitney
University of Minnesota

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The Rational Number Project (RNP)

The National Science Foundation initially funded the Rational Number Project in 1979. It started out as a cooperative project among three universities. The original members were Merlyn Behr, Northern Illinois University, Thomas Post, University of Minnesota, and Richard Lesh, Indiana University. Project personnel expanded over the years. Kathleen Cramer became involved in the RNP in 1980 and she along with Merlyn Behr developed RNP: *Fraction Lessons for the Middle Grades Level 1* module. We have updated that module and renamed it to better reflect its content: *RNP: Initial Fraction Ideas*. Terry Wyberg, University of Minnesota and Seth Leavitt, Middle School Mathematics Teacher from Minneapolis joined the RNP project with this latest funding cycle. The lessons in this module were created with NSF support from October 2006 through June 2009. They were created to be a companion module to the original one mentioned above.

The early work done by Rational Number Project personnel involved investigating how best to teach fourth and fifth graders initial fraction ideas. (Behr, Wachsmuth, Post, & Lesh, 1984; Bezuk & Cramer, 1989; Post). The curriculum mentioned above, *RNP: Fraction Lessons for the Middle Grades Level* (renamed as *RNP: Initial Fraction Ideas*) is the culminating product of this early work on teaching initial fraction ideas. Initial fraction learning includes developing meaning for fractions using a part-whole model; constructing informal ordering strategies based on mental representations for fractions; creating meaning for equivalence concretely; and adding and subtracting fractions using concrete models. Initial fraction understanding does not
include formal algorithms, and instruction with formal algorithms was not part of this earlier RNP curriculum module. The curriculum module, *RNP: Initial Fraction Ideas*, is available at the RNP website.

Studies have consistently shown that students using RNP lessons outperform students using traditional curricula on fraction assessments. Project personnel strongly believe that one reason the RNP fraction curriculum has been so successful is that the lessons emphasize translations within and between modes of representation. This idea is explained in more detail in the teacher guide.

Another factor is the extended amounts of time students spend with concrete models for fractions (Cramer & Henry, 2002). In addition, the curriculum invests more instructional time developing concepts, order, and equivalence ideas before working with the operations. Out of the 23 lessons in the first RNP module only five dealt explicitly with fraction addition and subtraction. Despite the limited time spent on fraction operations, assessments showed that RNP students performed as well as students using conventional texts on fraction addition and subtraction symbolic tasks while they consistently outperformed other students on estimation tasks involving fraction addition and subtraction (Cramer, Post, & delMas, 2002).

*RNP: Fraction Operations and Initial Decimals Ideas Curriculum Module* is a companion to this original set of lessons. This new module builds on experiences in the first set of lessons to help students build conceptual and procedural understanding for fraction operations. These lessons move into decimals and provide students
opportunities to develop meaning for decimals, order and equivalence ideas related to decimals, and ends with decimal addition and subtraction.

Both RNP curriculum modules were created using *Teaching Experiments*. In a teaching experiment, researchers go into the classroom as teachers and teach a classroom of students using an initial draft of the lessons based on a well thought-out theoretical framework. During the teaching experiment researchers study children’s learning as they progress through the lessons. This is done with classroom observations, student interviews, and written assessments. The lessons are then revised based on what is learned about children’s thinking. The *RNP: Fraction Operations and Initial Decimal Ideas* module went through two cycles of teaching experiments each 6 weeks in length. Then 8 sixth grade teachers in the same district implemented these lessons. Student interviews and written tests evaluated students’ learning. Our analysis of these data and input from the eight teachers led to another round of revisions.

We are thankful to the teachers and students who partnered with us on this project. With their help we were able to create a curriculum that helps students develop a deep understanding of fraction operations and decimals.

**Theoretical Framework**

Children using the RNP lessons use a variety of manipulative models. They work in small groups talking about mathematical ideas and interact with their teacher in large group settings. They draw pictures to record actions with different models. They solve story problems using manipulatives to model actions in the stories.
This model for teaching and learning reflects the theoretical framework suggested by Jean Piaget, Jerome Bruner, and Zoltan Dienes. Richard Lesh, a founding member of the RNP group, built on the work of these three theorists and created a model to guide curriculum developers. Lesh suggested an instructional model that clearly shows how to organize instruction so children are actively involved in their learning. Consider the picture below. Lesh suggests in this model that mathematical ideas can be represented in the five ways shown. Children learn by having opportunities to explore ideas in these different ways and by making connections or translations between the different representations.

In RNP lessons, for example, students will solve story problems for fraction multiplication using pictures. They record their actions with pictures as number sentences. This is an example of a real world to picture to symbol translation. We believe that students need many opportunities exploring fractions and decimals using multiple modes of representation and translating between different representations. In
this curriculum, the major translations for each lesson are identified at the end of each lesson.

**Lesson Format**

The lesson format reflects a classroom organization that values the important role teachers play in student learning as well as the need for students to work cooperatively, talk about ideas, and use manipulative or pictorial models to represent fractions and decimals. Each lesson includes an overview of the mathematical ideas developed. Materials needed by the teacher and students are noted. The lesson begins with a class **Warm Up**. Warm Ups are used to review ideas developed in previous lessons and should take only 5-10 minutes of class time. There is a **Large Group Introduction** section in each lesson. The teacher’s lesson plans provide problems and questions to generate discussion and target the exploration. **Small Group/Partner Work** is included in each lesson where students together continue the exploration of ideas introduced in the large group. The class ends with a **Wrap Up**. A final activity is presented to bring closure to the lesson. At times this will be a presentation by students of select problems from the group work. We found that students like to share their thinking. At other times the Wrap Up will be another problem to solve as a group.

Homework is provided in a separate section at the end of all 28 lessons. Homework sets have been developed for many of the lessons to allow students to have extra practice and to further develop their understandings. Many of the homework sets have problems relevant to the lesson designated on the worksheets, as well as review items from previous work. While each homework worksheet designates a lesson that
students should have had prior to doing the homework, it does not have to be given the exact day the lesson is used in the classroom. It was not the intent of the writers that students do all homework sets, the choice of when to assign homework is left to the discretion of the teacher.

**Manipulative Materials**

Fraction circles are an important manipulative model used in the RNP lessons. Our work over 20 years has shown that of all the manipulatives available for teaching about fractions, fraction circles are the most effective for building mental images for fractions. These mental images for fractions support students’ understanding of order, equivalence and fraction operations.

Students are introduced to other models in these lessons: paper folding for fraction equivalence, patty paper for fraction multiplication, 10 x 10 grid for decimals, and the number line for fractions and decimals. An important goal in creating this curriculum module was to determine effective models for teaching fractions and decimals and how to sequence the models. The models mentioned above proved effective in helping students build meaning for fraction and decimal concepts, order and equivalence, and operations with fractions and decimals.

**Special Notes on Children’s Thinking Related to Fractions**

We want students to develop number sense for fractions and decimals. Underlying number sense for fractions is the ability to judge the relative size of fractions using what the RNP group has labeled as “Students Informal Ordering Strategies”. Students using RNP materials have consistently constructed for themselves
powerful ordering strategies that are not based on common denominators. These ordering strategies are: same numerator, same denominator, transitive and residual.

When comparing two fractions like $\frac{3}{8}$ and $\frac{2}{8}$ (fractions with the same denominator) students using fraction circles will comment that both fractions use the same size pieces. Students will then reflect that 3 of the same-sized pieces are bigger than 2 of the same-sized pieces. This type of thinking is different than stating a rule given by the teacher; when students use fraction circles they construct the rule for themselves and use language closely tied to the manipulative model.

When comparing $\frac{2}{3}$ and $\frac{2}{6}$ (fractions with the same numerator) students can conclude that $\frac{2}{3}$ is larger because thirds are larger than sixths. Two of the larger pieces must be larger than two of the smaller pieces. This strategy involves understanding that an inverse relationship exists between the number of parts the unit is partitioned into and the size of the parts. This strategy also reflects the role of the numerator. An ordering decision based on the denominator only works if the numerators are the same. It is not enough for students to just say the denominator is smaller so the fraction is bigger. For example, when ordering $\frac{3}{4}$ and $\frac{1}{2}$, a student who only considered the denominators might conclude that $\frac{1}{2}$ is bigger because halves are bigger than fourths. This type of error is common among students who do not construct the same numerator strategy for themselves but uses a rule the teacher provides.
When comparing $\frac{3}{7}$ and $\frac{5}{9}$ a student can determine that $\frac{5}{9}$ is larger by using $\frac{1}{2}$ as a benchmark. $\frac{5}{9}$ is bigger than $\frac{1}{2}$ while $\frac{3}{7}$ is less than $\frac{1}{2}$. We found students using fraction circles often used $\frac{1}{2}$ as a benchmark to judge the relative size of other fractions.

This ordering strategy is termed the transitive strategy.

Students using fraction circles often construct one other strategy called the residual strategy. When comparing $\frac{3}{4}$ and $\frac{5}{6}$ students considered the amount away from one whole each fraction was to determine the larger fraction. $\frac{3}{4}$ is a fourth away from the whole while $\frac{5}{6}$ is a sixth away from the whole. Using the same numerator strategy students conclude that $\frac{1}{6}$ is less than $\frac{1}{4}$. $\frac{5}{6}$ is larger because it is closer to one whole. You should expect students in your class to construct these strategies. Students' construction of these strategies is one indicator of number sense.
**Goals**

The table below lists goals for the curriculum module:

<table>
<thead>
<tr>
<th>Fraction- Addition and Subtraction</th>
<th>Fraction- Multiplication and Division</th>
<th>Decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate sums and differences building on mental images for fractions and operations on them.</td>
<td>Use fraction circles, patty paper, number line models, and pictures to multiply fractions. Be able to explain the process.</td>
<td>Use 10 x 10 grids to develop meaning for decimals based on fractions and place value idea.</td>
</tr>
<tr>
<td>Use fraction circles and number line models to add and subtract fractions.</td>
<td>Record process for multiplying fractions with the patty paper model using symbols to develop meaning for algorithm.</td>
<td>Develop order strategies based on mental images of concrete models to judge the relative size of decimals.</td>
</tr>
<tr>
<td>Record process with concrete models using symbols to develop meaning for the common denominator algorithm.</td>
<td>Estimate products building on mental images for fractions and what happens when they are multiplied.</td>
<td>Use 10 x 10 grids to understand decimal equivalence.</td>
</tr>
<tr>
<td>Use common denominator algorithm to add and subtract fractions and be able to explain the process.</td>
<td>Use multiplication algorithm to make sense of multiplying two fractions on a number line.</td>
<td>Use Decimal +/- board to develop strategies for adding and subtracting decimals.</td>
</tr>
<tr>
<td>Use common denominator algorithm to make sense of addition and subtraction on a number line.</td>
<td>Use pictures to solve fraction division problems based on measurement interpretation and be able to explain the process.</td>
<td>Represent decimals and addition and subtraction of decimals on a number line.</td>
</tr>
<tr>
<td></td>
<td>Record the division process with pictures using symbols showing the same denominator algorithm for fraction division.</td>
<td>Solve decimal addition and subtraction problems symbolically and be able to judge reasonableness of results using estimation.</td>
</tr>
</tbody>
</table>

Our goals for fraction operations include being able to add and subtract fractions with like and unlike denominators using the standard algorithm. Students are considered to have this understanding if they can provide reasonable estimates to fraction addition and subtraction problems, if they can use fraction circles and the number line to add and subtract fractions, and if they can explain how the actions with those models corresponded to adding and subtracting fractions with symbols. We
believed that students’ prior experience with the RNP fraction lessons for initial fraction ideas provides the foundation upon which these new skills can be built.

The initial goals for fraction multiplication and division included students being able to model both operations embedded in story problems with concrete models and pictures and to be able to explain the process of multiplying and dividing fractions with concrete models and pictures.

We built on students’ understanding of whole number multiplication and extended it to whole number times a fraction, fraction times a whole number, and fraction times a fraction. We used fraction circles to help students construct understanding of whole number times a fraction; a number line to build meaning for fraction of a whole number; an area model (patty paper) to develop meaning for fraction times a fraction as well as the algorithm. We discovered that a number line model for fraction times a fraction is too complex a model for initially developing meaning and the algorithm for fraction times a fraction tasks. But we did find that once students learned the algorithm for fraction times a fraction, they were able to use that knowledge to make sense of fraction times a fraction on a number line. We believe this added translation to the number line makes the algorithm more meaningful to the students. We also learned that as students constructed meaning behind fraction multiplication, their understanding of what a fraction is evolved from a part-whole model or measurement construct to considering a fraction as an operator. This came naturally to students as they solve fraction multiplication tasks. For example when students multiplied $\frac{3}{4} \times 12$, they partitioned 12 into 4 parts and then multiplied this
result by three. When multiplying $\frac{3}{4} \times \frac{1}{2}$ on number line they would first show $\frac{1}{2}$ on then number line (measurement idea). They would partition this amount into 4 equal parts and iterate 3 of those parts; this is the operator idea.

Story problems for division reflect a measurement model. While students should know that there are other interpretations for fraction division, we believe that the measurement context provides students the support they need to build meaning to fraction division using a common denominator approach. By translating from story problems to pictures to language students develop mental images that support their number sense for fraction division as well as meaningful symbolic work with fractions division.

The development of decimals follows a similar trajectory as the one constructed for teaching fractions found in the original RNP fraction lessons. Time is invested in developing meaning for decimals using multiple models. We use a 10 x 10 grid model and a number line for decimals. Students develop order and equivalence ideas using these models. The 10 x 10 grid in particular helped students developed strong mental images for decimals that in turn supported their work with decimal operations. Addition and subtraction with decimals are developed using what we call a Decimal +-board based on the 10 x 10 grid. As you read through the table of contents you will see that fraction addition and subtraction procedure is developed first using fraction circle model. Decimals are introduced next using 10 x 10 grid and the number line. We return to fraction addition and subtraction using a number line after the work with decimals as we found students understood the number line model more easily with
decimals than with fractions. We build on students’ understanding of the number line model for decimals to support their work with the number line for fraction addition, subtraction, multiplication, and division. There are 28 lessons in all.

**Our Philosophy for Teaching the Algorithms**

Algorithms are tools for understanding and doing mathematics. In order to effectively use these tools students need to have meaning for the numbers they are operating upon and have a sense of what happens to the numbers when a particular operation is used. In our lessons students spend a time building meaning for fractions and decimals. We rely on different models to do that: fraction circles, paper folding, 10 x 10 grids, and number lines. We rely on different models and contexts to help students understand what happens to numbers when added, subtracted, multiplied and divided. We pay careful attention to finding out which models were most effective for helping students understand the impact different operations have on the numbers involved.

To teach the common denominator algorithm for addition and subtraction we carefully help students build an understanding as to why common denominators are needed. We used fraction circles to do this. Students’ understanding of equivalence (concretely and symbolically) also supports the steps to adding and subtracting fractions using common denominators. Students use their algorithm to solve problems in context. But the algorithm is not the end. In our lessons students use their common denominator algorithm to make sense of the number line model for adding and subtracting fractions. This step builds meaning for the algorithm and reinforces the procedure students have learned.
At the end of our lessons for fraction multiplication we believe students will have an extended view of what multiplication is. Students build on their understanding of multiplication of whole numbers and extend their understanding to fractions. We carefully sequenced the numbers involved and use of models so students move from multiplying whole number by a fraction to a fraction times a fraction. The models change as the numbers change. As with fraction addition and subtraction once students construct the multiplication algorithm they apply this procedure to a number line model. The procedure helps students make sense of this new model.

Overcoming whole number thinking students often apply to adding and subtracting decimals was a major goal of these lessons. Again, students first spend time building meaning for decimals before they operate on them. We constructed a new model to help students understanding how to add and subtract decimals. The model is a decimal +- board and is shown on the next page. We found that using this model helped students see the need for “lining up the decimal points”. While we never used that language with students, they constructed this idea for themselves partly because of the decimal +- board used in instruction. As with fraction operations, students applied their newly learned addition and subtraction algorithms to adding and subtracting on a number line. The connections students make among pictorial representations, the symbolic algorithm, and the number line only deepens students’ facility with decimal procedures.

Helping students develop algorithms for operating on fractions and decimals requires time. But we strongly believe the time invested in building meaning for
fraction and decimal operations is worthwhile if our goal is for students to use these tools to do mathematics.

**Decimal +- Board**

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**Special Notes on English Language Learners**

Martha Bigelow, Associate Professor of Second Languages and Cultures at the University of Minnesota, has read the curriculum and has shared some ideas of how to use the RNP lessons with English Language Learners (ELLs). She feels that the strength of the curriculum for ELLs is that it allows students to discuss their interpretation of a problem, formulate hypotheses, identify steps to finding a solution and give a rationale for their answer/ideas. These experiences will help them acquire the academic language of mathematics. The scaffolding of learning that takes place with the use of models and manipulatives are particularly beneficial to the students. The curriculum uses many of the things that literature suggests are good for ELLs when learning mathematics and other subjects (e.g., visualizing, using manipulatives).
With the above in mind, here are some ideas based on Dr. Bigelow’s review of these lessons for RNP teachers to keep in mind when using this curriculum with ELLs:

- Encourage students to talk with peers of the same native language to process concepts.
- Identify ways that students who display understanding, but cannot verbally articulate understanding, can participate in group work (e.g., draw pictures, put things in order, categorize things that are similar). Through the use of fraction circles, paper folding, number lines and pictures ELLs have many opportunities to show their thinking in ways other than words and symbols.
- The RNP lessons include many opportunities for class presentations. It is important that ELLs who seem to be speaking English and understanding concepts of the lesson are invited to give answers in whole-class discussion.
- Some ELLs have not had a great deal of experience with cooperative learning and need help with how to make this learning format work for them. Assigning roles and reminding them to use their peers to help answer questions, rather than always the teacher, is important.
- Though calculators are only used briefly in only one of the lessons, do not assume that all ELLs have experience using a calculator.
- It is important to invite ELLs to participate and give them the floor in math classes. You can let them know ahead of time that you are going to ask them to contribute at a certain point, if you think they need to prepare a little more.
• The reading that students need to do in mathematics classes has to be very close and careful. This is different than in other classes where they often read for the gist. ELLs often need to be reminded to look at every word and make sure they really understand every word in the word problems. When working on story problems there should be an emphasis on the action in the story to uncover the operation.

Assessment issues:

While the language in the curriculum is very important, it might present a barrier to assessing students because it is sometimes difficult to figure out if the answer is wrong because the word problem was in English or because the student does not understand the operation/concept. One way to be sure is to have a couple word problems offered to the student in the student’s native language, if he/she reads in that language.

Building confidence:

For students with limited or interrupted formal schooling, it may be necessary to do a bit more to remind them of what they already learned to give them more confidence that they know what is going on in class. You can ask them things like this: Have we had similar problems before? How did we solve them? Were any strategies useful? Remind them to use positive self-talk (e.g., Yes, I can do this. I did it before and I can do it again. I know when I’m stuck. I know how to ask for help.) The RNP group has found that ELLs do particularly well with the decimal models; teachers can build on these student successes in more challenging work.
*Language in the curriculum*

Some words in the curriculum have a specific meaning in Mathematics and also have an everyday word: *table, round, times*. This is tricky for ELLs because the word may seem familiar, but them in math class there is a new meaning. It is best to make the new meaning explicit.

There are a number of skills that are important across the language modalities. Items that the ESL teacher may wish to work on with students, in addition to the math teacher. It is important to be aware that these are the ways language is being used in the class:

*Listening:*
- Understand explanations with and without concrete referents.
- Understand oral numbers.
- Understand oral word problems.

*Reading:*
- Understand specialized vocabulary
- Understand word problems

*Speaking:*
- Answer questions and explain your answer
- Ask questions for clarification
- Explain problem-solving procedures (e.g., “...because”)
- Describe applications of concepts (e.g., “if...then”)
- Compare and contrast (e.g., uses “greater/less than”, “as...as”)

- Estimate and answer verbally
- **Writing:**
  - Write verbal input numerically
  - Write word problems
  - Estimate and answer in writing

Again, ELLs will benefit from the use of concrete models and the opportunities to interpret problems, develop hypotheses, work towards a solution, and justify their conclusions. Though it is important that the teacher invite ELLs to contribute to class discussions and use their successes as a way to connect future ideas.
Final Comments

At the end of each lesson you will find a form to record your adaptations for each lesson. Any curriculum will need to be “personalized” by the teacher who uses it so it best meets the needs of his/her students. This form will act as a reminder about changes you feel are important to make the next time you teach the lesson.

References


<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic Overview</th>
<th>Materials</th>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Students review how to order fractions using informal strategies. Students compare unit fractions, use $\frac{1}{2}$ as a benchmark and compare fractions close to one.</td>
<td>Fraction Circles</td>
<td>*</td>
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<tr>
<td>2</td>
<td>Students review equivalence ideas with paper folding. Students develop “multiplication” rule from paper folding.</td>
<td>Paper folding strips</td>
<td></td>
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<tr>
<td>3</td>
<td>Students use order and equivalence to estimate sums and differences.</td>
<td>Fraction Circles</td>
<td>*</td>
</tr>
<tr>
<td>4</td>
<td>Students use fraction circles to construct their own plan for adding two fractions. Students explain their plan and show a way to record the steps of their plan symbolically.</td>
<td>Fraction Circles</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Students transition from using fraction circles and symbols to adding fractions using only symbols. Connections between concrete model and symbols are emphasized.</td>
<td>Fraction Circles</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Students build on their understanding of adding fractions with symbols to subtract fractions using fraction circles and symbols.</td>
<td>Fraction Circles</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>Students solve story problems involving addition and subtraction. The subtraction stories go beyond the take away model used in lesson 7. How much more and compare problems are introduced.</td>
<td>Fraction Circles</td>
<td>*</td>
</tr>
<tr>
<td>8</td>
<td>Students have developed a strategy for adding and subtracting fractions using their equivalence ideas in previous lessons. This lesson extends their work to special cases: fractions $&gt; 1$; sums $&gt; 1$; differences with fractions $&gt; 1$; sums of more than two fractions.</td>
<td></td>
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* Homework available for use after this lesson is completed
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<table>
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<tbody>
<tr>
<td><strong>Decimals</strong></td>
<td></td>
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<tr>
<td>9</td>
<td>Students create a model for decimals using 10 x 10 grids to show tenths and hundredths. They record amounts in words and fractions.</td>
<td>10 x 10 grids</td>
</tr>
<tr>
<td>9.5</td>
<td>Students name models for decimals on 10 x 10 grid using decimal notation.</td>
<td></td>
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<tr>
<td>10</td>
<td>Students develop an understanding of thousandths and begin to look at equivalence among tenths, hundredths, and thousandths. Students develop decimal order strategies by identifying the larger of two decimals, by sorting sets of decimals and by finding a decimal between two decimals.</td>
<td>10 x 10 grids</td>
</tr>
<tr>
<td>11</td>
<td>Students estimate sums and differences using mental images of 10 x 10 grids. Students develop strategies for adding and subtracting decimals using Decimal + - boards. Students find exact answers to decimal addition and subtraction using mental math.</td>
<td>Decimal boards</td>
</tr>
<tr>
<td>12</td>
<td>Students review ordering and equivalence and practice adding and subtracting decimals in problem solving contexts.</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Students use a meter stick as a model for decimals by connecting this new model to the 10 x 10 grid model.</td>
<td>Meter stick</td>
</tr>
<tr>
<td>14</td>
<td>Students model decimal addition and subtraction problems using a number line, 10 x 10 grid and symbols.</td>
<td>Number line and decimal + - boards</td>
</tr>
<tr>
<td>15</td>
<td>Students learn to show fractions on a number line.</td>
<td>Number line</td>
</tr>
<tr>
<td>16</td>
<td>Students make connections between adding and subtracting fractions using a symbolic procedure to adding and fractions using a number line.</td>
<td>Number line</td>
</tr>
</tbody>
</table>

* Indicates an additional resource or activity.
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th></th>
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<tbody>
<tr>
<td>17</td>
<td>Students are able to use fraction circles to find the product of a whole number and a fraction. Students are able to explain that the expression axb can be read as “a groups of b”. (whole number x fraction)</td>
<td>Fraction circles</td>
</tr>
<tr>
<td>18</td>
<td>Students are able to multiply a whole number and a fraction using fraction circles, drawing pictures, and using mental images. (whole number x fraction)</td>
<td>Fraction circles</td>
</tr>
<tr>
<td>19</td>
<td>Students will use number lines to multiply a fraction and a whole number. (fraction x whole number)</td>
<td>Number line</td>
</tr>
<tr>
<td>20</td>
<td>Students will use number lines to multiply a whole number by a fraction. (fraction x whole number) Students will be able to explain the differences between multiplication involving a whole number of groups and a fractional number of groups.</td>
<td>Number line</td>
</tr>
<tr>
<td>21</td>
<td>Students will use patty paper (area model) to multiply two fractions. (fraction x fraction)</td>
<td>Patty paper</td>
</tr>
<tr>
<td>22</td>
<td>Students will use patty paper (area model) to multiply two fractions. (fraction x fraction) Students will develop the algorithm for multiplying fractions by noticing patterns.</td>
<td>Patty paper</td>
</tr>
<tr>
<td>23</td>
<td>Students will multiply fractions using a number line. (fraction x unit fraction)</td>
<td>Number line sheets.</td>
</tr>
<tr>
<td>24</td>
<td>Students will use a variety of models and the algorithm to multiply a fraction by another fraction. Students will describe connections among the number line, pictures, and the algorithm.</td>
<td>Patty paper and Number line</td>
</tr>
<tr>
<td>25</td>
<td>Students solve measurement division story problems by using fraction circles and pictures. Students explain their solution strategies. Story problems involve whole numbers divided by a fraction $&lt;1$; mixed numbers divided by a fraction $&lt;1$; fraction $&lt;1$ divided by another fraction $&lt;1$. All answers are whole numbers.</td>
<td>Pictures</td>
</tr>
<tr>
<td>26</td>
<td>Students solve measurement division story problems by using pictures. Students explain their solution strategies. Story problems involve mixed numbers divided by a fraction $&lt;1$; fraction $&lt;1$ divided by another fraction $&lt;1$. All answers include fractions.</td>
<td>Pictures</td>
</tr>
<tr>
<td>27</td>
<td>Students solve measurement division story problems by using pictures. Students write division sentences with common denominators. Students build on their experiences with different models and contexts to estimate quotients to fraction division problems.</td>
<td>Pictures</td>
</tr>
<tr>
<td>28</td>
<td>Students find estimates to fraction division problems.</td>
<td>Pictures</td>
</tr>
</tbody>
</table>

* Indicates that the corresponding page contains images or visual aids.
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 1: Overview

Students review how to model fractions with fraction circles by ordering unit fractions, using 1-half as a benchmark to order two fractions, and comparing fractions close to one.

Materials

- Fraction Circles for students and teacher
- Student Pages A, B, C, D

Teaching Actions

Warm Up

Order these fractions from smallest to largest. Be prepared to explain your thinking.

\[
\frac{3}{4}, \frac{1}{10}, \frac{14}{15}, \frac{3}{5}, \frac{5}{12}
\]

Large Group Introduction

1. Start the lesson by asking students to share their responses to the warm-up task. Record responses on the board and have students defend their answers. Encourage students to describe the mental images used to sort these fractions.

2. Questions to guide discussion:
   - Which is bigger \( \frac{3}{4} \) or \( \frac{3}{5} \)?
   - Which fraction is just under \( \frac{1}{2} \)?
   - Which fractions are just greater than \( \frac{1}{2} \)?

Comments

Your goal is to reinforce students’ informal order strategies and not to use common denominator rules.

Here is an example:

\[
\frac{1}{10}, \frac{5}{12}, \frac{3}{5}, \frac{3}{15}, \frac{3}{4}, \frac{14}{15}
\]

\( \frac{1}{10} \) is farthest from the whole. Five isn’t half of 12 yet. So it is under \( \frac{1}{2} \). Then \( \frac{3}{5} \). Then \( \frac{3}{4} \) and \( \frac{14}{15} \). Both are missing one piece but a fourth is a lot bigger than a fifteenth so it is missing a bigger piece.
Teaching Actions

• Which fractions are close to one whole?
• Why is $\frac{1}{10}$ the smallest?

3. Spend some time ordering $\frac{3}{4}$ and $\frac{14}{15}$.
   • Which fraction is closer to 1? How do you know?
   • One student suggested the two fractions are equal because the both are one piece away from the whole. What do you think?
   • Verify your conclusion by modeling each fraction with the fraction circles.

4. Test students understanding by asking them to order $\frac{4}{5}$ and $\frac{8}{9}$. Which is closer to a whole? Verify students’ responses by showing both fractions with fraction circles.

5. Summarize the main ideas from this lesson introduction:
   • You can judge the relative size of fractions by thinking about fraction circles.
   • Using $\frac{1}{2}$ as a benchmark is helpful.
   • Thinking about how close a fraction is to one whole is also helpful when comparing fractions like $\frac{3}{4}$ and $\frac{5}{6}$ or $\frac{4}{5}$ and $\frac{99}{100}$.

Comments

Expect some students to struggle with $\frac{3}{4}$ and $\frac{14}{15}$. Misunderstandings to expect are:
   • Equal, as both are one away from the whole.
   • $\frac{3}{4}$ is larger because the denominator is a lower number so it is going to have bigger pieces.

Behind this strategy is the ability to mentally represent both fractions using images of fraction circles. Here is an example of a 6th grader’s use of mental images to order $\frac{8}{9}$ and $\frac{4}{5}$.

$\frac{8}{9}$ is larger because if you get the fraction circles out, $\frac{1}{5}$ is bigger than $\frac{1}{9}$.

So if you put $\frac{8}{9}$ and $\frac{4}{5}$, $\frac{8}{9}$ would be bigger because it would have smaller pieces so there is going to be a small amount left and it’s going to be a bigger piece left for $\frac{4}{5}$.

Notice how this student uses his ability to order two unit fractions ($\frac{1}{5}$ and $\frac{1}{9}$) to answer this.

Small Group/Partner Work
Teaching Actions

6. Student pages A-D reinforce comparing fractions to \(\frac{1}{2}\), comparing fractions to unit fractions, and comparing fractions close to 1.

7. Present this problem: What do you think of this students’ reasoning?

\(\frac{6}{8}\) is greater than \(\frac{4}{11}\) because with \(\frac{6}{8}\) you need 2 to get to the whole and with \(\frac{4}{11}\) you need 7.

8. Does this student’s strategy work for \(\frac{9}{10}\) and \(\frac{98}{100}\)?

9. To assess their understanding of using \(\frac{1}{2}\) as a benchmark ask this question:

- Juanita said that \(\frac{7}{12} > \frac{1}{2}\) because she knew that \(\frac{6}{12}\) equals \(\frac{1}{2}\) and \(\frac{7}{12}\) is more than \(\frac{6}{12}\).
- Will said he knew \(\frac{7}{12} > \frac{1}{2}\) because he doubled the numerator and saw that it was greater than the denominator. That made \(\frac{7}{12}\) bigger than \(\frac{1}{2}\).
- I understand Juanita’s strategy but I don’t understand what Will meant. Can you help me? Can you use fraction circles to convince me his strategy is right?

Translations:
- Symbolic to Manipulative to Verbal;
- Symbolic to Verbal

Comments

This is a common error in student thinking. While the student gets a correct answer, her reasoning will not generalize to all fraction pairs. With fractions you have to consider the relative size of the amount away from one whole.

Possible explanations are as follows:
- Doubling the numerator gives you the size of the denominator if the fraction was equal to \(\frac{1}{2}\).
- If the numerator is 7 then \(\frac{7}{14} = \frac{1}{2}\). Because 12ths are bigger than 14ths, \(\frac{7}{12} > \frac{7}{14}\). From this comparison \(\frac{7}{12} > \frac{1}{2}\).
The RNP level 1 lessons support students’ development of informal ordering strategies. Four informal ordering strategies have been identified: same numerator, same denominator, transitive, and residual. These strategies are not symbolic ones, but strategies based on students’ mental representations for fractions. These mental representations are closely tied to the fraction circle model.

**Same denominator:** When comparing \( \frac{4}{5} \) and \( \frac{3}{5} \) students can conclude that \( \frac{4}{5} \) is larger because when comparing parts of a whole that are the same size (in this case 5ths) then 4 of those parts are bigger than 3 of them.

**Same numerator:** When comparing \( \frac{3}{5} \) and \( \frac{4}{6} \), students can conclude that \( \frac{3}{5} \) is bigger because 5ths are larger than 6ths and four of the larger pieces will be bigger than 4 of the smaller pieces. Students initially come this understanding by comparing unit fractions.

**Transitive:** When students use benchmark of \( \frac{1}{2} \) and one they are using the transitive property. When comparing \( \frac{5}{14} \) and \( \frac{9}{16} \) students can conclude that \( \frac{9}{16} \) is larger because \( \frac{5}{14} \) is a little less than \( \frac{1}{2} \) and \( \frac{9}{16} \) is a little more than \( \frac{1}{2} \).

**Residual:** When comparing fractions \( \frac{3}{5} \) and \( \frac{3}{4} \) students can decide on the relative size of each fraction by reflecting on the amount away from the whole. In this example, students can conclude that \( \frac{3}{4} \) is larger because the amount away from a whole is less than the amount away from the whole for \( \frac{3}{5} \). Notice that to use this strategy students rely on the same numerator strategy; they compare \( \frac{1}{4} \) and \( \frac{1}{3} \) to determine which of the original fractions have the largest amount away from one.

Students who do not have experiences with concrete models like fraction circles or students who may not have sufficient experiences with models to develop needed mental representations to judge the relative size of fractions using these informal strategies make consistent errors. On the next page we share with you examples of students’ errors based on written test given to students after their RNP level 1 review lessons. We also share examples of correct thinking among this group of sixth graders. In all questions students were asked to circle the larger of the two fractions.

**Misunderstandings**

Students often focus on the denominator only after internalizing the relationship between the size of the denominator and the size of the fractional part. To understand what a fraction means, students need to coordinate the numerator and denominator - an idea the following students did not do.
An underlying assumption when ordering fractions is that the units for both fractions must be the same. Not realizing the unit needs to be the same is a common error as shown in this student's picture.

Whole number thinking also dominates students thinking when they first start working with fractions. This is shown in different ways. Without models students might say that \( \frac{5}{6} > \frac{3}{4} \) because 6>3 and 8>4. But even after students use concrete models, their whole number thinking may still dominate. In the following examples, note that some students determine the larger fraction by deciding which fraction had the larger number of pieces. In other cases, students look at the difference between numerator and denominator to identify the larger fraction. In both instances, students have yet to focus on the relative size the fractional part being examined. These students need more time with concrete models to overcome their whole number thinking.
### Understandings

But with enough experiences with concrete models, students do overcome these misunderstandings. Below find student examples for the transitive and residual strategies:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Drawing</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{4})</td>
<td><img src="image" alt="6/8" /></td>
<td>has more pieces.</td>
</tr>
<tr>
<td>(\frac{4}{9})</td>
<td><img src="image" alt="4/15" /></td>
<td>has more pieces.</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td><img src="image" alt="2/3" /></td>
<td>These both equal because there both one fraction from a whole.</td>
</tr>
<tr>
<td>(\frac{3}{8})</td>
<td><img src="image" alt="6/8" /></td>
<td>It only need more to be half.</td>
</tr>
<tr>
<td>(\frac{6}{14})</td>
<td><img src="image" alt="9/16" /></td>
<td>(\frac{9}{16}) is a little less than (\frac{1}{2}) and (\frac{9}{16}) is a little more.</td>
</tr>
<tr>
<td>(\frac{6}{14})</td>
<td><img src="image" alt="9/16" /></td>
<td>(\frac{9}{16}) is bigger because it's over a half and (\frac{6}{14}) is less than a half.</td>
</tr>
</tbody>
</table>
When you teach lesson 3 you will notice students using these informal ordering strategies along with other benchmarks to estimate fraction and subtraction problems effectively.
Order these fractions from smallest to largest. Be prepared to explain your thinking.

\[
\frac{3}{4} \quad \frac{1}{10} \quad \frac{14}{15} \quad \frac{3}{5} \quad \frac{5}{12}
\]
**Fraction Estimation**

Picture these fractions in your mind. Is the fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Put a $>$ or $<$ sign in each box to show your answer. When in doubt use fraction circles or draw pictures to help you decide if the fraction is more or less than 1-half.

| $\frac{2}{3}$ | $\underline{\frac{1}{2}}$ | $\frac{1}{4}$ | $\underline{\frac{1}{2}}$ | $\frac{15}{18}$ | $\underline{\frac{1}{2}}$ |
| $\frac{3}{10}$ | $\underline{\frac{1}{2}}$ | $\frac{3}{9}$ | $\underline{\frac{1}{2}}$ | $\frac{6}{10}$ | $\underline{\frac{1}{2}}$ |
| $\frac{6}{11}$ | $\underline{\frac{1}{2}}$ | $\frac{5}{12}$ | $\underline{\frac{1}{2}}$ | $\frac{4}{6}$ | $\underline{\frac{1}{2}}$ |
| $\frac{9}{12}$ | $\underline{\frac{1}{2}}$ | $\frac{3}{7}$ | $\underline{\frac{1}{2}}$ | $\frac{9}{20}$ | $\underline{\frac{1}{2}}$ |

Complete these fractions so they are all close to $\frac{2}{3}$, but just a bit bigger.

\[
\frac{10}{10} \quad \frac{12}{12} \quad \frac{11}{11} \quad \frac{20}{20} \quad \frac{13}{13} \quad \frac{4}{4} \quad \frac{7}{7}
\]
**Fraction Estimation**

Work with a partner to order the fractions in each set from smallest to largest. Explain your thinking to each other.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{9}{16})</td>
<td>(\frac{8}{9})</td>
<td>(\frac{1}{14})</td>
<td>(\frac{19}{20})</td>
</tr>
<tr>
<td>(\frac{99}{100})</td>
<td>(\frac{6}{11})</td>
<td>(\frac{3}{100})</td>
<td>(\frac{6}{7})</td>
</tr>
<tr>
<td>(\frac{10}{13})</td>
<td>(\frac{2}{47})</td>
<td>(\frac{4}{7})</td>
<td>(\frac{3}{50})</td>
</tr>
</tbody>
</table>

Which fraction is the smallest? \(\frac{4}{7}\) or \(\frac{12}{33}\); \(\frac{2}{3}\) or \(\frac{1}{10}\); \(\frac{6}{14}\) or \(\frac{10}{18}\)

Which fraction is the largest? \(\frac{9}{10}\) or \(\frac{11}{12}\); \(\frac{2}{3}\) or \(\frac{3}{4}\); \(\frac{99}{100}\) or \(\frac{4}{5}\)

On the back of this paper, describe your strategies for ordering these last two fraction pairs.
Estimate the amount shaded in each example:

Shade to show about $\frac{4}{7}$

Shade to show about $\frac{18}{40}$
Order from smallest to largest. Explain your reasoning:

| 1/10 | 1/3 | 1/6 | 1/5 | 1/100 |

Picture each fraction. What fraction away from one whole is each one?

\[
\frac{10}{11} \quad \text{is ___ away from one whole.}
\]

\[
\frac{6}{7} \quad \text{is ___ away from one whole.}
\]

Which fraction is larger? \(\frac{10}{11}\) or \(\frac{6}{7}\)

<table>
<thead>
<tr>
<th>Margo and Joshua both had a candy bar (same size). Margo ate about (\frac{2}{3}) of her candy bar. Joshua ate about (\frac{5}{12}) of his candy bar. Who ate less? How do you know?</th>
<th>Ruby ran (1\frac{5}{6}) miles. Robert ran (1\frac{3}{4}) miles. How much further would Ruby need to run to run 2 miles? How much further would Robert need to run to run 2 miles? Who ran the furthest? Ruby or Robert?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margo and Joshua both had a candy bar (same size). Margo ate about (\frac{2}{3}) of her candy bar. Joshua ate about (\frac{5}{12}) of his candy bar. Who ate less? How do you know?</td>
<td>Ruby ran (1\frac{5}{6}) miles. Robert ran (1\frac{3}{4}) miles. How much further would Ruby need to run to run 2 miles? How much further would Robert need to run to run 2 miles? Who ran the furthest? Ruby or Robert?</td>
</tr>
</tbody>
</table>
| If you live \(\frac{2}{8}\) of a mile from school And you friend lives \(\frac{2}{5}\) of a mile, who lives the nearest to the school? Explain your thinking. | Order from smallest to largest. Explain your reasoning:

| 3/4 | 1 12 | 4 22 | 11 100 | 98 |
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 2: Overview

Students review equivalence ideas with paper folding. Students develop a symbolic rule for finding equivalent fractions.

Materials

- Paper folding strips for students and teacher (8.5” x 1”)
- Student Pages A, B, C
- Transparencies A-D

Teaching Actions

Warm Up

Order the fractions from smallest to largest. Be ready to explain your reasoning.

\[
\frac{3}{4}, \frac{1}{20}, \frac{4}{10}, \frac{6}{12}
\]

Large Group Introduction

1. Start this lesson by asking students to discuss this question in their work groups: What does it mean for two fractions to be equivalent?

2. After providing students time to discuss this question, ask students to share their definitions. Record their ideas on the board.

3. If no students use a model to explain equivalence, ask them to use fraction circles to help explain the idea of equivalence. [example: Show 2 fractions equal to \(\frac{2}{3}\) using fraction circles].

4. If no students use context to explain equivalence, ask them to create a story problem that uses the idea of fraction equivalence. (Ex: one person eats \(\frac{1}{4}\) of a small pizza while another ate \(\frac{2}{8}\) of a small pizza. They both ate the same amount, but the pieces were cut up differently).

Comments

Many lessons will have a warm-up question to use at the beginning of class. The question will reinforce an important idea from previous lessons. Plan on taking 5-7 minutes for the warm up to ensure you have enough class time for the major part of the lesson.

You want students to think about equivalence in more than one way. Some students may already have a rule to show that two fractions are equivalent but you also want students to be able to explain equivalence with concrete materials and meaningful contexts. If students do not use a rule, that is fine as it is developed using paper folding in this lesson.
### Teaching Actions

5. Summarize by stating that two fractions are equivalent if they represent the same amount even if the partitions differ.

6. Use the Fraction Equivalence transparencies A, B and C at this point to lead a discussion to help students see the connection between folding paper and finding equivalent fractions. The three examples on the transparency are:

\[
\begin{align*}
\frac{3}{4} & = \frac{6}{8} \\
\frac{2}{3} & = \frac{6}{9} \\
\frac{1}{3} & = \frac{2}{6}
\end{align*}
\]

7. For each example, record students’ explanations with symbols as follows:

\[
\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}
\]

**correct notation**

This recording shows that if number of total parts doubled after folding, then the total number of parts shaded also doubles. \(\frac{6}{8}\) is the fraction amount represented by the paper folding. As the paper folding did not change the size of the unit or the amount originally shaded, \(\frac{3}{4} \times \frac{6}{8}\).

[Note: While the student is multiplying the numerator and denominator by 2, they are not thinking of it as multiplying the fraction by a form of 1].

### Comments

Students have used paper folding to model fractions and find equivalent fractions in the RNP Level 1 module. This lesson is a review and a step towards a symbolic rule for finding equivalent fractions.

You are looking for multiplicative language to describe how the numerator and denominator change after folding:
- If I double the total parts then the shaded parts double.
- If the total number of parts is multiplied by 3, then the shaded parts are three times as many too.
- If the number of shaded parts is multiplied by 4 then total number of parts is multiplied by 4.

At this point, we are not interpreting equivalence as multiplying the fraction by one (such as \(\frac{2}{2}\) or \(\frac{3}{3}\), etc.). We do this in the homework after fraction multiplication is introduced.

It may not be obvious for many students how to go from thirds to ninths. They need to fold the paper into thirds again. Students have a natural inclination to keep doubling.

Look for this doubling error when completing the second example; ask students to explain how to go from thirds into ninths using paper folding. [Student explanation: Go back to the 3rd fold. Since 3 x 3 = 9, you fold into 3rd/s].

Some students may record this idea as shown below. Correct notation encourages students to communicate they are acting on both the
Teaching Actions

Small Group/Partner Work

8. Students complete Student Pages A, B and C. On pages A and B students draw in the folding lines to show fraction equivalence. On page C students rely on their symbolic rule.

9. Do this problem together first. \( \frac{1}{3} = \frac{2}{6} \)

Students need to partition each third into two equal parts to show 6ths.

Wrap Up

10. Show transparency D.

Kia said she could show \( \frac{1}{2} = \frac{1}{3} \) using these pictures.

Ask: Does this make sense?

Comments

numerator and denominator.

\[
\frac{3}{4} \times \frac{3}{4} = \frac{9}{12}
\]

incorrect notation

Solving problems using pictures of paper folding is a prelude to showing fractions on the number line. Students will likely approach the problems on Student Pages A and B from two perspectives.

Students who have not internalized the symbolic rule will partition first to find the missing value. Students who have internalized the symbolic rule will find the missing value first and then determine how to partition the picture.

This second strategy is an example of a symbolic to picture translation. The process of partitioning after finding the answer will reinforce for these students what this abstract rule means.

Example of student thinking: \( \frac{1}{3} = \frac{9}{9} \)

Student said: that’s 3 ninths so I needed 3 parts shaded. He partitions \( \frac{1}{3} \) into 3 equal parts and partitions the other un-shaded thirds into 3 equal parts each.

In this wrap up example you are making it explicit that when you compare fractions you assume that the unit is the same for both fractions.

You might ask this question too:
Rachel and William both spent \( \frac{1}{4} \) of their allowances. Is it possible that Rachel spent more than William?
Teaching Actions

11. Ask students how they can undo their multiplication rule for equivalent fractions to find these equivalences:

\[
\frac{2}{5} = \frac{12}{15}, \quad \frac{4}{8} = \frac{6}{9} = \frac{3}{3}, \quad \frac{8}{12} = \frac{3}{3}
\]

Translations:
- Symbols to concrete to verbal to symbols
- Symbols to pictures to symbols

Comments

Reducing fractions is reinforced in extended practice and warm ups throughout the lessons. It is important that students know common equivalences just as they know their basic whole number facts. The following equivalences are important:

\[
\begin{align*}
\frac{1}{2} &= \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \\
\frac{1}{3} &= \frac{2}{6} = \frac{3}{9} \\
\frac{2}{3} &= \frac{4}{6} = \frac{6}{9} \\
\frac{1}{4} &= \frac{2}{8} = \frac{3}{12} \\
\frac{3}{4} &= \frac{6}{8} = \frac{9}{12}
\end{align*}
\]
• Fold a paper strip into fourths and shade 3-fourths.

• Fold the paper strip to find how many 8ths $\frac{3}{4}$ equals.

\[
\begin{array}{c}
3 \\
- \\
4 \\
= \\
- \\
8
\end{array}
\]

Questions:

1. How did you fold the paper strips to make 8ths?

2. The number of total parts increased from 4 to __________. Describe how the total parts increased.

3. The number of total parts shaded increased from 3 to __________. Describe how the shaded parts increased.
• Fold a paper strip into thirds and shade 2-thirds.

• Fold the paper strip to find how many 9ths \( \frac{2}{3} \) equals.

\[
\frac{2}{3} = \frac{9}{9}
\]

Questions:

1. How did you fold the paper strips to make 9ths?

2. The number of total parts increased from 3 to \__________. Describe how the total parts increased.

3. The number of shaded parts increased from 2 to \__________. Describe how the shaded parts increased.
Lesson 2/Transparency C

• Fold a paper strip into thirds and shade 1-third.

• Fold the paper strip to find the missing denominator.

\[
\frac{1}{3} = \frac{4}{\_}
\]

Questions:

1. How did you fold the paper strips to make 4 parts shaded?

2. The number of shaded parts increased from 1 to \[\underline{\_}\]. Describe how the shaded parts increased.

3. The number of total parts increased from 3 to \[\underline{\_}\]. Describe how the total parts increased.
Final Problem

Kia said she could show that \( \frac{1}{2} = \frac{1}{3} \)

Here is her picture:

What do you think?
Order the fractions from smallest to largest. Be ready to explain your reasoning.

\[
\frac{3}{4} \quad \frac{1}{20} \quad \frac{4}{10} \quad \frac{6}{12}
\]
Looking for Equivalences

Draw the needed lines on each picture of a paper-folding strip to find the equivalent fraction. The lines should show the folds if you used a paper-folding strip to model each example.

\[
\frac{1}{3} = \frac{9}{9}
\]

\[
\frac{4}{6} = \frac{12}{12}
\]

\[
\frac{3}{5} = \frac{15}{15}
\]
Solve using your multiplication rule:

\[
\frac{3}{4} = \frac{6}{8}
\]

\[
\frac{1}{2} = \frac{6}{6}
\]

\[
\frac{1}{2} = \frac{8}{8}
\]
Looking for Equivalences

Working in your groups, solve each problem. Be ready to explain your thinking when the teacher stops by your group.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Equivalent Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dfrac{2}{12} )</td>
<td>( \dfrac{1}{6} )</td>
</tr>
<tr>
<td>( \dfrac{4}{45} )</td>
<td>( \dfrac{12}{135} )</td>
</tr>
<tr>
<td>( \dfrac{6}{8} )</td>
<td>( \dfrac{9}{12} )</td>
</tr>
<tr>
<td>( \dfrac{1}{8} )</td>
<td>( \dfrac{2}{16} )</td>
</tr>
<tr>
<td>( \dfrac{10}{4} )</td>
<td>( \dfrac{5}{2} )</td>
</tr>
</tbody>
</table>

Record 3 fractions equal to \( \dfrac{2}{7} \)

Record three fractions equal to \( \dfrac{4}{9} \)

Record 6 fractions equal to \( \dfrac{1}{2} \)

Record 4 fractions equal to \( \dfrac{6}{8} \)
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 3: Overview
Students use their order and equivalence skills to estimate fraction sums and differences.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Student Pages A - D</td>
</tr>
</tbody>
</table>

Teaching Actions

Warm Up

Estimate the sum: \( \frac{12}{13} + \frac{7}{8} \). The choices are: 1, 2, 19, or 21.

Large Group Introduction

1. Explain to the students that in this lesson they will use their understanding of order and equivalence to make sense of fraction addition.

2. Ask students what order ideas they used to do the warm up problem (both fractions are close to 1.)

3. Present this task and ask students to consider if this solution makes sense.

\[ \frac{1}{3} + \frac{8}{9} = \frac{9}{12} \]  Do you agree?

Comments

Explain that only 24% of eighth graders on a national test could do this correctly. Ask: What is a reasonable estimate? Why did so many students choose 19 or 21? Why is addition and subtraction of fractions so hard for so many students?

When students estimate fraction sums and differences they will rely on their mental images for fractions, their order and equivalence ideas, and their understanding of what addition and subtraction means. The order strategies used most often are: same numerator, residual, and using \( \frac{1}{2} \) as a benchmark.

After students share their ideas guide the discussion with these questions:

- Let’s use our fraction order ideas to judge the reasonableness of this answer: Is \( \frac{8}{9} > \) or \( < \frac{1}{2} \)? [Using \( \frac{1}{2} \) as a benchmark]

- What would you add to \( \frac{8}{9} \) to equal one whole? [Residual idea]

- Is \( \frac{1}{3} > \) or \( < \frac{1}{9} \)?
### Teaching Actions

4. If you placed the exact answer on this number line would it be between 0 and $\frac{1}{2}$, $\frac{1}{2}$ and 1; 1 and $1 \frac{1}{2}$; or $1 \frac{1}{2}$ and 2?

![Number Line](image)

5. Explain: We all agree that the answer will be greater than one but less than $1 \frac{1}{2}$. We will find the exact answer at another time.

6. Repeat for this subtraction problem.

   Ramla had $1 \frac{3}{4}$ pounds of candy hearts. She gave her sister about $1 \frac{1}{6}$ of a pound. About how many pounds did she have left? Do not find the exact answer, just make a reasonable estimate and explain your thinking.

   • Possible explanation: $\frac{1}{6}$ is less than $\frac{1}{4}$; $\frac{2}{4}$ equals $\frac{1}{2}$; take away less than $\frac{1}{4}$ from $\frac{3}{4}$ you are left with more than $\frac{1}{2}$.
   
   • Another possible response: $\frac{3}{4}$ is 3 blues. A pink is smaller than 2 blues. If you cover 3 blues with 1 pink you will have more than 2 blues uncovered. Answer is $> \frac{1}{2}$.

### Comments

- Is $\frac{9}{12}$ $>$ or $< 1$?

Explain to students you are interested in only the range, not the exact spot on the number line.

### Small Group/Partner Work

7. Ask students to work with a partner to complete problems on Student Pages A and B. Explain that they should prepare to explain their answers when called on in the large group discussion.

8. Student pages C and D are extensions to be used at the teacher’s discretion.

### Wrap Up

9. Share thinking strategies for each problem. Each time a student uses an ordering idea, record that

When calling on students to share their thinking make sure you call on students who struggle in math. Use
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>example on the board. Examine the list at the end of the lesson to show the different ways students can think about fractions.</td>
<td>this opportunity to do an informal assessment of the strength of students’ mental images for fractions. Students with strong mental images for fractions are successful estimators. But low achieving students may need help using these mental images to consider what happens when you operate on fractions. Don’t be surprised if some students can use their informal ordering strategies to order fractions but do not realize that these strategies can help them estimate. Some students will still resort to whole number thinking when estimating.</td>
</tr>
</tbody>
</table>

Translations:
- Symbolic to verbal to symbolic
- Real life to verbal to symbolic
Below find examples of students’ written explanations for estimating addition and subtraction problems for the homework labeled Lesson 3. Notice how students use the strategies, same numerator, residual, and $\frac{1}{2}$ as a benchmark to compare fractions. These are the type of explanations you want to encourage.

- $\frac{11}{12} - \frac{1}{3} = \frac{10}{12}$. This doesn’t make sense. $\frac{11}{12}$ is close to 1; so $1 - \frac{1}{2} = \frac{1}{2}$. $\frac{10}{12}$ is closer to 1 than $\frac{1}{2}$.

- $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$. Yes, I do think this is reasonable. $\frac{1}{3}$ is close to 1 and $\frac{1}{4}$ is close to $\frac{1}{3}$. If you add them you are close to 1. Also $\frac{11}{12}$ is close to 1.

- $\frac{1}{5} + \frac{2}{3} = \frac{3}{5}$. Doesn’t make sense. $\frac{2}{3}$ is bigger than $\frac{2}{5}$. It would be closer to $\frac{4}{5}$.

- $\frac{2}{3} - \frac{1}{4} = \frac{1}{12}$. Doesn’t make sense. $\frac{1}{4}$ is smaller than $\frac{1}{3}$. $\frac{1}{12}$ is smaller than $\frac{1}{4}$. More than $\frac{1}{3}$ is left.

- $\frac{8}{15} - \frac{1}{3} = \frac{7}{12}$. $\frac{8}{15}$ is practically the same thing as $\frac{7}{12}$; both a little larger than $\frac{1}{2}$. And you’re taking away a little bit less than $\frac{1}{2}$. So, no, it doesn’t make sense.

Below are examples of students’ estimation errors and examples where students’ explanations could be more precise. Teachers can build on these types of responses and with appropriate questions help students become better estimators.

Are the students over using the “close to 1 and 0” idea and not using other order strategies? Is this the reason for their incorrect estimates?

- $\frac{9}{10} - \frac{2}{100} = \frac{7}{10}$. This makes sense. $\frac{9}{10}$ is almost 1. $\frac{2}{100}$ is almost 0. $\frac{7}{10}$ is almost 1. 1-0=1 [Student could be more precise. 2/100 < 1/10 so the answer must be greater than 8/10; 7/10 is really too small.]

- $\frac{9}{10} - \frac{2}{100} = \frac{7}{10}$. This looks right because $\frac{9}{10}$ is close to one and $\frac{2}{100}$ is very close to 0. So the answer would be between 1/2 and one.
In the examples below students are considering the size of the fractions operated on but not the size of the given answer. In what ways could students be more thoughtful in their estimation?

- $\frac{3}{4} - \frac{2}{100} = \frac{1}{3}$. Doesn’t make sense because $\frac{3}{4}$ is less than $\frac{1}{2}$ and $\frac{2}{100}$ is almost 0. (*Student could have commented that $\frac{3}{4} > \frac{1}{2}$.*)

- $\frac{1}{4} - \frac{2}{100} = \frac{1}{3}$. Wrong! $\frac{2}{100}$ is nothing. $\frac{1}{4}$ - nothing does not equal $\frac{1}{3}$. (*Student could have noted that $\frac{1}{4} > \frac{1}{3}$.*)

- $\frac{8}{15} - \frac{1}{3} = \frac{7}{12}$. This does not make sense. $\frac{8}{15}$ is a little bit more than $\frac{1}{2}$. And $\frac{1}{3}$ is a little less than $\frac{1}{2}$. (*Student could have commented that $\frac{7}{12} > \frac{1}{2}$.*
Estimate the sum: \( \frac{12}{13} + \frac{7}{8} \).

The choices are: 1, 2, 19, or 21.
Fraction Estimation

For each problem, imagine the fractions using fraction circles. Estimate the value of each sum or difference. Put an X in the interval where you think the actual answer will be.

\[
\frac{5}{6} + \frac{11}{12}
\]

\[
\frac{4}{7} + \frac{6}{11}
\]

\[
\frac{7}{16} + \frac{5}{12}
\]

\[
\frac{1}{15} + \frac{1}{30}
\]

\[
\frac{1}{4} + \frac{1}{3}
\]
\[
\frac{\frac{9}{10}}{\frac{3}{4}}
\]

\[
\frac{\frac{8}{10}}{\frac{1}{100}}
\]

\[
\frac{\frac{8}{9}}{\frac{7}{8}}
\]

\[
\frac{\frac{11}{4}}{\frac{1}{10}}
\]

\[
\frac{\frac{7}{14}}{\frac{9}{10}}
\]

\[
\frac{\frac{10}{20}}{\frac{3}{10}}
\]
Estimation and Story Problems

1. After the party, there was $1 \frac{8}{9}$ of a pizza left. Then Brenna ate an amount equal to $\frac{7}{8}$ of a whole pizza. About how much of one pizza was left?

Provide a reasonable estimate with a clear explanation of your thinking. Exact answer is not needed!

2. Joshkin ran $15 \frac{3}{4}$ laps around the track. Caylee ran $14 \frac{1}{5}$ laps. Approximately how many more laps did Joshkin run than Caylee?

Provide a reasonable estimate with a clear explanation of your thinking. Exact answer is not needed!
Extensions

1. Pirate Jack buried \( \frac{1}{2} \) of his treasure. He gave \( \frac{1}{3} \) of the remaining treasure to his trusty mate Pirate Joe. Pirate Joe received $3000 in gold. Exactly how much gold was in Pirate Jack’s whole treasure? Draw a picture to show the solution.

2. Joshkin built a tower using blocks that linked together. I noticed that he had 27 blocks in \( \frac{3}{7} \) of his tower. Exactly how many blocks were in this entire tower? Provide a clear description of your solution strategy.

3. The line below is \( \frac{3}{4} \) as long as a ribbon I have. Draw a line the same length as my ribbon and another line that is \( 1 \frac{1}{6} \) as long as my ribbon. Label the lines.
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 4: Overview

Students use fraction circles to construct their own plan for adding two fractions. Students explain their plan and show a way to record the steps of their plan symbolically.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Fraction Circles for students and teacher</td>
</tr>
<tr>
<td>- Student Pages A, B, C, D</td>
</tr>
<tr>
<td>- Transparency 1</td>
</tr>
</tbody>
</table>

| Teaching Actions |
| Comments |
|---|---|
| **Warm Up** |
| Find a fraction between \( \frac{4}{5} \) and 1. Explain how you know that it is a fraction bigger than \( \frac{4}{5} \) and less than 1. |
| A deep understanding of fraction addition and subtraction involves the following: |
| - Student understands the need for a common denominator |
| - Student is able to model the operation with fraction circles and explain her actions with the circles |
| - Student can connect actions with the circles to symbols, explaining the connections |
| - Student can work in a meaningful way with symbols only |
| - Student can estimate a reasonable sum or difference |
| Note: Different students will need different amounts of time to develop a deep understanding for fraction addition/subtraction. |

<table>
<thead>
<tr>
<th>Large Group Introduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Explain that the important part of today’s lesson is for them to be able to solve addition problems using fraction circles, to explain what they are doing with fraction circles to add fractions, and to record their actions with the circles using symbols.</td>
</tr>
<tr>
<td>2. Present this story problem to the class:</td>
</tr>
<tr>
<td>Raven ate 1 piece of pizza from a personal size pizza cut into 6 equal parts. Later on she ate a 1 more piece from a personal size pizza cut into 4 equal parts. [Assume same-size pizzas]. How much of one personal size pizza did she eat?</td>
</tr>
<tr>
<td>3. Before showing this problem with fraction circles, ask students to estimate a reasonable range for this sum: Is the answer &gt; ( \frac{1}{2} ) or &lt; ( \frac{1}{2} )? &gt;1 or &lt; 1?</td>
</tr>
<tr>
<td>The sum is &lt; ( \frac{1}{2} ), ( \frac{1}{6} ) &lt; ( \frac{1}{4} ) and another ( \frac{1}{4} ) is needed for ( \frac{1}{2} ).</td>
</tr>
<tr>
<td>4. Suggest that some students might say the sum is ( \frac{2}{10} ). Ask why that is unreasonable.</td>
</tr>
<tr>
<td>Address the incorrect strategy of adding numerators and denominators right away. Don’t settle for students saying, “It’s wrong because you have to find common denominators”. Determine it is an</td>
</tr>
</tbody>
</table>

Lesson 4 ©RNP 2009
## Teaching Actions

5. Model the problem using fraction circles by placing 1 blue piece and 1 pink on the black circle. Ask students to use their fraction circles to set up the problem.

6. Ask: How can you find the exact amount of the whole circle covered by one pink and one blue piece? State: Right now I see that $\frac{1}{6}$ and $\frac{1}{4}$ together cover part of the circle, but I want to know what this amount is as one fraction.

7. Lead class discussion to these steps for using fraction circles to add fractions:
   - Set up the problem with the fraction circles using the black circle as the unit.
   - Look for a way to show both fractions using the same color pieces.
   - Building on fraction equivalence ideas, show $\frac{1}{6}$ using 12ths (reds) and show $\frac{1}{4}$ using 12ths (reds).
   - Cover the black circle with the amount equivalent to each fraction. From that picture see that $\frac{5}{12}$ of the circle is covered.

8. Ask students how to record each step with symbols. Phrase the request as wanting a record of what they did with the fraction circles. Elicit several ways for using symbols.

9. Repeat for $\frac{1}{3} + \frac{1}{4}$ and $\frac{3}{5} + \frac{1}{15}$ if needed. [Some students may be ready for independent practice while others may need to see a few more examples worked out with your guidance].

## Comments

unreasonable sum by estimation.

You are building a need for finding common denominators.

To do this, students hopefully will suggest that you exchange pink and blue pieces for equal amounts using the same color.

The idea of the same denominator is represented concretely by using the same color pieces to show each fraction in the problem.

$\frac{1}{6} + \frac{1}{4}$

![Diagram of fraction circles](image)

1 pink = 2 reds
1 blue = 3 reds
2 reds + 3 reds = 5 reds

Some students may keep the problem in horizontal form and write the equivalent fractions underneath them; others may rewrite the problem in vertical form with the equivalent fractions to the right.

Some students may use arrows to show change from the original fraction to the equivalent one. At this point be open to different ways, but encourage students to be neat to avoid errors.

The major goal is to use fraction
### Teaching Actions

#### Small Group/Partner Work

10. Student Pages A, B, and C provide students practice adding fractions with fraction circles. While students work on this, teacher should move from group to group and have students explain how to use the fraction circles to add fractions. Use Student Page D as extensions for those who finish before others.

11. Take note as to how students record their actions with fraction circles with symbols. Select different ways to share during large group discussion.

#### Wrap Up

12. Work through selected problems. For each problem have students show how they recorded the steps with the circles symbolically.

### Comments

- Circles to add fractions and to verbalize this process.
- Students’ idea of recording with symbols may be interpreted in different ways on Student Page 4A. Some will write in words; some will draw pictures; some will write a number sentence. See Additional Notes for the Teacher Lesson 4 for examples of students’ work.
- It is important that students can verbalize how to use the fraction circles and make connections to symbols.
- At this point make it explicit that recording in symbols means writing a number sentence.

### Translations:

- Real life to verbal
- Real life to concrete to verbal to symbols
- Symbols to verbal to concrete to symbols
These examples of students’ work come from Student Page A. Students interpreted the directions to record what they did with the circles using symbols differently. Students used pictures, words and symbols. For students who did use symbols note the different ways students recorded the steps to add fractions. Expect variety, but do encourage students to be neat so they are clearly communicating the steps taken to solve the problem.

\[
\frac{3}{4} + \frac{5}{8} = \frac{11}{8}
\]

Solve with the fraction circles and then record what you did with circles using symbols.

\[
\frac{11}{6} \lor \frac{3}{6} = \frac{4}{6}
\]

\[
\frac{3}{8} + \frac{5}{8} = \frac{13}{8}
\]

\[
\frac{3}{4} + \frac{2}{8} = 1 \quad \frac{3}{8} = 1\frac{3}{8}
\]
\[ \frac{1}{3} + \frac{1}{12} = \]

\[ \frac{4 \times 1}{4 \times 3} + \frac{1}{12} = \frac{5}{12} \]

\[ \text{I tried matching how many } \frac{1}{12} \text{ths} \]

\[ \frac{5}{12} \text{ would fit in } \frac{1}{3} \]

\[ \frac{1}{3} + \frac{1}{12} \]

\[ = \frac{5}{12} \]

I put \( \frac{1}{3} \) and \( \frac{1}{12} \) on the circle and added them and they equaled \( \frac{5}{12} \).
Below find examples of student work from class work involving adding fractions. These students need to be encouraged to show their work more clearly as this type of recording easily leads to errors. One way to do this is to show how other students record their work.

Solve this problem. Show your work.

$$\frac{3}{5} + \frac{1}{3} = $$

$$\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$$

Solve this problem using equivalent fractions with 16ths as the denominator.

$$\frac{5}{8} + \frac{1}{4} = $$

$$\frac{10}{16} + \frac{4}{16} = \frac{14}{16}$$
Raven ate 1 piece of pizza from a personal size pizza cut into 6 equal parts. Later on she ate 1 more piece from a personal size pizza cut into 4 equal parts. [Assume same-size pizzas].

How much of one personal size pizza did she eat?

Estimate:

Record of what we did with the fraction circles:
Find a fraction between $\frac{4}{5}$ and 1.

Explain how you know that it is bigger than $\frac{4}{5}$ and less than 1.
Adding Fractions with Fraction Circles

\[
\frac{1}{3} + \frac{1}{12} =
\]

Estimate first by putting an X on the number line showing about where the answer would be on this line.

Solve with the fraction circles and then record what you did with circles using symbols.

\[
\frac{3}{4} + \frac{5}{8} =
\]

Estimate first by putting an X on the number line showing about where the answer would be on this line.

Solve with the fraction circles and then record what you did with circles using symbols.
## Fraction Addition

Use your fraction circles to model each problem. Record what you do with the circles with symbols.

1. Mario ran $\frac{1}{4}$ of a mile and then rested a few minutes. He then ran $\frac{3}{8}$ of a mile more. How far did he run altogether?

2. $\frac{1}{6} + \frac{1}{4}$

3. $\frac{5}{12}$ of the whole class finished the spelling test and went to the playground. $\frac{1}{4}$ of the whole class finished soon after and joined the others in the playground. What fraction of the class is now on the playground? What fraction of the class is left inside?

4. $\frac{1}{3} + \frac{1}{6}$
5. \[ \frac{2}{3} + \frac{1}{4} = \]

6. JoAnna spent \( \frac{2}{9} \) of her whole allowance at the movie and \( \frac{1}{3} \) of her whole allowance on a new shirt. She spent \( \frac{1}{9} \) of her whole allowance on candy. What fraction of her allowance did she spend in all?

7. \[ \frac{3}{5} + \frac{1}{2} = \]

8. \[ \frac{1}{12} + \frac{1}{6} = \]
Extensions

1. You have 3 pounds of candy hearts. You plan on putting them into baggies that are \( \frac{1}{3} \) pound each. How many baggies can you make? Draw a picture that represents the problem.

2. You have $25. You plan on spending \( \frac{3}{5} \) of your money on a birthday present for your mom. How much will you spend on her present? Draw a picture that represents the problem.

3. I spent $24 on a present for my sister. This was \( \frac{3}{4} \) of all the money I had in my wallet? How much money did I have before I bought the present? Draw a picture that represents the problem.
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas

Lesson 5: Overview

Students transition from using fraction circles and symbols to adding fractions using only symbols. Connections between concrete model and symbols are emphasized.

Materials

- Fraction Circles for students and teacher
- Student Pages A and B

Teaching Actions

Warm Up

\[
\frac{2}{3} + \frac{1}{5}
\]

Is the sum > 1 or < 1?
Find the exact sum.

Large Group Introduction

1. Explain: You have been solving fraction addition problems using fraction circles. You have explained how to add fractions with the circles. You have recorded your steps with symbols.

2. Suggest that students imagine that they are using the fraction circles to add \( \frac{2}{6} + \frac{1}{4} \). Ask: What color pieces are you putting on the black circle?

3. Ask: Do you think the answer will be greater than \( \frac{1}{2} \) or less than \( \frac{1}{2} \) ?

4. Ask: What would you do to find the exact answer using fraction circles? As students explain their steps record them on the board. They should be something like the list that follows:
   - Show \( \frac{2}{6} \) on the black circle
   - Show \( \frac{1}{4} \) on the black circle.
   - Look for a way to show both fractions using the

Comments

Students are not using the circles but they are just imagining that they are.

You want students to be able to verbalize this process with the manipulatives.
### Teaching Actions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>same color pieces</td>
<td></td>
</tr>
<tr>
<td>• Make the exchanges and then add the fractions to find how many pieces of that color I have.</td>
<td></td>
</tr>
</tbody>
</table>

5. Explain that they stated that they would need to show each fraction using the same color. Ask: What color do you think we can use to show both fractions? What denominator would that be? (12ths)

6. State: Using your multiplication rule for equivalence instead of using the fraction circles, rewrite \( \frac{2}{6} \) as 12ths and \( \frac{1}{4} \) in 12ths. Explain to a partner at your table how to do this.

7. Ask: When you rewrite each fraction as 12ths, what would the action be using the circles? (Exchanging to reds).

8. Now actually solve the problems with fraction circles and record each step with symbols.

9. Repeat for \( \frac{1}{12} + \frac{3}{4} \) and \( \frac{1}{2} + \frac{2}{5} \)

### Comments

You are asking students to connect symbols back to the concrete model.

Students might comment that the problems on Student Page B where they listed equivalent fractions helped them solve the addition problems on that page.

Check in with students who struggle. Encourage them to use fraction circles to act out each problem and explain to you what they are doing. Once they have the language help them make the connections to symbols. If when working with symbols only, students add numerator and denominators, encourage them to imagine the fraction circles. Question if their picture matches the answer.

### Small Group/Partner Work

10. Students work in groups to complete Student Pages A and B. They are working through different translations: symbol to fraction circles to symbols; symbols to symbols to fraction circles.

   Students might comment that the
Teaching Actions

Wrap Up

11. Ask students to explain their strategy for finding a common denominator. Possible strategies are: just multiply the two denominators; use guess and check method; list equivalent fractions for both fractions added.

12. Ask students to present to the class their solutions to the two problems where they had to create their own addition problems.

13. End the lesson by asking students to solve these two problems using “mental math”. Have students explain how they can do this without circles or paper and pencil.

\[
\begin{align*}
\frac{1}{2} + \frac{1}{4} &= \frac{3}{8} + \frac{1}{4} \\
\end{align*}
\]

Comments

We are not committed to any one way to find the common denominator. Students seemed to construct their own ways. Many found the common denominators and used the traditional method to find equivalent fractions. The listing idea is presented for those students who do not think of their own way to find equivalent fractions with common denominators.

Below find examples of ways students record their number sentences:

\[\frac{1}{3} + \frac{1}{2} = \]

\[
\begin{align*}
\frac{3}{11} + \frac{5}{11} &= \frac{8}{11} \\
\end{align*}
\]

Solve this problem using equivalent fractions with 16ths as the denominator:

\[\frac{4}{16} + \frac{10}{16} = \frac{14}{16}\]

\[
\begin{align*}
\frac{5}{8} + \frac{1}{4} &= \frac{1}{2} \\
\frac{1}{2} \times 4 &= \frac{4}{2} = \frac{10}{16} \\
\frac{1}{2} \times 8 &= \frac{8}{16} \\
\end{align*}
\]

\[
\begin{align*}
\frac{9}{15} \times \frac{5}{3} &= \frac{14}{15} \\
\frac{15}{15} &= 1 \\
\end{align*}
\]
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3} + \frac{4}{9} = \frac{10}{9}$</td>
<td>$\frac{2}{3} \cdot \frac{4}{9} = \frac{8}{27}$ $\frac{10}{9} - \frac{8}{27} = \frac{20}{27}$ $20 \div 2 = \frac{10}{\frac{20}{27}}$</td>
</tr>
<tr>
<td>Translations:</td>
<td></td>
</tr>
<tr>
<td>• Symbols to verbal</td>
<td></td>
</tr>
<tr>
<td>• Symbols to symbols to verbal</td>
<td></td>
</tr>
</tbody>
</table>
\[ \frac{2}{3} + \frac{1}{5} \]

Is the sum >1 or <1?
Find the exact sum.
# Adding Fractions with Fraction Circles

<table>
<thead>
<tr>
<th>Do you think the answer will be greater than one or less than one?</th>
<th>Now record the steps you used with the fraction circles to solve this problem with symbols.</th>
</tr>
</thead>
</table>
| \[
\frac{1}{3} + \frac{1}{2} =
\] | \[
\frac{1}{3} + \frac{1}{2} =
\] |

Use fractions circles to solve and then describe what you did with the circles to find the answer.

<table>
<thead>
<tr>
<th>Solve this problem using equivalent fractions with 8ths as the common denominator.</th>
<th>Solve in two ways. Use 9ths as a common denominator and use 18ths as the common denominator. Why are the answers equivalent?</th>
</tr>
</thead>
</table>
| \[
\frac{5}{8} + \frac{1}{4} =
\] | \[
\frac{2}{3} + \frac{4}{9} =
\] |

Verify using the fraction circles.
<table>
<thead>
<tr>
<th>List at least 4 fractions equivalent to ( \frac{3}{5} )</th>
<th>Solve this problem. Show your work. ( \frac{3}{5} + \frac{1}{3} = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>List at least 4 fractions equivalent to ( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>List at least 5 fractions equivalent to ( \frac{1}{6} )</td>
<td>Solve this problem. Show your work. ( \frac{1}{6} + \frac{1}{10} = )</td>
</tr>
<tr>
<td>List at least 5 fractions equivalent to ( \frac{1}{10} )</td>
<td></td>
</tr>
<tr>
<td>Create your own addition problem using two fractions with different denominators whose sum is greater than ( \frac{1}{2} ) but less than one.</td>
<td>Create your own addition problem using two fractions with different denominators whose sum is greater than 1 but less than ( 1 \frac{1}{2} ).</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Teaching Actions

Warm Up

Explain how you would find the answer to the following problem using fraction circles.

\[
\frac{5}{6} + \frac{1}{3} = \]

Large Group Introduction

1. Present these three story problems. Ask students to imagine using fraction circles to model each one. Which two problems have the same action? Which one is different? How is it different?

   - Kia spent \( \frac{1}{2} \) of her total allowance on a movie and \( \frac{1}{4} \) of her total allowance on a candy. What fraction of her allowance did she spend in all?

   - Ty had a bag of peanuts weighing about \( \frac{3}{4} \) of a pound. He ate about \( \frac{1}{3} \) of a pound. How much did he have left?

   - India ran \( \frac{1}{2} \) mile and stopped for a few minutes to catch her breath. Then she ran another \( \frac{1}{8} \) of a mile. How far did she run in all?

Comments

The main goal is for students to develop a deep understanding of how to add and subtract fractions using symbols. This knowledge with symbols should be connected to actions with the fraction circles. This understanding should be reflected in their ability to estimate and judge reasonableness of answers.
### Teaching Actions

2. Suggest that you all solve the two addition problems using common denominators. But first ask students to please remind you as to why you decided to use common denominators to add fractions.

3. Record solutions using symbols and then verify the solutions using fraction circles.

4. Solve the subtraction problem using the fraction circles. The first step would be to show $\frac{3}{4}$ of a pound of peanuts using 3 blues. The whole circle is the unit. (Do this at the overhead). As it is difficult to take away $\frac{1}{3}$ of a pound of peanuts from $\frac{3}{4}$, ask how can they use equivalence ideas and common denominators to make this problem easier to solve.

5. Suggest that they list some fractions equivalent to $\frac{3}{4}$ and make another list of fractions equivalent to $\frac{1}{3}$. Ask: What do you notice? ($\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{3} = \frac{4}{12}$)

6. Explain that with circles, that means if you change the 3 blues to 9 reds it will be easier to take away $\frac{1}{3}$. It is possible to take away $\frac{1}{3}$ as $\frac{4}{12}$. (Do this at the overhead)

7. Propose that they show what you did with the circles using symbols. Ask students to describe each step which you will record with symbols: ($\frac{3}{4} - \frac{1}{3}$ is the original task; change $\frac{3}{4}$ to $\frac{9}{12}$; change $\frac{1}{3}$ to $\frac{4}{12}$.

Take $\frac{4}{12}$ away from $\frac{9}{12}$. The answer is $\frac{5}{12}$).

8. $\frac{3}{4} = \frac{9}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$

9. Repeat for this problem: $\frac{5}{9} - \frac{1}{3}$. Show with the circles; make a list of equivalences for each fraction. Decide on changing $\frac{1}{3}$ to $\frac{3}{9}$. Subtract. Record steps with symbols.

### Comments

The idea is to reinforce the notion that using common denominators makes it possible to name the amount as a single fraction.

Important Idea: What you are doing here is helping students use their symbolic skills to guide their use of manipulatives to act out this subtraction problem.
Teaching Actions

10. Summarize by asking students to help you write a class plan for subtracting fractions using symbols. Ask: What should this plan be?

- Find equivalent fractions for each fraction pair with the denominators the same.
- Subtract the numerators.
- Do not subtract the denominators.

Small Group/Partner Work

11. Give directions to students for pages A and B before students start to work on them.

- Direct students to work with a partner to complete Student Page A. Students are still using fraction circles but are using a symbolic method for finding common denominators.
- Read directions to Student Page B to students. Encourage them to imagine using the fraction circles to reinforce the need to find common denominators before subtracting.

Wrap Up

12. Ask: Do the answers you found for these problems make sense? Let’s estimate a reasonable answer and check with your exact answer.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3} - \frac{1}{4}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{2}{3} - \frac{1}{2}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{4}{5} - \frac{1}{2}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{4}{7} - \frac{4}{14}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{11}{12} - \frac{9}{12}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{3}{4} - \frac{1}{5}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
<tr>
<td>$\frac{6}{10} - \frac{1}{3}$</td>
<td>Is the answer $\frac{1}{2}$ or $&lt; \frac{1}{2}$?</td>
</tr>
</tbody>
</table>

As you work with students on estimation you may notice some students have an understanding of the relative size of the fractions in the problem, but still have difficulty putting it altogether. Consider this discussion between a teacher and a student. The student stuck to his initial estimate even though he showed an understanding of the sizes of the fractions:

Estimate $\frac{14}{15} - \frac{5}{7}$

S: A little bit bigger than $\frac{1}{2}$. Because $\frac{14}{15}$ is almost a whole and $\frac{5}{7}$ is a little bit over a half.

T: Is $\frac{14}{15}$ bigger than 1 or less than 1?

S: Less than 1.

T: and $\frac{5}{7}$
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>S: Close to $\frac{1}{2}$.</td>
<td>S: Close to $\frac{1}{2}$.</td>
</tr>
<tr>
<td>T: Is it bigger or less than $\frac{1}{2}$?</td>
<td>T: Is it bigger or less than $\frac{1}{2}$?</td>
</tr>
<tr>
<td>S: Probably bigger</td>
<td>S: Probably bigger</td>
</tr>
<tr>
<td>T: So you are taking 1 and taking away more than $\frac{1}{2}$. Is the answer going to be bigger than $\frac{1}{2}$ or less than $\frac{1}{2}$?</td>
<td>T: So you are taking 1 and taking away more than $\frac{1}{2}$. Is the answer going to be bigger than $\frac{1}{2}$ or less than $\frac{1}{2}$?</td>
</tr>
<tr>
<td>S: Bigger than $\frac{1}{2}$.</td>
<td>S: Bigger than $\frac{1}{2}$.</td>
</tr>
</tbody>
</table>

In this next example, the teacher’s questioning helped the student put together his understanding of the size of the fractions to provide a reasonable estimate.

Estimate $\frac{14}{15} - \frac{5}{7}$

S: It would be between 0 and $\frac{1}{2}$. | S: It would be between 0 and $\frac{1}{2}$. |
| T: Can you explain your thinking? | T: Can you explain your thinking? |
| S: Since $\frac{14}{15}$ is basically one whole. $\frac{5}{7}$ I can’t figure out. | S: Since $\frac{14}{15}$ is basically one whole. $\frac{5}{7}$ I can’t figure out. |
| T: In terms of $\frac{1}{2}$, how big is it? Is it bigger than $\frac{1}{2}$ or less than $\frac{1}{2}$? | T: In terms of $\frac{1}{2}$, how big is it? Is it bigger than $\frac{1}{2}$ or less than $\frac{1}{2}$? |
| S: More than $\frac{1}{2}$ | S: More than $\frac{1}{2}$ |
| T: Now put it all together for me. | T: Now put it all together for me. |
| S: So $\frac{14}{15}$ is basically a whole and $\frac{5}{7}$ is basically $\frac{1}{2}$ so it would be between 0 and $\frac{1}{2}$. | S: So $\frac{14}{15}$ is basically a whole and $\frac{5}{7}$ is basically $\frac{1}{2}$ so it would be between 0 and $\frac{1}{2}$. |

**Translations:**
- Real Life to concrete to verbal to symbols
- Symbols to symbols to verbal
- Real life to symbols to verbal
Explain how you would find the answer to the following problem using fraction circles:

\[ \frac{5}{6} + \frac{1}{3} = \]
Adding and Subtracting Fractions using Fraction Circles

Chee lives $\frac{2}{3}$ a mile from school. After going just about $\frac{1}{4}$ of a mile, his bike broke down and he has to walk the rest of the way. What fraction of one mile does Chee have to walk to get to school?

1. Show $\frac{2}{3}$ on your fraction circles.
2. List fractions equivalent to $\frac{2}{3}$.
3. List fractions equivalent to $\frac{1}{4}$.
4. Describe how you will change the two browns on your fraction circles so you can easily subtract $\frac{1}{4}$.
5. Describe how you will change $\frac{1}{4}$ so you can easily take away that amount from your fraction circle display.
6. What is your final answer?

India has $\frac{2}{3}$ of a cup of brown sugar left in the sugar bowl. Her recipe for chocolate chip cookies requires $\frac{1}{2}$ cup of brown sugar. How much brown sugar will she have left after making her chocolate chip cookies?

1. Show $\frac{2}{3}$ with your fraction circles.
2. List fractions equivalent to $\frac{2}{3}$.
3. List fractions equivalent to $\frac{1}{2}$.
4. Describe how you will change the two browns on your fraction circles so you can easily subtract $\frac{1}{2}$.
5. Describe how you will change $\frac{1}{2}$ so you can easily take away that amount from your fraction circle display.
6. What is your final answer?
Imagine setting up each problem using your fraction circles. Do you need to make changes to take away the amount shown in the problem? What will those changes be? Now solve using only symbols.

\[
\begin{array}{c}
\frac{4}{5} - \frac{1}{2} \\
\frac{1}{3} - \frac{4}{15}
\end{array}
\]

\[
\begin{array}{c}
\frac{5}{6} - \frac{1}{2} \\
\frac{11}{12} - \frac{9}{12}
\end{array}
\]

\[
\begin{array}{c}
\frac{3}{4} - \frac{1}{5} \\
\frac{6}{10} - \frac{1}{3}
\end{array}
\]

Which problem was the easiest to solve? Why?

Which problems were the most difficult to solve? Why?

Were there any problems with different denominators that you could still solve easily? Why?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ________________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 7: Overview

| Students solve story problems involving addition and subtraction. The subtraction stories go beyond the take away model used in lesson 6. |

Materials

- Calculators for students
- Transparencies 1 and 2
- Student Pages A and B

Teaching Actions

Warm Up

\[
\frac{2}{5} - \frac{1}{4} = \frac{7}{8} - \frac{1}{4} = \frac{4}{5} - \frac{2}{3} =
\]

For each problem estimate to determine if the answer would be > 1 or < 1. Find the exact answers to each problem.

Large Group Introduction

1. Explain that today they will continue to solve fraction addition and subtraction problems using their common denominator approach. All problems will be story problems and students need to determine if they should add or subtract to solve the problem.

2. Show the four problems to the students involving whole numbers. Cover up the side with the fractions. Ask students to use their calculators to solve and to keep track whether they added or subtracted the numbers in the problems.

3. Ask students to explain why they added or subtracted. Pay particular attention to why they subtracted in problem 1 (compare) and problem 4 (how many more needed).

4. Show the problems with fractions. Ask: Compare each pair of problems. How are they alike? Different? Does changing the numbers to fractions change the operation?

Comments

Our previous work has shown that students had more difficulty with subtraction than addition even after they could solve addition problems by finding common denominators. Part of the problem was the difficulty they had in identifying a story problem as subtraction.

In this lesson we go beyond the take away model to include other contexts for subtraction.

The reason we are asking students to use a calculator is to focus on the operation between the numbers as opposed to doing the arithmetic.

Students may find common denominators in several ways:
- If the denominators are familiar they use simple recall based on previous experience. For
Teaching Actions

5. Ask students to solve each problem using their common denominator strategy. Share answers.

Comments

example, \( \frac{1}{3} + \frac{1}{2} \). This problem is so familiar students just know 6ths is the common denominator;
• Students will just multiply denominators and use this product as the common denominator;
• Students will create lists of equivalent fractions

To find equivalent fractions with same denominators we found some students constructed this strategy based on the unit fraction:

\( \frac{3}{4} = \frac{3}{12} \). They know \( \frac{1}{4} = \frac{3}{12} \) so \( \frac{3}{4} \) would be three times as many twelfths: \( \frac{9}{12} \)

Small Group/Partner Work

6. Students work on Student Page A. They can use a calculator if they are dealing with whole numbers; but they are to use a common denominator approach with the fractions.

Problem 6 on Student Page A involves adding two fractions whose sum >1. This is the first time students encounter this. Watch how they solve the problem and select students to share their strategies.

Here is an example of a student estimating in subtraction who was able to coordinate what he knew about the relative size of the fractions in the problem and what happens when you operate on them. This is an indication for fraction number sense:

\( 1\frac{3}{5} - \frac{3}{8} \) You are going to have at least 1 left over because it is not

1 1 left over because it is not

\( 1\frac{3}{5} - 1\frac{3}{8} \). So then \( \frac{3}{5} \) is more than \( \frac{1}{2} \) and \( \frac{3}{8} \) is less so if you took those two away I think you would be between 1 and \( 1\frac{1}{2} \).
Teaching Actions

Wrap Up

7. Go over the fraction problems. Ask students if the whole number problem helped them decide if the fraction problem was addition or subtraction.

8. Estimation should be embedded in this sharing too. Consider the following example;

\[ \frac{67}{6} - \frac{65}{3} \]

- Based on the whole numbers only, the answer is about ____
- Is the answer >2 or <2?
- What do you know about \( \frac{5}{6} \) and \( \frac{3}{4} \)?
- Which fraction is bigger? How do you know?

Translations:
- Real life to symbols to verbal
<table>
<thead>
<tr>
<th>Christina is 162 cm tall.</th>
<th>Christina is 63 in. tall.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leah is 156 cm tall.</td>
<td>Leah is 61 in. tall.</td>
</tr>
<tr>
<td>How much taller is Christina?</td>
<td>How much taller is Christina?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>The distance from Los Angeles to New York City is about 2451 miles. After driving 1135 miles, how many more miles do you have to drive?</td>
<td>The race is $5\frac{1}{2}$ miles long. After running $2\frac{3}{8}$ miles, how many more miles do you have to run to finish the race?</td>
</tr>
<tr>
<td>On a long distance trip you drove 345 miles in one day and 567 miles the next day. How many miles did you drive in two days?</td>
<td>You have taken up jogging. On the first day you ran $2 \frac{2}{5}$ miles. On the next day you ran $1 \frac{1}{3}$ miles. How far did you run in two days?</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>You drove 375 miles in one day on your way to NYC. NYC is 1028 miles away. How much farther do you have to drive?</td>
<td>India ran $3 \frac{1}{4}$ miles so far in the race. The race is $5 \frac{5}{8}$ miles long. How much farther does she have to run?</td>
</tr>
</tbody>
</table>
For each problem estimate to determine if the answer to the above problem be >1 or <1.

Find the exact answers to the above problems.
First determine if you need to add or subtract the numbers in the problem. Use a calculator if the numbers are whole numbers. Use a common denominator approach if the numbers are fractions. Record your work on the student record page.

<table>
<thead>
<tr>
<th>(1)</th>
<th>Hamdi lives 452 miles from her grandparents. She lives 135 miles from her cousins. How much further away are her grandparents than her cousins?</th>
<th>(2)</th>
<th>Paola is $65\frac{3}{4}$ inches tall. Ty is $67\frac{5}{6}$ inches tall. How much taller is Ty than Paola?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is this addition or subtraction?</td>
<td></td>
<td>Is this addition or subtraction?</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>You have $3456 saved to buy a car. The used car you want costs $6785. How much more money do you need to save?</td>
<td>(4)</td>
<td>You live about $15\frac{3}{4}$ miles from school. You rode your bike $9\frac{3}{8}$ miles. How many more miles do you have to ride to get to school?</td>
</tr>
<tr>
<td>Is this addition or subtraction?</td>
<td></td>
<td>Is this addition or subtraction?</td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>You are driving to Los Angeles from Minneapolis. The distance is 2436 km. On the first day you drove 456 km. On the second day you drove 557 km. How much further do you have to go?</td>
<td>(6)</td>
<td>You are riding your bike in a 3-day bike-a-thon. The total distance is $26\frac{1}{2}$ miles. On the first day you rode $8\frac{6}{8}$ miles; On the second day you rode $9\frac{1}{2}$ miles. How far did you ride on the third day to cover all $26\frac{1}{2}$ miles.</td>
</tr>
<tr>
<td>Is this addition or subtraction?</td>
<td></td>
<td>Is this addition or subtraction?</td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>You have $1358 in the bank. You spend $398 on a plane ticket to California. How much money do you have left in the bank?</td>
<td>(8)</td>
<td>Hamdi lives $2\frac{4}{9}$ miles from school. After going about $\frac{1}{4}$ of a mile, her bike broke down and she walked the rest of the way. How far did Hamdi walk?</td>
</tr>
<tr>
<td>Is this addition or subtraction?</td>
<td></td>
<td>Is this addition or subtraction?</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>(3)</td>
<td>(4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>(8)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ________________

2) Student Pages used: ________________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 8: Overview

Students have developed a strategy for adding and subtracting fractions using their equivalence ideas in previous lessons. This lesson extends their work to special cases: fractions > 1; sums > 1; differences with fractions > 1; sums of more than two fractions.

Materials

- Problem Set duplicated for each group and cut into 8 separate problems and put in an envelope.
- Student Pages A - D

Teaching Actions

Warm Up

Without using your pencil, estimate which expression is greater: \( \frac{6}{15} + \frac{1}{5} \) or \( \frac{7}{8} - \frac{1}{10} \).

Large Group Introduction

1. Review main ideas in large group using these problems:
   - Estimate this sum: \( \frac{3}{4} + \frac{4}{9} \).
   - Find three fractions equal to \( \frac{4}{5} \).
   - Caylee ran \( \frac{4}{5} \) of a mile while Emilio ran \( \frac{7}{8} \) of a mile. Who ran the farther? If Emilio ran \( \frac{5}{8} \) of a mile, who ran the lesser amount?

2. Challenge the students with this problem: Evelyn rode her bike \( 3\frac{1}{2} \) miles on Monday. On Tuesday she rode \( 2\frac{1}{4} \) miles. On Friday she rode \( 5\frac{3}{8} \) miles. How far did she ride in all?

Comments

These items review order ideas.

- When adding \( \frac{3}{4} + \frac{4}{9} \), students compare both fractions to \( \frac{1}{2} \).
- When comparing \( \frac{4}{5} \) and \( \frac{7}{8} \), students rely on the residual strategy.
- When students order \( \frac{4}{5} \) and \( \frac{6}{8} \), they can use the residual strategy if they reduce \( \frac{6}{8} \) to \( \frac{3}{4} \).
Teaching Actions

Small Group/Partner Work

3. Set the stage for this lesson by explaining that each pair of students will receive an envelope with 8 problems. (*Problems from Student Pages A and B*). These problems are a bit more difficult than the ones they have solved before. Explain that they are similar to the one they just solved and if they work together they will be able to solve the problems.

4. As students work on the problems keep track of their different strategies. Identify the strategies you want all the class to learn. (Identify students ahead of time; give them a transparency to record their solution. This way they are ready to share).

Wrap Up

5. When students are finished bring class together to go over each problem.

6. Summarize the big ideas together that show how the students were able to solve more complex problems using the basic information they had for estimating and adding and subtracting fractions.

7. Reinforce these problems with Student Page C and D which could be a homework assignment or more class work.

Comments

Information on class work problems:

- #1 For fractions greater than one, students should be able to imagine $\frac{6}{5}$ and see that it would equal one whole circle and $\frac{1}{5}$ more.

- #2 Students may solve the clock problem by reducing $\frac{20}{60}$ to $\frac{1}{3}$.

- #3 Students can verbalize the need to work through two steps in a problem based on skills they practiced separately in the past.

- #5 Students can use their common denominator strategy to add or subtract more than two fractions. Students may add mixed numbers by adding or subtracting the fraction part and then working with the whole number part. (See examples below for the problem $\frac{3}{2} + \frac{1}{4} + \frac{5}{8}$)

\[
\frac{3}{2} + \frac{1}{4} + \frac{5}{8} = \frac{12}{8} + \frac{2}{8} + \frac{5}{8} = \frac{29}{8} = 3 \frac{5}{8}
\]

Translations:
- Real life to symbols
- Symbols to symbols
- Symbols to verbal
Without using your pencil, estimate which expression is greater:

\[
\frac{6}{12} + \frac{1}{5} \quad \text{or} \quad \frac{7}{8} - \frac{1}{10}
\]
Problem Solving

(1) Draw a picture to rewrite $1 \frac{3}{4}$ as an improper fraction. Explain what you did.

(2) Chee arrived at the dentist’s office at 2:30. He didn’t get to see the dentist until 2:45. What fraction of an hour did he wait? Reduce to lowest terms. Explain how you solved this problem.

(3) Brenna is making three different cookie recipes. One recipe asks for $2 \frac{1}{4}$ cups of flour. The second recipe calls for $2 \frac{1}{2}$ cups. The last recipe calls for $\frac{3}{4}$ cup of flour. How much did she use in all? Explain how you solved this problem. Can you do this mentally without paper and pencil?

(4) $\frac{15}{3} - \frac{12}{4} = \frac{2}{3} - \frac{1}{4}$ Explain your work.
(5) Draw a picture to rewrite $\frac{9}{8}$ as a whole number and a fraction. Explain what you did.

(6) Solve with symbols and show how you can solve with pictures:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} =$$

(7) Mitzy ran $3\frac{3}{4}$ of a mile in the morning. That afternoon she ran another $2\frac{2}{3}$ of a mile. How much further did she run in the morning? Explain how you solved the problem.

(8) The math team celebrated their victory at the national math contest. The principal gave them 2 sheet cakes of the same size for their celebration. One was a chocolate cake and the other one was a carrot cake. They ate $\frac{4}{5}$ of the chocolate cake and $\frac{3}{4}$ of the carrot cake. How much cake did they eat in all? How much cake was left over? Show your work.
Lesson 8/Problem Set Page A

Problem Set

Duplicate these problems for each group. Cut into eight separate problems. Students work on each problem and record their work on their own paper.

**Problem 1:** How is $\frac{6}{5}$ different from $\frac{5}{6}$? Picture $\frac{6}{5}$ in your mind. Imagine building that fraction with the circle pieces. Now, rewrite $\frac{6}{5}$ as a whole number and a fraction. Repeat for $\frac{4}{3}$, $\frac{9}{5}$, $\frac{8}{4}$, $\frac{6}{4}$. Explain how you think about changing fractions like these.

**Problem 2:** What fraction of an hour passes from 1:10 AM to 1:30 AM? Is the fraction amount $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, or $\frac{3}{4}$? Explain how you solved this problem.

**Problem 3:** Roger and Joe took a handful of candy from the candy bowl. Roger took $\frac{1}{3}$ of all the candies. Joe’s handful equaled $\frac{1}{6}$ of all the candy in the bowl. What fraction of the candy is left?

**Problem 4:** Show your work

\[
\frac{4}{5} - \frac{1}{3} = \quad \frac{1}{3} + \frac{1}{4} + \frac{2}{6} =
\]
### Problem 5:
Erin rode her bike $\frac{3}{2}$ miles on Monday. She rode her bike $\frac{3}{4}$ miles on Wednesday and $5\frac{1}{8}$ miles on Saturday. How far did she ride her bike in all three days? Show your work.

### Problem 6:
Marcus is $67\frac{3}{8}$ inches tall. Trina is $61\frac{1}{4}$ inches tall. How much taller is Marcus? Show your work.

### Problem 7:
Carlos lives $\frac{2}{3}$ mile from school. After riding about $\frac{1}{8}$ mile on his bike, the bike broke down. Carlos walked the rest of the way to school. Estimate: Did Carlos walk more or less than $\frac{1}{2}$ mile? Explain without finding the exact answer.

### Problem 8:
Addis lives $\frac{3}{5}$ miles from school. Her bike broke down along the way. She had to walk $2\frac{1}{3}$ miles to school after the bike broke down. How far did she ride her bike?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: _____________

2) Student Pages used: _____________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 9: Overview

Students create a model for decimals using a 10 x 10 grid to show tenths and hundredths. They record amounts in words and fractions with denominators of 10 and 100.

Materials
• Rulers for students
• Student Pages A - B
• Orange and yellow colored crayons, pencils or markers for each student

Teaching Actions

Warm Up

Order the fractions from smallest to largest. Be ready to explain your reasoning.

\[
\begin{array}{cccc}
5 & 87 & 49 & 5 \\
10 & 100 & 100 & 100 \\
\end{array}
\]

Large Group Introduction

1. In a large group setting, project the square (Student Page A) for all to see. Students should have their own copies.

2. Explain: Using the notches at the top and bottom of the square, use your ruler to draw lines down the square.

3. Ask: What did you do to the square? If the square is our whole unit, and I color three of the ten parts orange, what fraction of the whole square is orange? What amount is not shaded? (At this point, emphasize verbal mode; don’t record with symbols).

Comments

Create a 10 x 10 grid on your Smart Board or create a large poster size 10 x 10 grid on card stock, laminated. Cut out from poster board orange strips to represent 10ths and yellow squares to represent 100ths. Have this available for each decimal lesson.

Color is important as students construct mental images for tenths and hundredths. Students will remember not only the size of tenths and hundredths in relation to the 10 x 10 grid but also the color. That is why we recommend that students use orange and yellow crayons, pencils or markers to do the shading activities in the decimal part of the module.

An important skill for students to develop is the ability to compose and decompose decimals. This skill involves being able to interpret a decimal flexibly. For example .34 can be viewed as 3 tenths and 4 hundredths. But also students should be able to reconfigure or compose numbers if they are given 3 tenths and 4 hundredths to .34. Students who can do this are able to order decimals conceptually as well as build meaning for adding and subtracting decimals in a meaningful way.
4. Return to the grid and draw lines from left to right.
   • Ask: How has the whole changed?
   • How many small squares in all? If I color 5 small squares yellow, what fraction of the whole square is yellow?
   • How many small squares in 1-tenth of the square? What can you say about 10-hundredths and 1-tenth?
   • Name the amount shaded in orange and yellow combined (Encourage a variety of ways to do this: 3/10 + 5/100; 35 out of 100; 3 orange strips and 5 small yellow squares; if all in yellow it would be 35 yellows; 35/100).

5. Show the grid with 35 small squares in yellow. Ask students how this model is similar and different to the one with 3 orange strips and 5 yellow squares.

6. Ask: Where are the 3-tenths in the model all in yellow? (Outline that amount with orange). Name the total amount shaded in a variety of ways and note how they are the same as the model in orange and yellow.

Small Group/Partner Work
7. Ask students to use their crayons to show the amounts found in the table below in two different ways. Use Student Page B.

<table>
<thead>
<tr>
<th>Show this amount in two different ways on two different 10 x 10 grids</th>
<th>Potential ways of describing the pictures</th>
</tr>
</thead>
<tbody>
<tr>
<td>34 out of 100 equal parts</td>
<td>34, (\frac{3}{10}) and (\frac{4}{100}); 34-hundredths; 34 yellow squares; 3 orange strips and 4 yellow squares more</td>
</tr>
<tr>
<td>2 out of 10 equal parts and 6 out of 100 equal parts</td>
<td>(\frac{2}{10}) and (\frac{6}{100}); (\frac{26}{100}); 2 orange strips and 6 yellow squares; 26 yellow squares</td>
</tr>
<tr>
<td>55 hundredths</td>
<td>(\frac{55}{100}); 55 yellow squares; 5 orange strips and 5 yellow squares; (\frac{5}{10}) and (\frac{5}{100})</td>
</tr>
</tbody>
</table>

You can see how composing and decomposing helps students order decimals in this example. Consider how the grids and the colors support these students’ understanding of decimal size.

T: Which is bigger .75 or .9? What do you picture when you see the 75-hundredths?
S: I picture 7 oranges and 5 yellow squares.
T: And the tenths?
S: I see 9 oranges.
T: So then which is bigger?
S: 9-tenths.

Consider one more example of student’s thinking based on the 10 x 10 grid and the ability to be flexible on how he interprets a decimal amount:

2.3 - .05. I imagine this one with a grid so 2 full grids and then there’s 3 tenths so you minus 5 of the hundredths because 3 tenths is the same as 30 hundredths. Cross out 5 hundredths which leaves you with 25. Two and 25-hundredths.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wrap Up</strong></td>
<td></td>
</tr>
<tr>
<td>8. When students are done, share their pictures and record different ways to describe each representation. (See the table on the previous page for ideas). Encourage a variety of ways to describe the amounts on the board. You want students to be flexible in how they interpret the each visual. Record what students say on the board.</td>
<td></td>
</tr>
<tr>
<td>9. Explain that in the next lesson they will record amounts on the 10 x 10 grid using decimal notation. (For this lesson, use verbal descriptions based on color, fraction language (3-tenths and 5-hundredths; 35-hundredths) and fraction symbols (3/10 + 5/100; 35/100).</td>
<td></td>
</tr>
</tbody>
</table>

**Translations:**
- Concrete to verbal
- Concrete to verbal to symbols
- Concrete to verbal to pictures to symbols
- Symbols to pictures to symbols
Order the fractions from smallest to largest. Be ready to explain your reasoning.

\[
\frac{5}{10} \quad \frac{87}{100} \quad \frac{49}{100} \quad \frac{5}{100}
\]
Post Lesson Reflection

Lesson________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas

Lesson 9.5: Overview

Students name models for decimals on a 10 x 10 grid using decimal notation.

Materials

• 10 x10 grid with 3 orange strips and 4 yellow squares shaded and one with 34 yellow squares shaded
• Student Pages A, B
• Orange and yellow colored crayons, pencils or markers for each student

Teaching Actions

Warm Up

Using a two 10 x 10 grids represent 47-hundredths in two different ways. Name this amount in at least two ways.

Large Group Introduction

1. Show students a 10 x 10 grid that has 3 orange strips shaded plus 4 yellow squares. Ask: What part of the grid is covered? What are the different ways we can describe that amount:
   • \( \frac{3}{10} \) and \( \frac{4}{100} \)
   • 3-tenths and 4 hundredths more

   Show the amount using 34 yellow squares, and ask: how can you describe this amount?
   • 34 out of 100
   • \( \frac{34}{100} \)
   • 34-hundredths

2. Explain: You can also record that amount covered in this way: 0.34

3. Ask: How can you make sense of this way of naming that amount by comparing 0.34 to this way of writing the amount? \( \frac{3}{10} + \frac{4}{100} \) (Connect this to

Comments

As you work through these decimal lessons consider these 5 indicators of what it means for students to understand initial decimal ideas.

(1) use accurate language for decimal numbers, (2) use models with understanding to represent decimal numbers, (3) order decimals by composing and decomposing decimals using mental images of the models, (4) use their understanding of the relative size of decimals to guide their estimation with operations with decimals, and (5) interpret addition and subtraction operations with a model and using the model to build meaning for work with symbols.

The decimal lessons develop these understandings. Please note that using decimal language is important. Avoid the “point 2, 3” language for 23-hundredths. Students who can use accurate decimal language and be flexible on naming decimals avoid typical errors students make when comparing decimals or operating with decimals.

As you monitor students’ learning, ask questions related to the 5 indicators. Some students will need more time than others to build mental
Teaching Actions

showing 3 orange strips and 4 yellow squares).

4. Ask: How can you make sense of this way of naming that amount by comparing .34 to 34 out of 100 or \( \frac{34}{100} \)? (Connect this to showing the amount as 34 yellow squares).

12. Possible explanations:
   - 3 is in the tenths place so it is \( \frac{3}{10} \); 4 is in the 100ths place so it is \( \frac{4}{100} \).
   - If you consider how many 100ths are covered then it’s 34-hundredths.

13. Repeat for the other examples on from lesson 9, Student Page B. Students should record the different ways discussed directly on their papers:
   - Name verbally the amount on the grid
   - Describe verbally to their partner what they see
   - Record as fractions in two ways
   - Write as a decimal

Small Group/Partner Work

14. Students complete Student Pages A and B. They are to represent each number given in two ways. They are to record as a decimal and in at least two other ways.

Wrap Up

15. As students complete the student pages, monitor students to see if they are using accurate decimal images for decimals; they may struggle to not use whole number language for decimals; they may misinterpret the model and focus only on color (.5 is 5 orange squares while .06 is 6 yellow squares).

During class discussions look for students who do use language appropriately and understand the decimal model to share their thinking. Be purposeful in selecting students so examples of the 5 indicators are made public for all students to consider.

As you work through naming decimals remember that you want students to compose and decompose decimals. For example students should see .34 as the sum of two parts:
\[ \frac{3}{10} \text{ and } \frac{4}{100} \]
You want students to see the number as a single entity as well: \( \frac{34}{100} \).

\[ .34 = \frac{3}{10} + \frac{4}{100} \]

\[ .34 = 34 \text{ out of } 100 \text{ or } \frac{34}{100} \]
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>language and if they can explain the meaning for the decimal by connecting to each 10 x 10 grid.</td>
<td></td>
</tr>
</tbody>
</table>

16. Select students purposefully to present how they drew two representations for each number and explain what the decimal symbol means by connecting back to the 10 x 10 grids |

**Translations:**

- Concrete to verbal
- Concrete to verbal to symbols
- Concrete to verbal to pictures to symbols
- Symbols to pictures to symbols
Using a two 10 x 10 grids represent 47-hundredths in two different ways. Name this amount in at least two ways.
Naming Tenths and Hundredths on a 10 x 10 Grid
Show each number in two different ways on the 10 x 10 Grid. Then name the amount as a
decimal and in two other ways of your choice.

47-hundredths

\[ \frac{4}{10} + \frac{3}{100} \]
9-tenths and 8 hundredths
Post Lesson Reflection

Lesson________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas

Lesson 10: Overview

Students develop an understanding of thousandths and begin to look at equivalence among tenths, hundredths, and thousandths. Students develop decimal order strategies by identifying the larger of two decimals, by sorting sets of decimals and by finding a decimal between two decimals.

Materials

- Classroom 10 x 10 Grid
- Student Pages A - F
- Orange and yellow crayons, pencils or markers for students.
- Completed Student Page A from Lesson 9

Teaching Actions

Warm Up

Name a decimal close to zero, one close to $\frac{1}{2}$ and one close to 1. Describe the picture in your mind that helps you find the fraction.

Large Group Introduction

1. Explain: You may have encountered decimals in your science class when you measured volume using graduated cylinders, when you measured differences in mass before and after you added another substance, or when you measured the growth of a plant. Often in science you need a great deal of accuracy in your measurements.

2. Notice that these decimals have more digits to the right of the decimal point than the examples we have examined. 0.432; 0.003; 0.106

3. Let’s consider what they mean.

4. Ask students to take out their 10 x 10 grid (Student Page A from Lesson 9) from last lesson that they

Comments

This is a 2-day lesson.

Language is an important part of understanding decimals. Students struggle keeping track of whether the decimal is read as tenths, hundredths or thousandths. Unfortunately this issue isn’t helped when students name a decimal like .23 as “point 23” instead of 23-hundredths.

Accurate use of decimal language is one indicator of decimal understanding.
Teaching Actions

partitioned into 100 equal parts. Ask: If the large square is our unit, then what is one-tenth? What is 1-hundredth? To show hundredths, what did you do to each tenth?

5. Ask: If I want to shade in 4-thousandths of the grid I need to show thousandths. How can we partition the 10 x 10 grid to show thousandths?

6. After allowing time for students to determine a strategy to partition the grid into thousandths, summarize the steps: Divide one of the small squares into 10 equal parts. Do this on the class grid. Ask: How many parts would a tenth be partitioned into if you did that for each small square in 1-tenth? How do you know that? Now imagine that you did this for each tenth, how many equal parts in the whole grid? (You might want to record this information on the 10 x 10 grid showing that 100-thousandths = 1-tenth; there are 100 in each tenth; there are 1000 equal parts in all)

7. Now show 4-thousandths by dividing one small square into 4 equal parts, shading 4 of them black.

8. Shade 423-thousandths on the class grid. Here we want students to color the grid using 3 colors to emphasize the place value components of the decimal. Explain: Look at the classroom grid and examine the amount I have shaded (0.423). Ask: Describe that amount in terms of the number of tenths, hundredths and thousandths shaded in.

9. Record students’ descriptions using fractions:

\[
\frac{4}{10} + \frac{2}{100} + \frac{3}{1000}
\]

10. Now imagine if each square was divided into 10 equal parts. How many thousandths in the 2 small squares? How many thousandths in the 4 bars?

11. How many thousandths is that in all? \[
\frac{3}{1000} \cdot \text{Record as a single fraction} \quad \frac{423}{1000}
\]

Comments

This might not be obvious to students. In our experience students first suggested to divide each column in half. One student’s strategy was to partition one square into 4 equal parts then 8 parts and then realized that each small square needed to be divided into 10 equal parts if the total was to be 1000.

Classes have used different ways to “color” 1000ths. Most encourage students to use a pencil to partition 1 small square into 10 equal parts and to shade number of 1000ths needed with pencil. Another class developed their own language for 1000ths: “itty-bitty yellows”. They used the same crayon for 1000ths as 100ths but added the description that it was the “itty-bitty” yellow.

You aren’t going to partition all 100 squares into 10 equal parts. To show .423 shade in 4 tenths as orange; 2 hundredths in yellow. Then partition one of the hundredths into 10 equal parts and shade in 3 of those very small parts using the color black or whatever color your class decides to use.

Here is a sample of student work showing a decimal in thousandths
Teaching Actions

12. Ask: How can we write that as a decimal? (.423)
   Help students make connections between the decimal and their initial description using the sum of three fractions and to the single fraction using thousandths.

13. Write these decimals on the board. Direct students to represent each decimal on the 10x10 grid using orange, yellow and a color for thousandths. Under each grid have them record the amount shaded as a sum or 2 or 3 fractions with denominators of 10, 100 and 1000. (Student Pages A and B)
   a. 0.304
   b. .034
   c. 0.004
   d. .119
   e. .109
   f. 0.019
   g. 0.82
   h. .012

Wrap Up

14. As students work on Student Page A and B monitor student work to see that students are interpreting the grid model correctly.
   • Ask students to describe the model
   • Ask students to name the amount shaded using place value language (3 tenths + 0 hundredths and 4 thousandths) and fraction language (304-thousandths).

15. During class discussion to wrap up the lesson, make public any errors you see in the model. Purposefully select students who do model the decimals correctly and can use accurate language to describe their work to others.

Day 2 of Lesson 10

Small Group/Partner work

1. Explain that this lesson is a day to practice their decimal work. As they solve the problems, students are asked to informally order decimals based on

Comments

Another indicator of decimal understanding is the ability to accurately use a model to represent decimals.

Another indicator of decimal understanding is the ability to order decimals by composing and decomposing decimals using mental images of the 10 x 10 grid model.

In these problems students are using the grids to show decimals and to order decimals. As with fractions,
Teaching Actions

their representations on the 10 x 10 grid.

2. As student work on Student Pages C – F, ask them to explain how the model is used to solve the tasks.

3. Look to see if students can explain how they ordered decimals. Do they break the decimal into its components and compare? (See student thinking in the comments section)

Wrap up

4. Present these decimal pairs to the class. Ask students to imagine each decimal on the 10 x 10 grid. From that mental image: Which is the larger decimal?

<table>
<thead>
<tr>
<th>0.3</th>
<th>.025</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>.09</td>
</tr>
<tr>
<td>.15</td>
<td>.015</td>
</tr>
<tr>
<td>.8</td>
<td>.80</td>
</tr>
</tbody>
</table>

Comments

order ideas are important part of understanding the relative size of decimals. Students without mental images for decimals will bring whole number thinking to decimal order tasks. When ordering 0.75 and 0.9, a student might say .75 is bigger because as whole numbers 75>9. Below see how two students reasoned through this order task using mental images of 10 x 10 grid:

75-hundredths and 9-tenths.
Seven rows and 5 hundredths.
Nine rows. 9-tenths is bigger.

9-tenths would be greater than 75-hundredths. I see 9 orange bars and 7 orange bars and 5 yellow squares.

Consider the role of the 10 x 10 grid in these students’ explanations.

Which is bigger .5 or .055?
I see 5 hundredths and half of a square since there are 10 thousandths in each square and 5 is half of it. And I picture \( \frac{1}{2} \) of a whole square, the grid .5 is bigger.

Which is bigger .9 or .009?
9 tenths. I picture that there are 100 boxes and 90 of them are full. With 9 thousandths it would be almost one little box. 9 tenths is bigger.

In this next example, the teacher helped the student overcome a misunderstanding by asking her to think of the 10 x 10 grid and to describe what she saw in her mind. Initially when ordering .245, .025, .249, .3 the student said .3 was the smallest even though she ordered the other three decimals correctly.

T: I was wondering about 3-tenths. Do you think the 3-tenths is the
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
</table>
| smallest? How many tenths are in .245?  
S: 2 tenths  
T: Can you picture that decimal on the grid? How many orange strips would this be?  
S: There would be 2 oranges, 4 yellows and ½ of a square.  
T: And how many oranges for .3  
S: 3. 3 tenths is the biggest.  
Modeling many decimals with the 10 x 10 grid enabled these students and others to overcome any whole number thinking most students bring to decimal tasks. |

Translations:
- Symbols to pictures to verbal
- Symbols to pictures to symbols
Name a decimal close to zero, one close to $\frac{1}{2}$ and one close to 1.

Describe the picture in your mind that helps you find the fraction.
Naming Tenths, Hundredths and Thousandths on a $10 \times 10$ Grid

Show each decimal that your teacher lists on the board on a $10 \times 10$ grid. Use orange for 10ths, yellow for 100ths and whatever color your class decided upon for 1000ths. Then record the amount shaded as a sum of 2 or more fractions.
Decimals

Be ready to explain your work to the class.

1) Use the grid below to find three numbers between 0.07 and 0.08. Explain how you know the numbers are between these two.

2) Circle the smaller number: 0.025  0.03. Explain how to use the grids below to support your answer.
3) Determine if the sentences are true or false. Use the grids to support each answer.

0.40 = 0.4

0.880 = 0.8
.098 = .980

.234 = .24
Imagine each decimal on a 10 x 10 grid. Describe each decimal. Then order each set from greatest to least:

2.32  3.082  2.157

Name three decimals less than one but greater than $\frac{1}{2}$. Describe what they would look like on a 10 x 10 grid. How do you know that they are greater than $\frac{1}{2}$?

Imagine each decimal on a 10 x 10 grid. Describe each decimal. Which number is the smallest? How do you know?

0.625
0.25
0.675
0.8

Name three decimals equal to $\frac{1}{2}$. Describe what they would look like on a 10 x 10 grid. How do you know that they are equal to $\frac{1}{2}$?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: _____________

2) Student Pages used: ________________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 11: Overview

Students estimate sums and differences using mental images of the 10 x 10 grid. Students develop strategies for adding and subtracting decimals using decimal +/− board. Students find exact answers to decimal addition and subtraction using mental math.

Materials

• Student Pages A -D
• Orange and yellow crayons
• Teacher Pages 1 and 2

Teaching Actions

Warm up

Imagine adding \( \frac{1}{2} + \frac{1}{4} \) using fraction circles.
Describe how you would solve this.
Imagine \( \frac{1}{2} \) as a decimal, how much of the 10 x 10 grid is shaded? Describe this amount if you used only orange. Imagine \( \frac{1}{4} \) as a decimal, How much of the 10 x 10 grid is shaded? How many orange bars and yellow squares would be shaded to show \( \frac{1}{4} \)?

Large Group Introduction

1. Show the top half of the Addition/Subtract board (Teacher Page 1. Explain: I want you to imagine the decimal, 0.37 on the left grid. What does it look like? Imagine 4 −tenths on the right grid. If you combined the two amounts would you have more or less than \( \frac{1}{2} \) of a grid shaded? (Show the bottom of the board with 5 orange bars shaded). Would you have more or less than a whole grid shaded? Explain your thinking. [At this point you are not showing the decimals on the board. This is all mental math at this point].

2. Repeat for the 3 problems shown on Teacher Page 2. Emphasize using \( \frac{1}{2} \) as a benchmark.
   • .56 take off .009

Comments

This is a 2-day lesson.
The initial model for decimals in this module is the 10 x 10 grid.

In this lesson you are developing estimation strategies based on students’ mental images related to the 10 x 10 grid.

Another indicator of decimal understanding is students’ ability to use their understanding of the relative size of decimals to guide their estimation with operations with decimals.

Students use their mental images for decimals to guide their estimation. Consider how these two students demonstrated this ability:

“Picture 57-hundredths on the Decimal +/− Board. If you took away 9-thousandths would the amount left shaded be more than ½ or less than ½? Explain without finding the exact answer”.

Student responded:
S: More than ½.
I: And how do you know that?
S: Because these 9 thousandths would only take up one of the hundredths because you just split one of those into 10 and it would be 9 and so it would be 5 hun...yeah, 561. thousandths.
### Teaching Actions

- .16 add .7
- .758; take off .6

### Comments

Solving the same task another student relied on his skill in decomposing decimals, his mental images of the 10 x 10 grid, and then reconfiguring (composing) the decimal parts into an estimate of the final amount.

S: Bigger. Because 9-thousandths is like, like 9 itty-bitty yellows and that’s only taking away like 1 (hundredth) away from that (57 hundredths). Probably not even one but then you would, it would still be 56-hundredths and something.

From our first teaching experiment we found this model to be quite effective to build mental representations for decimals. On the decimal assessment, the percent correct on the 6 order items ranged from 84% to 100%.

But the assessment also showed that students did not do as well on decimal addition and subtraction items. Whole number thinking was still prevalent among many students. With the first group of 6th graders the percentage correct for 6 decimal operation problems was from 35% to 59%.

We considered reasons for this and decided that the 10 x 10 grid model needed to be adapted to more explicitly show the action of addition and subtraction. We used in the second teaching experiment a “decimal + - board”. (see Student page A). We found that in the second teaching experiment students resonated to this model; they used it to solve the problems; they were able to describe verbally how to add and subtract decimals using this board. On the decimal assessment students’ performance increased. With the second group of 6th graders the percentage correct for decimal

### Small Group/Partner Work

3. Have students use the Decimal + - board on Student Page A to model each estimation problem and find the exact answer. Let students find their own way to use the board to add and subtract decimals. See the teacher notes for this lesson for examples of how students might do these types of problems.

4. It is less obvious ho to use the board to subtract. Step back and see how students use the board for subtraction. (See Teacher Notes for this lesson to see how students use the decimal + - board).

### Wrap Up

5. Have students share their strategies for solving each addition and subtraction problem. Be purposeful in selecting students so correct strategies are modeled. Look for different ways in which students used the boards to add and subtract.

6. If you see common errors make those public as well.
Teaching Actions

Day 2

Large Group Instruction

7. Return to each problem from the previous lesson. Ask students how they might record what they did with the board using written symbols. Compare the exact answer using symbols with the answer obtained using the board.

8. Record like this:
   \[
   \begin{array}{c}
   \frac{3}{10} \\
   \frac{4}{10}
   \end{array}
   \]

   Students should verbalize that putting tenths below tenths in the example above matches adding 3 tenths to 4 tenths on their addition/subtraction board (or 3 orange strips to 4 orange strips). Please do not tell students the rule: add zero and add as whole numbers!! Students will construct meaningful strategies and do not need a rote rule.

9. Explain to students that another student said that the answer to: \(.34 + .5 = .39\). Ask: Is that reasonable? What is the error?

Small Group/Partner Work

10. Ask students to complete Student Pages B and C. Remind students to estimate before they find the exact answer. Encourage students to represent each decimal based on its place value components (ex: .75 would be shown as 7 orange bars and 5 yellow squares).

11. Students who have been successful with adding and subtracting decimals are comfortable working with decimals in their place value components. They are able to decompose the decimal to operate on them and then compose the parts to name the decimal using accurate decimal language. This is another indicator of decimal understanding: Using a model and their ability to compose and decompose decimals to interpret addition and subtraction operations and build meaning for work with symbols.

Comments

operations ranged from 93% to 100% on 4 operations problems using the decimal board.

We believe this board was helpful because students were able to clearly show each number in the problem before they added or subtracted them.

The mental images students have of the addition/subtraction grid help students overcome whole number thinking. Below find two 3 examples of students’ thinking on subtraction problems students were asked to solve mentally:

\[.55 - .3\] So this would be 25 hundredths. I just know 5 tenths minus 3 tenths would be 2 tenths and this extra 5 hundredths.

\[2.3 - .05\] I don’t think I have done a problem like this. (Encouraged to imagine the grid) 2 and 3 tenths would be 2 things colored in and 3 tenths and then 5…. Now I got it. It would be 2 and 25 hundredths. I was imagining this being 2 and 30 hundredths and 30 hundredths minus 5 hundredths would be 25 hundredths.

\[2.3 - .05\] I imagine this one with the grid so 2 full grids and then there’s 3 tenths so you minus 5 of the hundredths, so then you cross out 5 of the hundredths which leaves you with 25. 2 and 25-hundredths.
Teaching Actions

Wrap Up

5. End class with mental math problems. Ask students to imagine the addition/subtraction board to mentally determine the exact answer to each problem. Ask students to explain their thinking.

- \(.37 + .6\)
- \(1.02 + .08\)
- \(.46 + .15\)
- \(.50 - .05\)
- \(.82 - .09\)

Comments

Encourage students to be precise in their language. While this student was able to estimate and find the exact answer mentally, her language was imprecise and reflected whole number thinking.

28 hundredths + 6-hundredths. That would be less than 1-half because 8 plus 6 equals 14 and there’s no tenths. It is going to make it 34.

This next example shows a student thinking in terms of decimals. The problem was 2.3 - .05

I imagine this one with the grid so 2 full grids and there’s 3 tenths so then you minus 5 of the hundredths, so then you cross out 5 of the hundredths which leaves you with 25. 2 and 25 hundredths.

Below find examples of a sixth grader’s thinking on addition and subtraction tasks. This student was identified by her classroom teacher as one who struggles in math class. Notice how the use of the decimal +-board helped her develop understanding for decimal addition and subtraction. Student still reverted to “Point” language that should be addressed.

Imagine the decimal board. Use that image to solve this problem: .26 + .5

26- hundredths. That would be 2 of the tenths columns and 6 of the small hundredths pieces, the small boxes. You add to it a half of the other grid. Put them together. Point 75.

.55-.3

That’s half a grid and 5 more
### Teaching Actions

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>squares. Then take 3 of the columns, tenths columns, minus that. So that would be point 25. There’s half of the grid and 5 pieces and I take away 3 from the half and just keep the 5.</td>
</tr>
<tr>
<td>2.3 -.05</td>
</tr>
<tr>
<td>Still going to be 2 so don’t bother with that. Point 3 is 3 tenths columns minus point zero five is 5 little squares. So 3 tenths columns minus 5 little squares (pauses to think) is point 25.</td>
</tr>
</tbody>
</table>

### Translations:
- Symbolic to verbal
- Symbolic to picture to verbal to symbolic
- Symbolic to picture
The examples below show students’ use of the Addition/Subtraction boards. These examples came from student interviews conducted after completing the decimal lessons.

**Decimal Addition**

\[ .28 + .06 \]

To solve using the board the student first showed .28 on the left grid as 2 orange strips and 8 yellow squares. He showed .06 as 6 yellow squares on the right grid.

He then said, “I take 2 of those to make whole, 3 tenths. Then if you minus 2 from these (6 yellow squares) you have 4 left.

He shows the final answer on the bottom grid as 3 orange strips and 4 yellows.
Notice that the student mentally operated on the display of the two numbers in the problem to determine how to represent the answer on the bottom grid. In the next example, the student solving the same addition problem physically acted out the addition on the grid and then showed the result on the bottom grid.

To solve using the board the student first showed .28 on the left grid as 2 orange strips and 8 yellow squares. He showed .06 as 6 yellow squares on the right grid.

The student then crosses off 2 of the yellow squares from 6-hundredths. She then shades in 3 orange strips and 4 yellows on the bottom grid. “This is 28 so I shade in 2 oranges and 8 yellows. Then there’s only 6-hundredths so shade in 6 then take away 2. And add them to that. I am going to make tens so I can make 3. And there is still 4 left over.
Subtraction:

In this example the student acts out the subtraction as take-away marking off the needed number of tenths on the grid on the left side.

\[.67 - .5\]

To solve using the board the student first showed .67 on the left grid as 6 orange strips and 7 yellow squares. He showed .5 as 5 orange strips on the right grid.

He takes away 5 tenths by crossing out 5 orange strips on the left grid. He then shades the answer on the bottom grid as 1 orange strip and 7 yellow squares.

“And I take the 5 oranges away from this and I cross them out over here and then I know when I come down over here I have 1 tenth and 7 hundredths”.

![Diagram of grid showing subtraction process](image)
Another student showed the two amounts on the board but mentally subtracted 5-tenths and then showed the final answer on the bottom grid. “OK so you would subtract 6 bars from 5 bars which would give you one left and there’s 7 yellows, there’s 7 squares and no squares so you would just put these squares over there and there you go.” (When asked what his final answer was he stated: 17 hundredths.”

Notice that this student rewrote the problem vertically to show his work with the grid. He lined up the 5 tenths under the 6 tenths without having to rewrite .5 as .50.
To solve using the board the student first showed .67 on the left grid as 6 orange strips and 7 yellow squares. He showed .09 as 9 yellow squares on the right grid.

“For this one you have to subtract 7 to 9 which I am going to just use a bar and you would add a little circle there and get rid of one of the bars so it would be 5 bars and then go 1, 2, ,,,8 and there you go.”

He then shaded in 5 oranges and 8 yellow on the bottom grid. “That would be 58 hundredths”.

The student reflected on the two representations for the numbers involved in the problem. He imagined taking away 1 tenth and leaving 1 yellow square. (He pointed to the grid on the left while explaining this).
representation. The reality is that multiple representations for the mathematical concepts/skills being developed enhance students understanding and the connections among different representations are varied.

Below find examples of student’s work from lesson 11. There are no indications on the addition/subtraction board that the student used them to act out the problem. But the problems are recorded correctly using symbols and correctly solved.
Decimal Addition and Subtraction Board
Student Page A
<table>
<thead>
<tr>
<th></th>
<th>Greater than $\frac{1}{2}$</th>
<th>Less than $\frac{1}{2}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>37-hundredths plus 4-tenths</td>
<td>Greater than $\frac{1}{2}$</td>
<td>Less than $\frac{1}{2}$</td>
<td>Less than 1</td>
</tr>
<tr>
<td>56-hundredths take away 9-thousandths</td>
<td>Greater than $\frac{1}{2}$</td>
<td>Less than $\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>1 and 6-hundredths plus 7-tenths</td>
<td>Greater than 1</td>
<td>Less than 1</td>
<td>Greater than 2</td>
</tr>
<tr>
<td>1 and 58-hundredths take away 6-tenths</td>
<td>Greater than 1</td>
<td>Less than 1</td>
<td>Greater than $\frac{1}{2}$</td>
</tr>
<tr>
<td>73-hundredths take away 6-hundredths</td>
<td>Greater than 1</td>
<td>Less than 1</td>
<td>Greater than $\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Imagine adding $\frac{1}{2} + \frac{1}{4}$ using fraction circles. Describe how you would solve this.

Imagine $\frac{1}{2}$ as a decimal, how much of the 10 x 10 grid is shaded?

Describe this amount if you used only orange.

Imagine $\frac{1}{4}$ as a decimal, how much of the 10 x 10 grid is shaded?

How many orange bars and yellow squares would be shaded to show $\frac{1}{4}$?
# Addition and Subtraction with 10 x 10 Grids

Solve each problem using the grid. Record the problem with symbols showing the final answer.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Estimate</th>
<th>Picture</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75 + 0.02 =</td>
<td></td>
<td><img src="image1.png" alt="Grid Image" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Greater than one or less than one?</td>
<td><img src="image2.png" alt="Grid Image" /></td>
<td></td>
</tr>
<tr>
<td>37-hundredths and 3-tenths more</td>
<td></td>
<td><img src="image3.png" alt="Grid Image" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?</td>
<td><img src="image4.png" alt="Grid Image" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Greater than one or less than one?</td>
<td><img src="image5.png" alt="Grid Image" /></td>
<td></td>
</tr>
<tr>
<td>15-hundredths minus 7-hundredths</td>
<td>Greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------------------</td>
<td>--------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greater than $\frac{1}{4}$ or less than $\frac{1}{4}$?</td>
<td><img src="image1" alt="Grid" /> <img src="image2" alt="Grid" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$0.75 - 0.4 =$</th>
<th>Greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Grid" /> <img src="image4" alt="Grid" /></td>
<td><img src="image5" alt="Grid" /> <img src="image6" alt="Grid" /></td>
</tr>
<tr>
<td>99-hundredths - 88-hundredths</td>
<td>Greater than ( \frac{1}{2} ) or less than ( \frac{1}{2} )?</td>
</tr>
<tr>
<td>------------------------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Greater than ( \frac{1}{4} ) or less than ( \frac{1}{4} )?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>57-hundredths minus 3-tenths</th>
<th>Greater than ( \frac{1}{2} ) or less than ( \frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greater than ( \frac{1}{4} ) or less than ( \frac{1}{4} )?</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Fraction Operations and Initial Decimal Ideas
Lesson 12: Overview

Students review ordering and equivalence and practice adding and subtracting decimals in problem solving contexts.

Materials

- Student problems A – G cut apart and put in envelopes for each group.
- Poster paper, markers, multiple copies of 10 x 10 grid and Decimal + - board.
- Yellow and orange crayons, pencils or markers.

Teaching Actions

Warm Up

Estimate:

.85 + .39 =
Greater than 1 or less then 1?

3.675 + 2.399 =
Greater then 6 or less than 6?

45.06 – 40.56 =
Greater than 5 or less than 5?

Large Group Introduction

1. In this lesson students have the opportunity to practice their decimal order and equivalence ideas to extend their work with decimal addition and subtraction using symbols.

2. Give each group a set of problems to solve. They can solve in any order they want

Comments

The decimal lessons have been developed so students construct these indicators of decimal understanding: (1) using precise mathematical language when working with decimals, (2) accurately using models to represent decimals, (3) describing how to compose and decompose decimals through mental images of the models as they ordered decimals, (4) using their understanding of the relative size of decimals to guide their estimation with operations with decimals, and (5) using a model and their ability to compose and decompose decimals to interpret addition and subtraction operations and build meaning for work with symbols.

The rest of the decimal lessons reinforce and extend students’ decimal concepts and procedures. You might find that some students may need more time completing lessons 9 – 11. Consider Lesson 12 as an extension for those students who are ready to go beyond Grade 4 and 5 Standards for Decimals.
Small Group/Partner Work

3. After students have finished half of the problems, assign a particular problem to each group to prepare and present to the whole class. At this point students may not finish the rest of the problems.

4. Provide poster paper, markers, and 10 x 10 grids that can be cut apart and put on posters. Students are to construct a large display of their solution.

Wrap Up

5. Explain that a good presentation clearly communicates not only the answer, but also how students reached the answer. Tell them to imagine that they are the teachers and they are to teach the rest of the students how to solve their problem.

6. Explain that a good audience listens politely to the presentation and asks questions if they don’t understand something.

Translations:

- Symbolic to pictures to verbal
- Story problem to symbols to verbal
- Symbolic to symbolic to verbal

See Additional Notes for the Teacher for Lesson 12 to see examples of students’ displays.
0.327

Three hundred seventy thousandths

3 + 0.02

327

1000

Sum of decimals = Sum of fractions

0.007

1000

327 + 0.07

327

327

Sum of decimals = Three hundred seventy thousandths
Order from Smallest to Largest.
Estimate

.85 + .39 =
Greater than 1 or less than 1?

3.675 + 2.399 =
Greater than 6 or less than 6?

45.06 – 40.56 =
Greater than 5 or less than 5?
### Problems 12A – 12D

**Copy and Cut out problems**  
Organize into envelopes for each group

<table>
<thead>
<tr>
<th>Problem 12A</th>
<th>Problem 12B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show how you can use decimal equivalents for each fraction to order the fractions from smallest to largest. Use your 10 x 10 grids in your solution.</td>
<td>Raven’s tomato plant measured 15.035 cm when she planted it in May. By August it was 85.16 cm tall. How much did her plant grow from May to August?</td>
</tr>
</tbody>
</table>
| ![Fractions](null) | • Draw a sketch showing the plant in May and in August.  
• Estimate the plant growth. Did it grow more or less than 50 cm? More or less than 100 cm? Explain why.  
• Show with symbols how to find the exact answer. |

<table>
<thead>
<tr>
<th>Problem 12C</th>
<th>Problem 12D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chee’s sunflower plant grew 120.54 cm over the summer. When he planted it in May it measured 12.009 cm in height. How tall was his sunflower plant at the end of the summer?</td>
<td>Use 10 x 10 grids to show how to order these decimals.</td>
</tr>
</tbody>
</table>
| • Draw a sketch showing the plant in May and at the end of the summer.  
• Estimate the plant height. Was it more than 150 cm or less than 150 cm by the end of the summer? Explain why.  
• Show with symbols how to find the exact answer. | .005 .50 .05 .495 .062 .5 |
Problems 12E – 12G

Copy and Cut out problems
Organize into envelopes for each group

Problem 12E

Estimate each answer before finding the exact answer. Explain your thinking. Then show how to rewrite the problems so finding the exact answer is easy to do. Find the exact answers.

4.7 + 3.94 + .034 =
Estimate: Is the answer greater than 10 or less than 10? Is it greater than 400 or less than 400? Is it greater than 6 or less than 6?

16.75 – 8.974 =
Estimate: Is it greater than 25 or less than 25? Is the answer greater than 10 or less than 10? Is it greater than 6 or less than 6?

53 – 4.95 =
Estimate: Is the answer greater than 10 or less than 10? Is it greater than 25 or less than 25? Is it greater than 50 or less than 50?

Problem 12F

Imagine a 10 x 10 grid. Picture each decimal below given in words on this grid. Describe what you see in your mind.

Now write each decimal in symbols.

- 12-tenths
- 105-hundredths
- 33-tenths
- 1002-thousandths

Problem 12G

Show this number in multiple ways (fraction, decimal, grid, words, as a sum of two or more decimals or fractions…)

0.327
Post Lesson Reflection

Lesson________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
### Teaching Actions

#### Warm Up

How long is your desk in meters?
- More than one meter?
- More than .5 meters?
- Less than .5 meters?
- Between .5 meters and .75 meters?

#### Large Group Introduction

1. Hand out Student Page A, a picture of the 10 x 10 grid, to each student and ask students to shade the decimal .74. Show this on the classroom grid as well. Review that if the unit is the whole square, then the vertical bar is a tenth of the square and the small square is 1-hundredth of a square. We can see that .74 is 7 full tenths and 4-hundredths more or 74 hundredths.

2. Explain that they are going to model decimals using a different manipulative. Give each pair a meter stick. Explain that the whole stick is the unit.

3. Direct students to cut the 10 x 10 grid showing .74 apart into vertical strips; tape the strips together to form one long strip. Match the paper strip to the meter stick.

### Comments

The purpose of this activity is to make the transition from a 10 by 10 grid to a number line (as represented by the meter stick.) As with all measuring activities, there will be inaccuracy. Be prepared for students’ taped lines to equal a little more or a little less than 0.74 meters. This is a good time to discuss the approximate nature of all measurements, the possibility of various answers, the possibility that the same person might get a slightly different result in a second measurement, etc.

See Additional Notes to the teacher for Lesson 13 for background information on students’ use of a number line.

This activity is transitioning from an area model (grid) to a length model (number line). The taped-together strips emphasize length characteristics even though the area idea is still present.
Teaching Actions

4. Ask: what do you notice? How are the two models alike and different?

5. Ask: What does each mark on the stick mean? How do these marks match up with the orange tenths and yellow hundredths from the 10 x 10 grid? If you partitioned one small square into ten parts, what mark on the meter stick would match that small amount?

6. Conclude that the meter stick is partitioned into 10 equal parts; 100 equal parts; and 1000 equal parts.

7. Have students find 74-hundredths on the meter stick and ask them why that is 7 tenths and 4 hundredths more. Refer to it as a distance from 0 to .74 on the meter stick.

8. Name the parts for the students or ask if they know the special names. Match each name with the markings on the meter stick. Record this information on the board.
   - Decimeter (dm) = 1-tenth of the meter
   - Centimeter (cm) = 1-hundredth of the meter
   - Millimeter (mm) = 1-thousandth of the meter

9. Ask: How many tenths make up the whole 10 x 10 grid? How many dm in one meter? How many hundredths make up the whole 10 x 10 grid? How many cm in one meter? How many thousandths make up the whole 10 x 10 grid? How many mm in one meter?

10. Ask: If the length of a table I have at home measures 4 dm, 3 cm and 5 extra mm, what part of one meter is the length of this table? Find that mark on your meter stick.

11. Repeat for other lengths. Find the point on the meter stick and name it as a decimal part of the whole meter.
   - 2 cm and 7 mm extra \( \left( \frac{2}{100} + \frac{7}{1000} ; 0.027 \right) \)
   - 4 dm, 5 mm extra \( \left( \frac{4}{10} + \frac{5}{1000} ; 0.405 \right) \)

Comments

You might want to draw a sketch of a meter stick on the board and label the dm, cm and mm.

Record the measurements in different ways: \( \frac{4}{10} + \frac{3}{100} + \frac{5}{1000} \) or \( \frac{435}{1000} = 0.435 \). Work from fraction notation to decimal notation.

Connect to the more common language: zero tenths, 2 hundredths, and 7 thousandths.
**Teaching Actions**

- 7 dm, 2 cm ($\frac{7}{10} + \frac{2}{100}; 0.72$)
- 8 dm, 2 cm, 7 mm ($\frac{8}{10} + \frac{2}{100} + \frac{7}{1000}; 0.827$)

**Small Group/Partner Work**

12. Assign Student Page B. Students measure lengths of objects in the room and record the amounts as dm, cm and mm; then translate those amounts to a decimal part of the meter.

**Wrap Up**

13. Explain that they now have used two models to show decimals – the 10 x 10 grid and a meter stick.

14. Show transparency of 0.38 with number line below it. Ask students to describe the number line and to find 0.38 on it. Draw a length to .38 on the number line.

15. Ask: How are the two models alike and different?

16. End lesson by completing Student Page C together. (Make a transparency; students should have their own copies).

17. Encourage students to explain how to translate from the symbols to the 10 x 10 grid to the number line.

**Translations:**

- Concrete to picture to verbal
- Symbolic to picture to verbal

The number line is partitioned to show tenths and hundredths.

Start with the 10 x 10 grid first and then translate to the number line.

In this example notice how the student used loops to jump over 4-tenths and then used smaller loops to count over 6-hundredths more. The connection between the 10 x 10 grid and the number line is clear in this student’s work.
Additional Notes to the Teacher

Lesson 13

In the first teaching experiment we completed all the lessons pertaining to fraction operations before beginning decimal lessons. We did notice from classroom observations and student interviews that the students had difficulty using the number line model for fraction addition and subtraction. But when the number line model was introduced to students during the decimal unit, they seemed comfortable using this model to identify decimal amounts. On the decimal posttest, 92% and 96% of the students correctly located or identified a decimal in hundredths on a line partitioned into tenths and hundredths. While students’ performance on decimal operations was less than we hoped, students were more successful showing addition on a number line (59%) than when solving just symbolic problems (35% - 48%). We addressed issue of the number line model and students performance on decimal operations in our second teaching experiment.

For the second teaching experiment we rearranged the sequence introducing our work with decimals prior to introducing the number line for fraction operations. We believed that if students had experience with number line for decimals, then the transition to using a number line for fraction operations would be easier. Students seemed to have an easier time with fraction number lines during the second teaching experiment. We also added more examples where students partitioned number lines to show different fractions before asking students to use number line to add and subtract. Students were quite adept at using the number line for fraction multiplication in certain situations.
0.38
How long is your desk in meters?

• More than one meter?
• More than .5 meters?
• Less than .5 meters?
• Between .5 meters and .75 meters?
**Using Your Meter Stick**

Complete the chart with a partner. The first length is given to you. Talk to each other as you complete each section of the table. Make sure you agree with each other on all the data you put in the table.

<table>
<thead>
<tr>
<th>Length</th>
<th>How many full dm?</th>
<th>How many full cm extra?</th>
<th>How many full mm extra?</th>
<th>How many mm in all?</th>
<th>What part of a meter stick is this length?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of a regular sheet of paper</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>279</td>
<td>0.279</td>
</tr>
<tr>
<td>Desk width</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Desk length</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Desk height</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find one or two lengths greater than one meter but less than 3 meters. Measure the lengths as accurately as you can. Use meters as your unit. Your measurements should include decimals. Record what you measured and its length below.
<table>
<thead>
<tr>
<th>Decimal</th>
<th>Grid</th>
<th>Number Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>![Grid](0.09 Grid.png)</td>
<td>![Number Line](0.09 Number Line.png)</td>
</tr>
<tr>
<td>0.46</td>
<td>![Grid](0.46 Grid.png)</td>
<td>![Number Line](0.46 Number Line.png)</td>
</tr>
<tr>
<td>0.12</td>
<td>![Grid](0.12 Grid.png)</td>
<td>![Number Line](0.12 Number Line.png)</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson______________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 14: Overview

Students model decimal addition and subtraction problems using a number line, 10 x 10 grid and symbols.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Several copies of Student Pages A and B for students</td>
</tr>
<tr>
<td>• Student Pages C-E</td>
</tr>
<tr>
<td>• Transparency 1 (optional)</td>
</tr>
</tbody>
</table>

Teaching Actions

Warm Up

Mental Math: Find the sum or difference without using fraction circles or paper and pencil

\[
\begin{align*}
0.35 + 2.05 &= 2.40 \\
0.90 + 0.15 &= 1.05 \\
0.90 - 0.14 &= 0.76 \\
2.85 - 1.05 &= 1.80 \\
\end{align*}
\]

Large Group Introduction

1. Explain: In a previous lesson you showed how to model addition and subtraction problems involving decimals with a 10 x 10 grid and with symbols. Today you will learn how to model similar problems using a number line.

2. By the end of today’s lesson you should be able to show how to add and subtract decimals in three ways: number line, grid and with symbols. You should also be able to explain similarities among the different ways.

3. Say: Let’s take a moment to examine the page of number lines. How many units on the number lines? How is each unit divided?

4. Using the first number line, let’s find these decimals: 0.97; 0.07; 1.35.

Comments

Note: This lesson may take 2 days.
Teaching Actions

5. Which is larger: .07 or .68? How can you use your number line to support your answer? Repeat for 1.45 and 1.405

6. Consider this problem: A plant grew .5 of a unit in one week; the next week the plant grew .65 of a unit. How much did the plant grow in two weeks? Do you think the plant is more or less than one unit?

7. Since our estimate is over 1 unit, we will need a Decimal +/- board with an extra grid at the bottom. (Hand out Student Page 14B).

8. Explain: A number line is a model that can include many units. So I will use a number line to model this addition problem. But you will use the Decimal +/- board. Let’s see how the two models are alike for adding decimals.

9. Show a number line at the board. Students use the Decimal +/- board. Ask students to show .5 on their grid. You show .5 on the number line by shading from 0 to .5. Say: We both have shown the growth for week one on our models. To show the growth of week two, we need to add to .65 to this .5 amount. Show .65 on the other grid at the top of your board. Combine the two amounts on the grids at the bottom of the boards. Based on the grids what is the answer?

10. State: Let’s show that on the number line. Starting at .5, I will extend the line 6-tenths more and then add on .05 to that. What is the value of the point on the number line at the end of my line? Is it the same as with the grid? How are the two models alike? Different?

11. Ask a student to model the problem using symbols. Guide the student to explain how the symbols match the actions with the number line and decimal board. Without using a rule (line up the decimal points) point out that she is matching tenths to tenths and hundredths to hundredths.

12. We need to show subtraction using the number line too. Consider this problem: Ada’s tomato plant

Comments

Encourage students to verbalize what each decimal means. For example: .97 is 9-tenths and 7 hundredths or 97-hundredths. Stating the decimal as tenths and hundredths will help students see how to find that amount on the number line.

We want students to see how adding .5 + .65 on the 10 x 10 grid is similar to adding .5 + .65 on the number line. We want students to verbalize that they need to add tenths to tenths and hundredths to hundredths.

You are modeling a picture-to-picture translation.
Lesson 14

Teaching Actions

was 1.4 units tall, while Diego’s tomato plant was 1.65 units tall. Which plant was the tallest? How much taller?

13. Let’s make a quick sketch to show the two plants. How can we solve the problem?

14. Because the numbers are greater than one we can’t use the Decimal +/- boards. But we can model this problem with the number lines. Show 1.65 on the large class number line by shading a line up to 1.65. Whose plant does this amount refer to? Where is Ada’s plant height on the number line? (Shade a line up to 1.4) What is the difference between these two amounts? How can we figure that out? (Count up from 1.4 to 1.65).

15. Record with symbols.

16. Explain: You have several pages of number lines at your tables as well as Decimal +/- boards. Student Page C contains a table of data related to plant growth. All data is measured in decimals as parts of the one unit of length. Student Pages D and E contain problems to be solved based on the data table on Student Page C.

Small Group/Partner Work

16. Work together in your groups to find out how to use number line, and grids to add or subtract decimals. You can draw on any number line. Be prepared to share your strategies in large group discussion.

17. Provide students in groups with several pages of the number lines, Decimal +/- boards, and Student Pages B – D.

Wrap Up

18. Ask select students to share their solutions to problem 5. This is a “take-away” subtraction problem. Discuss how they used the number line differently for this subtraction problem than for problem 1 (compare problem). Conclude that both are subtraction. For both problems, record using

Comments

Students seem to have difficulty identifying a compare problem as subtraction. We found having students do a sketch first helped them see the need to subtract.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>symbols in vertical form.</td>
<td></td>
</tr>
</tbody>
</table>

19. Ask student to go back to each problem and record with symbols (vertical format). They should verify that their answers are the same whether solving using number line, grids or symbols.

Translations:
- Picture to Picture to verbal
- Real-life to pictures to symbols to verbal
Look for examples among students’ work where students have been careful in recording their work on the number lines. Ask these students to share their ways of labeling the number lines with others.

Below find examples of students’ work that clearly informs the teacher how the student used the number line to solve a decimal addition or subtraction problem using a number line. These items came from a decimal test students in our teaching experiments took at the end of the decimal lessons.

Beth’s plan was 0.88 units tall. She cut off the flower part that was .45 units tall. How tall was her plant now? Use the number line below to solve the problem.

Chase’s plant was .25 units on Monday. It grew .3 units in two weeks. How tall was his plant at the end of two weeks? Show how to solve this problem on the number line.
Decimal Addition and Subtraction Board
Mental Math: Find the sum or difference without using fraction circles or paper and pencil

<table>
<thead>
<tr>
<th>0.35 + 2.05 =</th>
<th>0.90 + 0.15 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90 - 0.14 =</td>
<td>2.85 - 1.05 =</td>
</tr>
</tbody>
</table>
Addition/Subtraction Boards
Adding and Subtracting with Decimals

In science class a group of students planted sunflowers seeds. These plants grow quickly. The students kept very accurate measurements noting their growth from one week to the next week. The table shows the information four students in class collected.

Plant Growth for 4 Students in Mr. Leavitt’s Science Class

<table>
<thead>
<tr>
<th>Student’s Name</th>
<th>Height at the end of week 1</th>
<th>Height at the end of week 2</th>
<th>Height at the end of week 3</th>
<th>Height at the end of week 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guadalupe</td>
<td>.27 meters</td>
<td>.5 meters</td>
<td>.9 meters</td>
<td>1.45 meters</td>
</tr>
<tr>
<td>Malik</td>
<td>.18 meters</td>
<td>.35 meters</td>
<td>.72 meters</td>
<td>1.4 meters</td>
</tr>
<tr>
<td>Kelly</td>
<td>.3 meters</td>
<td>.55 meters</td>
<td>1.03 meters</td>
<td>1.95 meters</td>
</tr>
<tr>
<td>Mohamed</td>
<td>.2 meters</td>
<td>.43 meters</td>
<td>.86 meters</td>
<td>1.6 meters</td>
</tr>
<tr>
<td>PROBLEM 1</td>
<td>PROBLEM 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How tall was Guadalupe’s plant at the end of week 1? ________</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How tall was Guadalupe’s plant at the end of week 2? ________</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How much did Guadalupe’s plant grow from week 1 to week 2? ________</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE NUMBER LINE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How tall was Guadalupe’s plant at the end of week 4? ________</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guadalupe’s plant grew .27 meters between week 4 and week 5.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How tall was Guadalupe’s plant at the end of week 5? ________</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE DECIMAL + - BOARD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PROBLEM 3</th>
<th>PROBLEM 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>How tall was Malik’s plant at the end of week 3? ________</td>
<td></td>
</tr>
<tr>
<td>How tall was Kelly’s plant at the end of week 3? ________</td>
<td></td>
</tr>
<tr>
<td>How much taller was Laneesha’s plant than Malik’s? ________</td>
<td></td>
</tr>
<tr>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE NUMBER LINE</td>
<td></td>
</tr>
<tr>
<td>How tall was Mohamed’s plant at the end of week 4? ________</td>
<td></td>
</tr>
<tr>
<td>Mohamed’s plant grew .09 meters between week 4 and week 5.</td>
<td></td>
</tr>
<tr>
<td>How tall was Mohamed’s plant at the end of week 5? ________</td>
<td></td>
</tr>
<tr>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE NUMBER LINE</td>
<td></td>
</tr>
<tr>
<td>PROBLEM 5</td>
<td>PROBLEM 6</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>How tall was Malik’s plant at the end of week 4? ________</td>
<td>Which plant is the tallest at the end of week 4? How tall is it? ________</td>
</tr>
<tr>
<td>He cut the flower off to give to his mother. This was about 0.6 of a meter.</td>
<td>Which plant is the smallest at the end of week 4? How tall is it?</td>
</tr>
<tr>
<td>How tall is the stem that is left? _____</td>
<td>What is the difference in height?_______</td>
</tr>
<tr>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE NUMBER LINE</td>
<td>SHOW HOW TO SOLVE THIS PROBLEM USING THE DECIMAL + - BOARD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PROBLEM 7</th>
<th>PROBLEM 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create your own addition story problem and solve using a number line.</td>
<td>Create your own subtraction story problem and solve using the 10 x 10 grid</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas

Lesson 15: Overview

Students learn to show fractions on a number line by making a translation from paper folding to the number line.

Materials

- Paper strips for folding
- Students Pages A, B, and C
- Transparency of final problem

Teaching Actions

Warm Up

Salma was measuring the width of her desk. This is what she found:

If you were to estimate the fraction of the meter that is shaded, which would it be?

a. $\frac{1}{2}$  b. $\frac{3}{4}$  c. $\frac{9}{10}$  d. $\frac{1}{3}$

Large Group Introduction

1. Ask students to use paper strips to show the fraction $\frac{3}{4}$. Ask them to explain their steps (fold in half and half again; shade 3 of 4 equal parts).

2. Display a large number line on the board as noted below (students should follow along using Student Page A).

```
0 1 2 3 4
```

Comments

Before asking students to add and subtract fractions on a number line students need practice identifying fractions on this model.

The greatest problem students have is identifying the unit on the number line.

For example, when you ask students to show $\frac{3}{4}$ on the number line don’t be surprised if many students circle 3. If the whole number line is the unit then 3 is $\frac{3}{4}$ of 4. But with a number line the unit is a distance between 0 and 1. This unit is then iterated to show multiple units. This is unique to this model for fractions.
### Teaching Actions

3. Have students describe the number line to you.  
   Ask: What is the unit?

4. State: Let’s repeat the steps you just went through to show \( \frac{3}{4} \) with paper strips on the number line.
   - What did you do first with the paper strips? (Partition into 4 equal parts)
   - How can we do that on the number line? Where would numbers less than 1 but greater than 0 be on the number line?
   - How did you show 3 of 4 equal parts on the paper strips? How might we show 3 of 4 equal parts on the number line?
   - If you wanted to show \( \frac{2\frac{3}{4}}{2} \) with paper strips what would you need? (3 paper strips)  
     Where would \( \frac{2\frac{3}{4}}{2} \) be on the number line? (Between 2 and 3).

5. Have someone describe to the class how to locate \( \frac{1\frac{4}{5}}{2} \) would be on the first number line. Have everyone partition the first number line on Student Page A and mark where \( \frac{1\frac{4}{5}}{2} \) would be on the line. [Students should partition the unit between 1 and 2 into fifths].

6. Have students identify where \( \frac{2}{4} \) would be on the same line? Ask: Can you name \( \frac{2}{4} \) in another way? (\( \frac{1}{2} \)). Ask: To do this what lines must you ignore?

7. Explain: We know that \( \frac{2}{4} \) also equals \( \frac{4}{8} \). How can you change the number line from 0 to 1 to show that?

### Comments

Students need Student Page A for this large group development. Students use these number lines to follow along with what you show on the large number line on the board.

Possible ways to show \( \frac{3}{4} \):
Teaching Actions

8. Ask: how many 8ths equals $\frac{3}{4}$?

9. On the second number line on Student Page A, ask students to find $1\frac{3}{8}$. Have a discussion as to how to do this as the line is partitioned only into halves.

10. On the third number line ask students to show $2\frac{2}{3}$. Discuss how to do this given that the line is partitioned into 6ths.

Small Group/Partner Work

11. With partners, work through Student pages B and C. Select students to share their strategies at the overhead once the majority of students are done.

Wrap Up

12. After sharing responses to the group work, end class with this problem.

What is this point called. Is it $\frac{2}{6}$, $\frac{2}{7}$, 2, or $\frac{2}{4}$?

Translations:
- Manipulative to Picture to Verbal
- Symbol to Picture
- Symbol to Picture to Verbal

Comments

Make overheads of Student Pages B and C so students can show during the wrap up how they solved each problem. We found that students do like to present their solutions to the whole group. See additional notes for the teacher for sample student work.

- Students might say the arrow is pointed to $\frac{2}{7}$ if they count the number of partitions by the number of slash marks and not the number of equal parts between 0 and 1.
- Students might say the arrow is pointed to 2 if they count the slash marks as 0, 1, 2, 3...
- Students may say that the arrow is pointed to $\frac{2}{4}$ if they identify the point under the arrow as 2 and compares that amount to the 4 partitions from that point to 1.
Students approached locating a fractional amount on a number line in different ways. Consider these three examples from students’ work.

**Problem 7:** Locate $\frac{2}{3}$ and $1\frac{1}{3}$ on this number line. What is another name for both fractions?

Notice that the student re-partitioned the number line into thirds first and then labeled the location for $\frac{2}{3}$ and $1\frac{1}{3}$.

**Problem 7:** Locate $\frac{2}{3}$ and $1\frac{1}{3}$ on this number line. What is another name for both fractions? $\frac{4}{6} + \frac{4}{3}$

In this example the student found equivalent fraction for $\frac{2}{3}$ first. Then partitioned the number line to show thirds.

**Problem 4:** Locate $\frac{2}{3}$ and $2\frac{2}{6}$ on this number line. What is another name for $\frac{4}{6}$?

The student only estimated the location of the two fractions. She needs to show the partitions.
What is this point called?

Is it 2/6, 2/7, 2, or 2/4?
Salma was measuring the width of her desk. This is what she found:

If you were to estimate the fraction of the meter that is shaded, which would it be?

a. $\frac{1}{2}$   
b. $\frac{3}{4}$   
c. $\frac{9}{10}$   
d. $\frac{1}{3}$
Fractions and the Number Line

0 1 2 3 4

$0, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, 4$
Fractions and the Number Line

Problem 1: Locate $\frac{1}{2}$ and $\frac{3}{4}$ on this number line.

Problem 2: Locate $\frac{3}{8}$ and $\frac{6}{8}$ on this number line. What is another name for $\frac{6}{8}$?

Problem 3: Locate $\frac{1}{6}$ and $\frac{2}{6}$ on this number line. What is another name for $\frac{2}{6}$?

Problem 4: Locate $\frac{2}{3}$ and $\frac{2}{6}$ on this number line. What is another name for $\frac{2}{3}$?
Problem 5: Locate $\frac{6}{8}$ and $2\frac{2}{8}$ on this number line. What is another name for both fractions?

Problem 6: Locate $\frac{3}{4}$ and $1\frac{1}{2}$ on this number line. What is another name for both fractions?

Problem 7: Locate $\frac{2}{3}$ and $1\frac{1}{3}$ on this number line. What is another name for both fractions?

Problem 8: Locate $\frac{5}{6}$ and $1\frac{2}{4}$ on this number line. What is another name for both fractions?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Lesson 16: Fraction Operations and Initial Decimal Ideas

Students make connections between adding and subtracting fractions using a symbolic procedure to using a number line.

Materials
- Large classroom display of the first 6 number lines on Student Page A
- Red ribbon
- Student Pages A-H.
- Transparencies of large group story problems

Teaching Actions

Warm Up

Tom was in a hurry when he was doing his homework. He has a picture that is partially drawn. What number is shown on the number line?

Large Group Introduction

1. Remind students that they are able to add fractions using fraction circles and with symbols. Explain that in this lesson, they will see how to model adding and subtracting fractions on a number line.

2. State clearly that the goal is for them to explain how to use a number line to model fraction addition and subtraction.

3. Direct students’ attention to the 6 number lines on the board. Students should have Student Page A showing the same number lines.

Comments

The purpose of this lesson is to reinforce students’ understanding of the addition and subtraction procedure for fractions by making symbolic to picture translations using the number line as the picture.

Students understanding of the algorithm for adding and subtracting fractions is strengthened by asking them to make sense of adding and subtracting on a number line. Students use the algorithm to make sense of this new model. Consider what the students will need to do to use the number line to add and subtraction fractions:

- The decision as to what number line to use to add or subtract two fractions is equivalent to finding common denominator.

- Identifying each fraction as an equivalent amount on the number line relies on students’ ability to find equivalent fractions symbolically.

Students should note that the lines are equal in length; the distance from 0 to 1 is the same. The lines are partitioned into different number of
### Teaching Actions

Ask: How are these number lines alike and different?

4. Present this story problem: Jacob ran $\frac{2}{3}$ of a mile and stopped to tie his shoelaces. He then ran another $\frac{1}{2}$ of a mile. Did he run more or less than one mile?

5. Estimate: Is the amount $>$ or $<$ $1$? Greater or less than $2$? More or less than $1 \frac{1}{2}$?

6. Show the ribbons cut into $\frac{2}{3}$ length and $\frac{1}{2}$ length. Place the ribbons on the first number line (no fractional amounts shown). Mark the spot on the number line that shows the length of the two ribbons combined.

7. Comment on their estimate. Ask: What is the exact amount? You know how to do this with fraction circles. You know how to do this with symbols. Now you need to figure out how to model this on the number line.

8. Ask: Which number line might be the best one to show how much Jacob ran in all? Try their suggestions. Ask students to explain their reasoning.

9. Move the ribbons to the number line partitioned into sixths. Ask: $\frac{2}{3}$ is equal to how many 6ths? $\frac{1}{2}$ is equal to how many 6ths? What is the total number of miles that Jacob ran?

10. Ask: Why is the 6ths number line better for solving this problem than the number line showing 3rds? (Both fractions can be easily modeled on the number line showing 6ths).

11. Ask: How might you use symbols to show what you did on the number line?

### Comments

equal parts.

Prepare 2 lengths of ribbon. One is $\frac{2}{3}$ the length of the unit of the number line you drew on the board. The other is $\frac{1}{2}$ the length of the unit.

You may want at this point to step back and let students try to do this on their own as opposed to guiding them through the steps.

The idea is for students to build on their prior experience with equivalence and operating with fractions and symbols to see that the number line representing the common denominator is the best choice. For example a student working on $\frac{3}{4} + \frac{1}{3}$ explained that he would use the “12 number line because they both go into it.”

Number line for $\frac{2}{3} + \frac{1}{2}$
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\frac{2}{3} = 4 \frac{1}{6} = \frac{3}{6}, \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6})$</td>
<td></td>
</tr>
<tr>
<td>12. Repeat for $\frac{1}{4} + \frac{7}{8}$</td>
<td></td>
</tr>
<tr>
<td>13. Ask: In what ways is adding fractions using the number line the same as using fraction circles? How is it the same as using just symbols?</td>
<td></td>
</tr>
<tr>
<td>14. Suggest that they imagine adding $\frac{3}{7} + \frac{1}{4}$ using a number line. Ask: What type of number line would you need to add these two fractions?</td>
<td>Students will need extra copies of the number lines (Student Page A). As you watch students do the class work notice if they are labeling the number lines. If not, encourage students to label the number lines to clearly show the numbers involved.</td>
</tr>
</tbody>
</table>

**Small Group/Partner Work**

15. Provide practice using Student Pages B-H. As students work on these problems, stop by each group and ask students to explain what they are doing.

16. Student Pages F-H ask students to construct for themselves a way to model subtraction on the number line. On the first story problem students may draw in two lines and compare lengths; on the take away problem, students may draw one line and count back.

**Wrap Up**

17. End class by sharing students’ strategies for subtraction.

18. Help students to verbalize how finding answers on the number line is the same as adding or subtracting finding common denominators.

**Translations:**
- Real life to verbal to pictures to symbols
- Symbols to pictures to symbols
Tom was in a hurry when he was doing his homework. He has a picture that is partially drawn. What number is shown on the number line?
Adding Fractions on the Number Line

In this activity you are to show that you understand how to use a number line to model the sum of two fractions. In your groups, talk with each to answer each question. Record your responses on this page. You will need additional copies of the number lines on Student Page A.

Problem: \( \frac{3}{4} + \frac{1}{3} = \)

1. Which number line will you use to model this problem? Explain why you chose that number line.

2. Where is \( \frac{3}{4} \) on this number line? How do you know?

3. What will you do to add on the fraction \( \frac{1}{3} \)? How will you show that on the number line?

4. How can you read the number line to determine the exact answer?
Problem: \( \frac{2}{3} + \frac{2}{9} = \)

1. Which number line will you use to model this problem? Explain why you chose that number line.

2. Where is \( \frac{2}{3} \) on this number line? How do you know?

3. What will you do to add the fraction \( \frac{2}{9} \)? How will you show that on the number line?

4. How can you read the number line to determine the exact answer?
Adding Fractions on the Number Line

Show each problem on the best number line. Record your answer below each problem, showing the equivalent fractions you used.

\[
\frac{3}{12} + \frac{1}{6} \quad \frac{2}{3} + \frac{1}{2}
\]
\[
\frac{2}{5} + \frac{1}{2}
\]

\[
\frac{1\frac{1}{3}}{3} + \frac{2}{9}
\]
How can you subtract using the number line?
With your partner show how to solve each problem using the number line. Are you using the number line in the same way for each problem? Be ready to present your ideas to class.

Problem 1:  Addis lives $\frac{7}{8}$ of a mile from school. Xander lives $\frac{1}{4}$ of a mile from school. Who lives farther from school? How much farther? Solve this problem on the number line of your choice.

Describe how you used the number lines to solve this problem:
Problem 2: Hannah lives $\frac{4}{5}$ miles from school. She rides her bike everyday. On Monday her bike broke down after riding about $\frac{1}{2}$ of a mile. How far did she have to walk?

Describe how you solve the problem using the number line.
Problem 3: Solve these two problems on the number line.

\[
\begin{align*}
1\frac{3}{4} - 1\frac{1}{2} & \quad \quad \quad 1\frac{2}{3} - \frac{4}{6}
\end{align*}
\]
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
### Rational Number Project

#### Fraction Operations and Initial Decimal Ideas

**Lesson 17: Overview**

| Students use pictures and fraction circles to find the product of a whole number and a fraction. Students explain that the expression $a \times b$ can be read as “$a$ groups of $b$”. |

**Materials**

- Fraction circles
- Student Pages A and B
- Transparencies 1-3

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#### Teaching Actions

**Warm Up**

Draw a picture that you can use to solve the following word problem. Write a number sentence for the problem.

Riley wants to give five cookies to each of his three friends. How many cookies will he need?

---

**Large Group Introduction**

1. Go over the warm up problem asking students to share their pictures and number sentences. A possible picture might look like this. Students’ pictures should show three groups of five cookies.

2. Write $2 \times 4 = ___$ on the board or overhead. Ask the students to explain what the 2 and the 4 stand for in

---

**Comments**

It is important for students to notice that this problem can be modeled using the multiplication sentence: $3 \times 5 = 15$. Many students will have trouble with this notation, especially when using fractions. Help students make connections between the words 3 groups of 5 and the mathematical expression $3 \times 5$. These connections will have to be developed throughout the next few lessons.

Some students may write $5 \times 3 = 15$. This notation is correct but you may want your class to adopt the convention that $3 \times 5$ means “3 groups of 5” and $5 \times 3$ means “5 groups of 3”. This language helps give a deeper understanding of the mathematics involved in multiplying numbers and the vocabulary “groups of” will be emphasized throughout the lessons.

Some students may say suggest writing $5 + 5 + 5 = 15$ which is correct and should be introduced by the teacher if no student suggests this notation. Explain that $5 + 5 + 5$ is equivalent to $3 \times 5$. Students should be able to go forward and backwards when solving this
Teaching Actions

this problem. The 2 should stand for the number of groups and the 4 should stand for the number of objects in each group. Ask students to draw a picture to find the answer $2 \times 4 = \_\_$. 

A few students may draw a picture similar to the one below.

If you see this, ask the student how this picture shows two groups of four. You might ask, “Where are the two groups?”

3. Ask the students to solve the following problem with their fraction circles. Have them also write a multiplication sentence that would answer this problem. (Transparency 1)

Seth wants to give each of his 4 friends $\frac{2}{5}$ of a pizza. How much pizza will he give away?

The mathematical sentence will be $4 \times \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$ where the 4 represents the number of groups, the $\frac{2}{5}$ represents the amount in each group, and $\frac{8}{5}$ or $1\frac{3}{5}$ represents the total amount of pizza given away. Emphasize that the multiplication sentence means there are 4 groups of $\frac{2}{5}$. Write the words underneath the multiplication sentence.

Ask: How is this problem similar to the cookie problem we did previously?

Comments

problem. Question 1 asks students to go from real-world situation to picture and written symbols. This question asks them to go from written symbol to picture.

Walk around as students work and ask two or three students to prepare transparencies showing their pictures. Have the students show and explain their work. Be sure to emphasize that they draw 2 groups of 4 objects.

The picture to the left is a picture of the symbolic number sentence and does not model two groups of four. The students only need to show two groups of four, not an equation.

The students should put 4 groups of $\frac{2}{5}$ as shown below and the sum should be $\frac{8}{5}$ or $1\frac{3}{5}$.

The goal for this part of the lesson is for students to see that we are modeling a multiplication problem using fraction circles.

The number of groups for all the problems in this lesson will be whole numbers because it can be difficult for students to imagine a fractional number of groups. The amount in each group can be a whole number.
Teaching Actions

4. Ask the students to solve the following problem with their fraction circles. Again they will need two sets of fraction circles to be able to find an answer. (Transparency 2)

\[ 5 \times \frac{1}{3} \]

Ask what the 5 means in the sentence and what the \( \frac{1}{3} \) means.

5. Ask the students to individually find the product below using fraction circles. Make sure that they show 3 groups of \( \frac{2}{3} \).

\[ 3 \times \frac{2}{7} \]

Small Group/Partner Work

6. Assign Student Pages A and B.

Wrap Up

7. Review select problems from Student Pages A & B after most of the students are done. Be sure to emphasize the different representations (i.e. words, multiplication sentence, picture, real-world situation) for each multiplication problem. Students can come up and show how they did select problems.

8. Close the lesson by trying to get the students to find multiplication and addition sentences for both problems below, if appropriate. (Transparency 3)

Comments

or a fraction as is done in this example.

Students should see this as 5 groups of \( \frac{1}{3} \). The final answer is \( \frac{5}{3} \) or \( 1\frac{2}{3} \). It is OK if students use improper fractions here. Have students record the answer both as an improper fraction and a mixed number.

Be sure to emphasize that they are showing 3 groups of \( \frac{2}{3} \).

Make a transparency for both pages. Make sure to note some of the problems they struggle with so you can review at the end.

Look at the drawings the students are using for Student Page B. Some students may use circles while others may use rectangles. Be sure to show examples of each when reviewing. Emphasize that the unit for each picture needs to stay the same throughout a problem.
### Teaching Actions

| Addition Sentence: \( \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1 \)  
| Multiplication Sentence:  
| \( 3 \times \frac{1}{3} = \frac{6}{3} = 2 \)  

| Addition Sentence: \( \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \)  
| Multiplication Sentence:  
| Not appropriate. The amount in each group needs to be the same. |

### Translations:
- Real-world to picture
- Real-world to written symbol
- Picture to written symbol
- Real-world to manipulative
- Written symbol to manipulative
Additional Notes to the Teacher
Lesson 17

One goal of this lesson is for students to use pictures and fraction circles to multiply a whole number and a fraction. Another goal for this lesson is to help students make a connection between the notation a x b and the phrase “a groups of b”.

In this lesson we introduce clouds as a way to separate groups. We originally used circles for the groups and put fraction circles inside. Students seem to confuse the circle denoting the groups with the circles used for the fraction circle picture.

We created on a 10 by 12 piece of paper 6 blank pictures of clouds. Students used this sheet when they solved problems in lessons 17 and 18. The sheet provides an easy way for the students to keep track of groups and an easy place to organize their fraction circle pieces as they model multiplication problems. You may want to make up a similar page.

It is important for students to notice that a multiplication problem can be modeled using a multiplication sentence as for example: 2x4=8. Many students will have trouble with this notation when using fractions. Make sure you emphasize the connections between the words “2 groups of 4” and the mathematical expression 2x4. These connections will be developed throughout the next few lessons.

A convention that is typically used in classrooms in the United States is that the mathematical expression 3 x 5 represents “3 groups of 5”. In many European countries the expression 3 x 5 means “3 five times” or 5 groups of 3. This curriculum adopts the convention a x b means “a groups of b” but students who say that a x b means “b groups of a” are technically correct. You may want to explain that conventions are used in mathematics so that we communicate the same idea when using mathematical notation.
The three representations for multiplication in this and the next lesson are pictures, words, and symbols. The goal is for students to make connections among these representations as well as using these representations to make sense of situations involving multiplication.

A few students may draw a picture similar to the one below when you ask them to draw a picture for $2 \times 4$. This is a picture of the symbolic number sentence and does not model two groups of four. The students only need to show two groups of four, not an equation.

If you see this, ask the student how this picture shows two groups of four. You might ask, “Where are the two groups?” Some students may be able to correctly explain that the right hand side of the equation shows two groups of 4. The left hand side does not show this clearly and the notation should be discouraged.
Use Fraction Circles

Seth wants to give each of his 4 friends $\frac{2}{5}$ of a pizza. How much pizza will he give away?

Words:

Multiplication Sentence:
Find Using Fraction Circles

\[ 5 \times \frac{1}{3} = \______ \]

\[ 3 \times \frac{2}{7} = \______ \]
Write a multiplication sentence and an addition sentence for each problem below, if appropriate:
Draw a Picture

Riley wants to give five cookies to each of his three friends. How many cookies will he need?
Multiplying Fractions
(whole number x fraction)

Write a multiplication sentence for each picture shown below. Each cloud contains a group.

1. Unit is one cookie (🍪)

Words:
3 groups of ____

Multiplication Sentence:
____ x ____ = ____

2. Unit is

Words:

Multiplication Sentence:

3. Unit is

Words:

Multiplication Sentence:

4. Unit is

Words:

Multiplication Sentence:
Show how to find a solution by drawing a picture, writing out in words, and writing a multiplication sentence. Use fraction circles when necessary.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.</strong> Ginger the dog buries 4 bones in each hole that she digs. How many bones does she bury in all if she digs 6 holes?</td>
<td><strong>6.</strong> Ming uses $\frac{1}{4}$ a cup of brown sugar for each batch of chocolate chip cookies she makes. How much brown sugar will she need if she makes 3 batches?</td>
</tr>
<tr>
<td>Picture:</td>
<td>Picture:</td>
</tr>
<tr>
<td>Words:</td>
<td>Words:</td>
</tr>
<tr>
<td>Multiplication Sentence:</td>
<td>Multiplication Sentence:</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.</strong> Each serving of Brown-Sugar Oatmeal contains $\frac{3}{8}$ grams of sodium. How many grams of sodium will be in 4 servings of oatmeal?</td>
<td><strong>8.</strong> Mia swims $\frac{3}{2}$ of a mile every day. How many miles will she swim in 4 days?</td>
</tr>
<tr>
<td>Picture:</td>
<td>Picture:</td>
</tr>
<tr>
<td>Words:</td>
<td>Words:</td>
</tr>
<tr>
<td>Multiplication Sentence:</td>
<td>Multiplication Sentence:</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Students multiply a whole number and a fraction using fraction circles, pictures, and mental images.

Materials
- Fraction Circles
- Student Pages A, B, and C
- Transparency 1

Teaching Actions

Warm Up

Solve using your fraction circles.

Scout walks \( \frac{2}{10} \) of a mile each day on her treadmill. How many miles will she walk in 4 days?

Large Group Introduction

1. Write \( \frac{2}{3} \) on the board. Ask the students to picture this fraction in their minds. Ask a few students to describe what they are picturing.

2. Write a 5 and a multiplication sign before the \( \frac{2}{3} \). Ask the students to picture what the following statement would represent.

   \[ \frac{2}{3} \times \frac{5}{3} \]

   Ask the students to draw a picture of this problem to find the product.

3. Walk around as students work and ask two or three students to prepare transparencies showing their

Comments

After all students are able to display the answers individually, review this problem by writing out the words and multiplication sentences.

\[ 4 \times \frac{2}{10} = \frac{8}{10} \]

Some students may picture the brown fraction circle pieces while others may picture a rectangle similar to paper strips. Some may picture the numerals 2 over 3.

The goal for this lesson is for students to draw pictures for fraction multiplication and create mental images for counting the product.

For example, if a student is trying to determine what 5 groups of \( \frac{2}{3} \) would total they should picture the two-thirds as two brown pieces. They would imagine that 5 groups of 2 brown pieces would be 10 brown pieces and the total would be \( \frac{10}{3} \).
Teaching Actions

pictures. Have the students show and explain their work. Be sure to emphasize that they draw 5 groups of \( \frac{2}{3} \).

![Sample student work for 5 groups of two-thirds]

Small Group/Partner Work

4. Assign Student Pages A and B. Help students get started on the first problem by doing it with them. Have them complete the rest of the problems as you circulate and help individual groups of students.

Wrap Up

5. Review problems from Student Pages A, B, and C (as appropriate) after most of the students are done. Emphasize the different representations (i.e. words, multiplication sentence, picture, real-world situation) and translations for each multiplication problem you review.

6. Write the problem \( 3 \times \frac{1}{5} \) on the board. Cover up the 3 and the multiplication sign with your hand. Ask them to picture the fraction \( \frac{1}{5} \). Ask one student to tell the class what they are picturing. [Hopefully someone will state they see an orange fraction circle piece or one-fifth of a paper strip].

Comments

The 10 represents the total number of pieces and the 3 in the denominator represents the size of each piece.

Make sure to note some of the problems they struggle with so you know which problems you should review. Pass out Student page 18C to students if they successfully complete Student Pages A and B.

You do not need to review all of the problems from the sheet, just one or two that provide interesting discussion points.

Be sure to focus on how the students explain how they get their answers.

Some students may say that the answer to the problems can be found by multiplying the numerator of the fraction by the whole number. This is a good observation but make sure the focus is that this method counts...
## Teaching Actions

7. Uncover the multiplication problem and ask them to picture what this statement is telling them. [Hopefully some will be picturing three groups of one orange piece. They should state that three orange pieces are \( \frac{3}{5} \) of the black circle].

8. Complete the multiplication sentence by writing \( 3 \times \frac{1}{5} = \frac{3}{5} \).

You may want to ask the students why they did not multiply both the numerator and the denominator by 3 when doing this problem.

9. Repeat the same process one at a time with the following examples.

\[
\begin{align*}
4 \times \frac{3}{10} \\
3 \times \frac{7}{8} \\
7 \times \frac{5}{12}
\end{align*}
\]

## Comments

the number of pieces total.

The product of the whole number and the numerator in the fraction count the number of pieces while the denominator determines the size of the pieces.

We will be working on the algorithm for multiplying fractions in a couple lessons so resist writing a 1 under the whole number and using the fraction multiplication algorithm until later.

## Translations:

- Real-world to picture
- Symbol to picture to verbal
- Real-world to written symbol
- Picture to written symbol
- Real-world to manipulative
Lesson 18

A common mistake for some students on Student Page B is shown below. The student wrote the correct answer for $2 \times \frac{2}{5}$ in the top row but incorrectly stated the answers to the Multiplication Sentence problems on the next two rows. Both answers are missing the denominators. Note that the picture correctly show that 4 groups of $\frac{5}{6}$ is $\frac{20}{6}$ but this student and his partner chose to write 20 instead of $\frac{20}{6}$. Be sure to emphasize that the numerator gives the number of pieces and the denominator gives the number of equally-sized pieces that the unit is divided. Also emphasize that 20 represents 20 full circles and $\frac{20}{6}$ represents 20 pieces that are one-sixth of a circle in size.

<table>
<thead>
<tr>
<th>Multiplication Sentence</th>
<th>Words</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times \frac{2}{5} = \frac{4}{5}$</td>
<td>2 groups of $\frac{2}{5}$</td>
<td>![Picture 1]</td>
</tr>
<tr>
<td>$4 \times \frac{5}{6} = 20$</td>
<td>4 groups of $\frac{5}{6}$</td>
<td>![Picture 2]</td>
</tr>
<tr>
<td>$5 \times \frac{5}{4} = \frac{25}{4}$</td>
<td>5 groups of $\frac{5}{4}$</td>
<td>![Picture 3]</td>
</tr>
</tbody>
</table>

Student work from Student Page B
Mentally Picture

\[ 3 \times \frac{1}{5} \]

\[ 4 \times \frac{3}{10} \]

\[ 3 \times \frac{7}{8} \]

\[ 7 \times \frac{5}{12} \]
Use Fraction Circles

Scout walks \( \frac{2}{10} \) of a mile each day on her treadmill. How many miles will she walk in 4 days?
Write a multiplication sentence for each picture shown below. Each cloud contains a group.

1. Unit is
   
   Words: 
   _____ groups of _____
   
   Multiplication Sentence: 
   ____ x ____ = _____

2. Unit is
   
   Words: 
   
   Multiplication Sentence: 

Draw a picture and write a multiplication sentence for the given problems.

3. **5 groups of** \( \frac{3}{4} \)
   
   Picture:
   
   Multiplication Sentence: 

4. **4 groups of** \( \frac{1}{8} \)
   
   Picture:
   
   Multiplication Sentence:
Draw a picture for each mathematical sentence written below and complete the sentence.

5. \( 7 \times 5 = \) ___

6. \( 2 \times \frac{3}{4} = \) ___

Fill in the missing blanks.

<table>
<thead>
<tr>
<th>Multiplication Sentence</th>
<th>Words</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 \times \frac{2}{5} = ) ____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 groups of ( \frac{5}{6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 5 \times \frac{5}{4} = ) ____</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Write a story problem to represent each sentence below then draw a picture to find the answer and complete the sentence.

<table>
<thead>
<tr>
<th></th>
<th>Story Problem:</th>
<th>Picture:</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>$4 \times 7 = ___$</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>$5 \times \frac{1}{4} = ___$</td>
<td></td>
</tr>
</tbody>
</table>

---

Try to mentally picture the following problems. Write the product without drawing pictures. Explain a shortcut someone could use to find the answers quickly.

<table>
<thead>
<tr>
<th></th>
<th>9. $7 \times \frac{1}{8} =$</th>
<th>10. $8 \times \frac{2}{11} =$</th>
<th>11. $7 \times \frac{4}{5} =$</th>
</tr>
</thead>
</table>

Describe your shortcut for getting the answers to problems 9 to 11:
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Lesson 19: Overview

Students use number lines to multiply a whole number by a fraction.

Materials

• Student Pages A, B, and C
• Transparencies 1 and 2

Teaching Actions

Warm Up

Faisel makes $12 an hour. How much does he make in 3 hours of work? Write a multiplication sentence that describes how to find the answer.

3 x 12 = 36

number of groups (hours)  number in each group ($/hour)  amount earned in 3 hours ($)

Ask the students the following questions:

• What does the 3 represent in the multiplication sentence? The 3 represents the number of hours Faisel worked.
• What does the 12 represent in the multiplication sentence? The dollars earned per hour.
• What does the 36 represent in the multiplication sentence? The amount of money earned in 3 hours.
• Does the multiplication sentence represent 3 groups of 12 or 12 groups of 3? Why?

In the previous two lessons we tried to develop meaning for the factors in a multiplication sentence. The first factor is typically the number of groups. The second factor is the number of objects in each group. Many real world situations make it difficult to have a fractional number of groups.

When we work with rate multiplication situations as shown in the first problem, the first factor is the amount of time (# of groups), the second factor is the rate (amount in each group), and the result represents the amount of money earned. It is easier to make either factor a fraction in a rate situation.
Teaching Actions

Large Group Introduction

1. Show the students how they can solve the word problem from the warm-up using a number line.

   Explain that a number line can be used to show the amount of money that Faisel earns when he works a certain number of hours. Take out transparency 1 and give students Student Page A.

<table>
<thead>
<tr>
<th>0</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>amount of money earned (dollars)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Fill in the amounts for the first two hours with the students. Explain that a jump from 0 to the first tick mark represents one hour of work and the $12 represents the amount Faisel will earn in one hour. The $24 represents how much Faisel will earn in two hours of work.

<table>
<thead>
<tr>
<th>0</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>amount of money earned (dollars)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Ask the students to fill in the numbers below the tick marks of the number line on their paper.

<table>
<thead>
<tr>
<th>0</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
<th>72</th>
<th>84</th>
<th>96</th>
<th>108</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>amount of money earned (dollars)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Ask the students to explain how the number line can be used to find the amount of money Faisel will earn if he works 3 hours.

2. Ask individual students how the number line can be used to show how much Faisel will earn if he works 2 hours, 7 hours, 9 hours. Have the students use their pencils or fingers to trace along the same number line they have been using. Have them start tracing at 0 and make jumps for each hour worked.

   Students typically have a hard time using a number line when representing numbers. Emphasize the length characteristics of the number line by starting at 0 and showing the jumps of 12.
3. Ask the students to use number line I on Student Page A to find the amount of money Faisel will earn if he works 5 hours. Encourage them to draw in the jumps on the number line and then write a multiplication sentence that solves the problem. The multiplication sentence should be $5 \times 12 = 60$.

4. Ask the students to use the number line II to find the exact amount of money that Faisel will earn if he works $6 \frac{1}{2}$ hours.

Have a student or two come to the front of the classroom and explain how they solved the problem using the number line.

Ask them to write a multiplication sentence for this problem and explain the meaning of each factor:

$$6 \frac{1}{2} \times 12 = 78$$
Write the meaning of each factor on the board as shown below.

\[
\begin{array}{cccc}
6 \frac{1}{2} & \times & 12 & = 78 \\
\text{# of hours worked} & \text{amount earned per hour} & \text{amount of money earned} \\
\end{array}
\]

5. Present to the students: “Suppose Faisel works \(\frac{3}{4}\) of an hour.” Ask the students to estimate in their heads how much money Faisel will earn. Ask the students as a group whether this amount would be more than $12 or less than $12. Ask a few students to explain their estimate.

6. Ask the students to use the number line to find the exact amount of money that Faisel will earn if he works \(\frac{3}{4}\) of an hour and write a multiplication sentence. Have them try it on number line III on Student Page A.

\[
\begin{array}{cccc}
0 & 12 & 24 \\
\text{amount of money earned (dollars)} \\
\end{array}
\]

Students work will look something like what is shown below. Many of the students will be able to do this with little prompting.

\[
3 \times \left(\frac{1}{4} \times 12\right) = 9
\]

7. Have several students come to the board and show how they used the number line to solve this problem. Be sure that they show how they divided
Teaching Actions

their number line and explain how found their answer. Also make sure they explain the meaning of each number in the multiplication sentence.

Sample student presentation

Small Group/Partner Work

8. Assign Student Pages B and C.

Wrap Up

9. Review select problems from Student Pages B and C. Be sure to focus on how students multiply a whole number by a fraction by dividing the whole number by the denominator of the fraction then multiplying this result by the numerator.

10. Have the students solve the following problem using a blank number line and write a multiplication sentence. (Transparency 2)

Tyanna earns $20 an hour growing flowers. How much does she earn if she works \( \frac{3}{5} \) of an hour?

amount of money earned (dollars)

Ask the following questions:
• What does a jump from 0 to 20 mean on

Comments

explained that each length represented $3; therefore Faisel will earn $9 if he works for \( \frac{3}{4} \) of an hour.

Make sure to emphasize the translations among symbols, number line, and real world situations as you help students.

Have various students explain how they partitioned the number line.

Students have not created their own number line from scratch. Watch to see if they naturally write tick marks for every $20 and partition the space on the number line between $0 and $20 into fifths.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>the number line? (1 hour of work)</td>
<td></td>
</tr>
<tr>
<td>• How do you jump three-fourths of an hour? (Partition the number line into</td>
<td>(Partition the number line into fifths and jump 3 of these)</td>
</tr>
<tr>
<td>fifths and jump 3 of these)</td>
<td></td>
</tr>
<tr>
<td>• How could you calculate $\frac{3}{5} \times 20$ without using the number line? (Divide 20 by 5 and multiply by 3)</td>
<td></td>
</tr>
</tbody>
</table>

Translations:
- Real world to verbal
- Real world to picture to verbal
- Symbol to picture
Additional Notes to the Teacher
Lesson 19

Partitioning
The main goal of this lesson is for students to use partitioning to find a fractional part of a whole number. For example, when looking at $\frac{5}{6} \times 18$ the students will learn to find the product by dividing 18 into $6^{th}$s to get 3 and multiplying this result by 5 to get 15. This partitioning idea is an essential part of what it means to multiply fractions and will be emphasized throughout the next several lessons.

Number Line for Multiplication
Another goal of the lesson is for the students to learn how the number line can be used to do multiplication of whole numbers. This is emphasized at the beginning of the lesson. The number line is a representation that makes sense to partition. Once students understand how multiplication can be modeled using the number line with whole numbers it makes it easier for them to make sense of expressions like $\frac{3}{4} \times 20$. As students begin to understand that $3 \times 20$ can be modeled on the number line as 3 jumps of 20 they will have a mental image of multiplication that allows them to think about $\frac{3}{4} \times 20$ in a meaningful way. Some students may struggle with relating the symbolic representation with the number line once fractions are introduced.

Labeling the Number Line
We found that students labeled the number lines in two different ways when multiplying a fraction and a whole number. Although the differences between the methods are subtle, the two interpretations of the number line are important to highlight. Both methods involve making tick marks on the number line but the students labeled the amounts in two different ways. Some students labeled the tick marks (Method I) with the amounts while others labeled the intervals (Method II) with the amounts earned.

Method I: Points on a Number Line
The tick marks on the number line represent the amounts of money earned at specific times. The student’s work shown below shows this approach.

![Student work using Method I (with context)](image)

The tick mark labeled 3 represents a passage of time of $\frac{1}{4}$ of an hour. This tick mark is a specific point on the number line and has meaning when referenced to time 0.
Method II: Value of Interval

The student whose work is shown below labeled the intervals with the values.

Notice that this student labeled each interval as being worth 5. This labeling system highlights the value of the interval between two tick marks and does not necessarily reference 0 as above.

The student work below still emphasizes the value of the interval. The interval labeled 3 represents the first \( \frac{1}{4} \) of an hour worked. This student seems to show that if Faisel works the first interval worth of time he will earn $3. The second interval that is labeled 6 might be more accurately labeled 3 since the second interval of time is one-quarter hour that is worth $3. This interval seems to represent a running total.

Although this recording scheme is somewhat misleading (i.e. the value of the interval labeled 6 is $3) it still represents correct thinking and does not need to be corrected. It is important that children share thinking and recording strategies with each other during class discussions.
Tyanna earns $20 an hour growing flowers. How much does she earn if she works $\frac{3}{5}$ of an hour?

- What does a jump from 0 to 20 mean on the number line?
- How do you jump three-fifths of an hour?
- How could you calculate $\frac{3}{5} \times 20$ without using the number line?
Faisel makes $12 an hour. How much does he make in 3 hours of work? Write a multiplication sentence that describes how to find the answer.
Faisel makes $12 an hour.

I. 5 hours  
Multiplication Sentence: ____ x ____ = ____

II. ____ hours  
Multiplication Sentence: ____ x ____ = ____

III. ____ hours  
Multiplication Sentence: ____ x ____ = ____
Multiplying Fractions on Number Lines

Use a number line and write a multiplication sentence to answer each of the questions below.

<table>
<thead>
<tr>
<th>Question</th>
<th>Number Line</th>
<th>Multiplication Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nicki earns $20 an hour fixing cars. Use the number line to determine how much she earns if she works $\frac{1}{2}$ an hour.</td>
<td><img src="image" alt="Number Line" /></td>
<td>$\text{amount of money earned} = \text{amount earned per hour} \times # \text{ of hours worked}$</td>
</tr>
<tr>
<td>2. Joshkin earns $15 an hour cleaning bird cages. How much will he earn if he works $\frac{2}{3}$ of an hour?</td>
<td><img src="image" alt="Number Line" /></td>
<td>$\text{amount of money earned} = \text{amount earned per hour} \times # \text{ of hours worked}$</td>
</tr>
<tr>
<td>3. Emily makes $24 an hour. How much will she make if she works $\frac{1}{8}$ of an hour?</td>
<td><img src="image" alt="Number Line" /></td>
<td>$\text{amount of money earned} = \text{amount earned per hour} \times # \text{ of hours worked}$</td>
</tr>
<tr>
<td>4. Ty earns $24 an hour mowing lawns? How much will he earn if he works $\frac{3}{4}$ of an hour?</td>
<td><img src="image" alt="Number Line" /></td>
<td>$\text{amount of money earned} = \text{amount earned per hour} \times # \text{ of hours worked}$</td>
</tr>
</tbody>
</table>
Use the number line to find the answer to the following problems.

5. \( \frac{1}{2} \times 6 = \) ____

6. \( \frac{3}{8} \times 40 = \) ____

7. \( \frac{2}{3} \times 12 = \) ____

8. \( \frac{5}{4} \times 12 = \) ____
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Lesson 20

Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 20: Overview

Students use double number lines to multiply a whole number by a fraction. Students write a multiplication sentence that represents a cloud or number line picture.

Materials
- Transparencies 1-8
- Student Pages A and B

Teaching Actions

Warm Up

A) Each tray of Gerber daisies weighs \( \frac{3}{8} \) of a pound. How much do 5 trays of Gerber daisies weigh?

Use clouds to show how to find the answer.

B) India earns $18 an hour working at the plant sale. How much will she earn if she works \( \frac{5}{6} \) of an hour?

Show how to solve problem on a number line.

Comments

Have students do the Warm-Up on a piece of paper. The goal of the warm up is for students to review the two different types of fraction multiplication problems they learned in the previous lessons (i.e. whole number x fraction using clouds and fraction x whole number using a number line).

Large Group Introduction

1. Review the two warm-up problems with the students. Look for students who made a double number line similar to one shown below:

   The students seemed to solve warm-up problem A the same way using either circles or rectangles for the fraction.

   The students solved warm-up problem B in quite different but still correct ways. Please see the Teacher Notes for more explanation.
Teaching Actions

If any of your students made a double number line as above, please have them show and explain what they did.

2. Double Number Line
   Model how to make a double number line using problem B from the Warm-Up if no student made one. The steps to building a double number line are shown above.

   ![Double Number Line Diagram]

   The number line shows the amount of money earned on the top and the number of hours on the bottom. You should encourage students to label the number line with a title and units.

   Partition the number line into sixths and label both the times and amounts earned as shown on the left.

   Riley earns $15 an hour raking leaves. How much will he earn if he works \( \frac{2}{3} \) of an hour?

Comments

Begin with a number line as shown to the left. Label the amount of money earned on the bottom. Explain how the time worked matches the amount of money earned (i.e. India earns $18 an hour).

Riley earns $15 an hour raking leaves. How much will he earn if he works \( \frac{2}{3} \) of an hour?
**Teaching Actions**

**Small Group/Partner Work**

4. Assign Student Pages A and B.

**Wrap Up**

5. Use problems A to G on Transparencies 2 – 8 to bring closure to the last 4 lessons on multiplication. Ask students to determine the multiplication sentence that matches the picture or number line. Be sure that students explain their answers.

A.  

![Diagram of 7 groups of 4](image)

\[ 7 \times 4 = 28 \]

B.  

![Number line diagram](image)

\[ \frac{2}{3} \times 24 = 16 \]

C.  

![Number line diagram](image)

\[ \frac{1}{2} \times 40 = 20 \]

**Comments**

Observe students as they do the class work. What types of pictures do they draw for problems 5 – 10?

This closing recaps all the different types of multiplication problems in the past 4 lessons. These problems can be done quickly but make sure to have the students explain how they determined their answers.
Teaching Actions

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>E. $6 \times \frac{2}{5} = \frac{12}{5}$</td>
</tr>
<tr>
<td>F. $5 \times \frac{4}{7} = \frac{20}{7}$</td>
</tr>
<tr>
<td>G. $\frac{3}{2} \times 40 = 60$</td>
</tr>
</tbody>
</table>

Translations:
- Real-world to picture to verbal
- Real-world to picture to written symbol
- Picture to written symbol
- Symbol to picture
Additional Notes to the Teacher
Lesson 20

Most students drew pictures to solve Warm-Up problem A similar to the one shown below. The only differences were in the way that the three-eighths is represented. Most students used circles but a few drew rectangles.

Sample answer for Warm-Up question A

There were several ways that students correctly showed how to solve Warm-Up problem B on the number line. These differences are important to notice and should help you as you teach this lesson.

The number line shown below is typical of what many students did and is the method that was emphasized in the previous lesson. Make sure that students label the units on the number line.

Sample answer for Warm-Up question B

The student drew the number line and labeled the end tick marks 0 and 18. She used the number line to help her partition the $18 into sixths and then determined that five-sixths of $18
is $15. This student shows the five-sixths of the $18 on the number line but does not specifically label these fraction amounts on her number line.

We found that several students naturally made double number lines as shown below.

![Double number line for Warm-Up question B]

This student labeled the number line with dollars on top but labeled the fractional amounts underneath. These fractional amounts show the amount of time in hours that India works and the amounts she would earn. Although no students who did this labeled the fraction amounts as hours worked they were able to explain what they represented. You should encourage the students to label both parts of the number line. The student above also did not label the 0 that represents working 0 hours. Many of the students we worked with forgot to label the 0 on the number line. You should encourage your students to label 0 on the number line when they use them.
Riley earns $15 an hour raking leaves. How much will he earn if he works $\frac{2}{3}$ of an hour?

Show how to do it on a double number line.
What is the multiplication sentence represented in each picture?

A. Unit:  ●
B. Unit: $24 an hour

(amount of money earned (dollars))

(time worked (hours))

0 8 16 24
C. Unit: $40 an hour

<table>
<thead>
<tr>
<th>time worked (hours)</th>
<th>amount of money earned (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
</tbody>
</table>
D. Unit:  

Diagram of flowers with shaded parts.
F. Unit: $40 an hour

<table>
<thead>
<tr>
<th>time worked (hours)</th>
<th>amount of money earned (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
</tbody>
</table>
G. Unit: 

![Diagram of four clouds with shaded sections]
A) Each tray of Gerber daisies weigh $\frac{3}{8}$ of a pound. How much do 5 trays of Gerber daisies weigh?

Use clouds to show how to find the answer.

B) India earns $18\text{ an hour working at the plant sale. How much will she earn if she works }\frac{5}{6}\text{ of an hour?}$

Show how to do it on number line.

amount of money earned (dollars)
Multiplying Fractions on Number Lines

Use a double number line and write a multiplication sentence to answer each of the questions below.

1. Maddie earns $24 an hour. How much will she make if she works $\frac{3}{8}$ of an hour?

   \[ \frac{\text{amount of money earned (\$)}}{\text{amount earned per hour}} = \frac{\text{amount of money earned}}{\text{# of hours worked}} \]

   Hours worked
   0 1
   0 24 48

   Multiplication Sentence:

2. Rachael earns $8 an hour selling popcorn. How much will she earn if she works $\frac{3}{4}$ of an hour?

   \[ \frac{\text{amount of money earned (\$)}}{\text{amount earned per hour}} = \frac{\text{amount of money earned}}{\text{# of hours worked}} \]

   Hours worked
   0

   Multiplication Sentence:

3. There are 24 students that went camping. $\frac{5}{6}$ of the students are girls. How many girls went camping?

   Multiplication Sentence:

4. Andrew earns $40 an hour building Legos. How much will he earn if he works $\frac{7}{4}$ hours?

   Multiplication Sentence:
Solve each problem by drawing a picture or a number line.

5. \[ \frac{1}{2} \times 26 = \]

6. \[ \frac{3}{8} \times 16 = \]

7. Shadow ate \( \frac{4}{4} \) of a brownie. A whole brownie has 120 calories. How many calories did Shadow eat?

8. There are 6 adults in this classroom. Matt wants to give \( \frac{5}{8} \) of a doughnut to each adult. How many doughnuts will he need?

9. Leah buys 36 flowers at the plant sale. She plants \( \frac{2}{9} \) of the flowers in the front yard. How many flowers does she plant in the front yard?

10. Lew wants to sell \( \frac{3}{4} \) of his baseball card collection on E-bay. If he has 224 baseball cards in his collection, how many cards will he be selling?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
**Rational Number Project**

**Fraction Operations and Initial Decimal Ideas**

**Lesson 21: Overview**

Students use patty paper (an area model) to multiply two fractions.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Patty Paper (5 per student)</td>
</tr>
<tr>
<td>• Student Pages A &amp; B</td>
</tr>
<tr>
<td>• Two colors of pencils (or markers) for each student</td>
</tr>
<tr>
<td>• Transparencies 1-5</td>
</tr>
</tbody>
</table>

**Teaching Actions**

**Warm Up**

Problem A:

Use a number line to find the product $\frac{2}{3} \times 21$.

Problem B:

Explain a way to multiply $\frac{3}{5} \times 40$ without using the number line.

Problem C:

Becky earns $12 an hour grading math homework. How much will she earn if she works $\frac{5}{6}$ of an hour?

**Large Group Introduction**

1. Ask the students to solve the following problem: (Transparency 1)

   Jed has $\frac{1}{2}$ of a tray of brownies left over from his birthday party. Jed is hungry and eats $\frac{2}{3}$ of the left over brownie. How much of one tray of brownies did Jed eat?

**Comments**

The goal of the warm up is to review the previous day of work and focus student thinking on partitioning.

When you discuss the warm-up Problem A emphasize that multiplying 21 by two-thirds can be done by dividing 21 by 3 and multiplying by 2. This in an operator interpretation for fractions.

Students may say that one-third of 21 is 7 so two-thirds of 21 is 14 (2x7).

For Problem B, one-fifth of 40 is 8 so three-fifths of 40 is 24 (3x8).

When you review the problems you may want to spend some time estimating results before asking kids to state solution strategies.

Let the students choose how they solve this problem. Encourage students to draw pictures to show how they found the answer. Have one or two students share their pictures how they solved the problem.
Teaching Actions

2. Work as a class to solve the problem again using patty paper. Begin by telling them that one piece of patty paper represents one tray of brownies. Ask them to take the patty paper and fold in half to represent one-half a tray of brownies. Have them shade one-half using the lighter shade of color.

![Patty paper folded in half]

Explain that the shaded region represents the leftover brownies.

Have them fold the patty paper so that only the shaded part is shown.

![Patty paper folded with shaded part shown]

Ask the students to shade in the amount that Jed ate using a darker color.

![Patty paper shaded darker]

This shaded amount represents two-thirds of the one-half. Ask them to explain how this shows the part of one tray of brownies that Jed ate. [Students need to open the patty paper and rename the double-shaded amount when compared to the whole]

Comments

Some students may fold the patty paper in half diagonally. Although this approach works and could work for doing multiplication it is difficult to find two-thirds of the one-half tray of brownies when folded this way.

![Patty paper folded diagonally]

In this problem it is important to focus on the unit. The original unit is the tray of brownies. When the students shade in one-half they shade in one-half of the patty paper square.

When the students shade in the two-thirds of this leftover piece they are using the one-half shaded region as the new unit.

They are shading in two-thirds of the one-half. This highlights the importance of students’ flexibility of unit. In this problem the unit is changing.
### Teaching Actions

patty paper].

Ask a few students to explain how the patty paper shows the answer. Be sure to focus on the unit. [It may be easier for students to see the answer if they draw in lines to show that the patty paper is folded into 6 equal parts].

Write the multiplication sentence that can be used to represent this problem.

\[ \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} \text{ or } \frac{1}{3} \]

3. Ask students to solve the following problem using patty paper. (Transparency 2)

Ernesto is very particular about the pan pizza he makes for his family. He puts pepperoni on three-fourths of the pizza. He puts sausage on one-fourth of the pepperoni part. How much of the pizza has both pepperoni and sausage?

Use patty paper to find the answer and complete the multiplication sentence below.

\[ _____ \times \frac{3}{4} = _____ \]

### Comments

When students answer this question and state that \( \frac{2}{6} \) or \( \frac{1}{3} \) of one tray of brownies are shaded they are now using the entire piece of patty paper as the unit.

Some students may fold the paper differently but should still get the same answer.

Be sure to explain that the multiplication sentence states that they are finding two-thirds of a group of one-half.

Students will fold the paper in different ways.

One way to show the solution is to represent the three-fourths using vertical folds.

Fold paper so that only the three-fourths is showing (the amount of pizza with pepperoni). This helps focus attention on the new unit.

(new unit: three-fourths of a pizza)
Teaching Actions

(black is one-fourth of the three-fourths pepperoni part of the pizza)

\[
\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}
\]

You may need to extend the three horizontal lines to show the 16 equal sized pieces that the unit is partitioned.

Small Group/Partner Work

4. Assign Student Pages A & B.

Wrap Up

5. Review select problems from the Student Pages.

A) Show the picture below. Tell the students that they will be finding a multiplication problem for what you will draw next. (Transparency 3)

Separate the two-thirds into 5 equal sized pieces by drawing four horizontal lines. Shade in 4-fifths of the two-thirds. Ask the

Comments

Then fold horizontally to find one-fourth of the pepperoni part of the pizza.

A) Your picture should look like this when you draw in the horizontal lines:

\[
\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}
\]
### Teaching Actions

students to write down the multiplication sentence that corresponds to this picture.  
\[
\left( \frac{4}{5} \times \frac{2}{3} = \frac{8}{15} \right)
\]

B) Ask the students to write down a multiplication sentence when you shade in two-thirds of the picture of one-sixth shown below. (Transparency 4)

![Diagram](image)

\[
\frac{2}{3} \times \frac{1}{6} = \frac{2}{18}
\]

### Comments

B) The picture should look like this:

![Diagram](image)

### Translations:
- Real-world to picture
- Real-world to written symbol
- Picture to written symbol
- Real-world to manipulative
- Written symbol to manipulative
Jed Problem
Jed has $\frac{1}{2}$ of a tray of brownies left over from his birthday party. Jed is hungry and eats $\frac{2}{3}$ of the left over brownie. How much of one tray of brownies did Jed eat?

The Jed problem will be new to the students. This is the first time they will multiply a fraction by a fraction. The solution strategy that you will want to look for is when students partition the one-half tray of brownies into thirds. This results in pieces that are one-sixth the size of the original tray of brownies. Jed eats two of these pieces so he ends up eating two-sixths of a tray of brownies.

Many of the students were able to solve the problem by drawing a picture. Two of the pictures are shown below. Note how both students took two-thirds of the half tray that was remaining. They also partitioned the other half of the tray into thirds to make all the pieces the same size. They then had to make the entire tray of brownies the unit to answer the amount of one tray that Jed ate.

Ernesto Problem
Ernesto is very particular about the pan pizza he makes for his family. He puts pepperoni on three-fourths of the pizza. He puts sausage on one-fourth of the pepperoni part. How much of the pizza has both pepperoni and sausage?

Some students may fold and shade the patty paper as shown below when they solve the Ernesto problem.
They can still correctly find one-fourth of three-fourths using this picture. Students who draw the three-fourths this way typically partition each fourth into fourths again and shade in one fourth of each fourth. They will still be able to find the product as three-sixteenths of one pizza. In the next lesson the students will spend more time partitioning pictures that already are partitioned using vertical lines for the fraction amount in each group. You will encourage students to partition these amounts by drawing horizontal lines. The purpose of this approach is that it is easier to see why multiplying numerators and denominators help determine the product.

In this lesson you may allow the students to partition the patty paper in any way that makes sense to them.
Jed has $\frac{1}{2}$ of a tray of brownies left over from his birthday party. Jed is hungry and eats $\frac{2}{3}$ of the left over brownie. How much of one tray of brownies did Jed eat?
Ernesto is very particular about the pan pizza he makes for his family. He puts pepperoni on three-fourths of the pizza. He puts sausage on one-fourth of the pepperoni part. How much of the pizza has both pepperoni and sausage?
Write the multiplication sentence:
Write the multiplication sentence:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
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</tbody>
</table>
Problem A:

Use a number line to find the product \( \frac{2}{3} \times 21 \)

Problem B:

Explain a way to multiply \( \frac{3}{5} \times 40 \) without using the number line.

Problem C:

Becky earns $12 per hour grading math homework. How much will she earn if she works \( \frac{5}{6} \) of an hour?
Multiplying Fractions Using Patty-Paper

1. Shade in the fractional amounts of patty-paper pictures shown below.

   \[
   \begin{array}{ccc}
   & \frac{1}{3} & \\
   \frac{3}{4} & & \\
   \frac{3}{5} & & \\
   \end{array}
   \]

2. Chimeng finds \( \frac{1}{3} \) of a square cake out on the counter of his kitchen. He eats \( \frac{1}{2} \) of this piece. What fraction of the entire cake did Chimeng eat? Use patty-paper to find your answer then shade in the picture below to show how you got your answer.

   \[
   \begin{array}{c}
   \text{cake} \\
   \end{array}
   \]

   Multiplication Sentence:
3. Use your patty paper to find the answer to the following question:

Christina is making a big square cookie. She puts blue M&M’s on $\frac{1}{2}$ of the cookie. She puts peanuts on $\frac{2}{3}$ of the part of the cookie with blue M&M’s. How much of the big cookie has both peanuts and blue M&M’s?

Draw a picture to record your steps:

```
cookie
```

Multiplication Sentence:

4. Find the answer to the following problems using patty-paper. Write the multiplication sentences as word phrases and draw pictures to show answers.

$$\frac{1}{2} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{3}{4} =$$

___ group of $\frac{1}{4}$  ___ groups of ___  ___ groups of ___
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Students use patty paper (an area model) to multiply fractions. Students develop the algorithm for multiplying fractions by noticing patterns related to the patty paper model.

Materials
- Patty paper
- Two color pencils or markers
- Student sheets A and B
- Transparencies 1 and 2

Teaching Actions

Warm Up

Determine the fraction of the square that is colored gray.

Show A and B together then discuss. Show B and C together and discuss.

The students should be able to do the warm up problems mentally. Please give them a minute to think of the fractional amounts and have them write the fractions on a piece of scratch paper.

The purpose of squares A and B is to focus the students' attention on getting equal sized pieces. Students may suggest that you draw in lines on the transparency to show equal sized pieces when you discuss the answers.

The emphasis for Square C is on multiplication. Try to get students to explain how they calculated the fraction colored gray. Some students may notice that the total number of equal sized pieces (denominator) can be found by multiplying 7 by 4 and the number of colored in pieces can be found by multiplying 4 by 2 (numerator).

The emphasis on Square D is a combination of strategies for the previous squares. Multiplication can be used to count both the number of shaded rectangles as well as the number of total rectangles when determining the fraction of the square that is
### Teaching Actions

**Large Group Introduction**

1. Explain to the students that we are going to develop a rule for multiplying fractions today. To begin this exploration, ask them to look at the picture on transparency 1. Explain that the grayed in part represents the amount of cake Kathy has left from yesterday.

![Kathy’s Cake](image)

2. Ask the students to think about how much cake Kathy has left. Record the fractional amount below the cake.

3. Explain that Kathy wants to eat two-fifths of the remaining cake. Ask the students to think how Kathy could find two-fifths of the remaining cake.

4. Have a volunteer come up and shade in two-fifths of the three-sevenths of a cake with a dark marker. It may look something like what is shown below.

### Comments

shaded. This is an important part of explaining why the algorithm for multiplication of fractions works.

There is four-sevenths of a cake remaining.

The student may or may not draw a picture as shown to the left. The sevenths and the fifths were chosen for this example so the students will draw horizontal and vertical lines to show the total number of pieces. It is easier to explain why the multiplication algorithm works if students draw one fraction vertically and the fraction of that fraction horizontally.
Teaching Actions

5. Ask the students to explain what fraction of the whole cake will Kathy eat ($\frac{8}{35}$). Ask a few students how they determined this fraction. [Students most likely will draw in the horizontal lines across the whole square to explain why the total number of parts is 35].

6. Ask: What multiplication sentence matches the actions we made on the picture of the patty paper?

$$\frac{2}{5} \times \frac{4}{7} = \frac{6}{35}$$

Small Group/Partner Work

7. Assign Student Pages A & B. You may want to go over the first question with the students so they know how to complete the class work.

Wrap Up

8. Review select problems from the Student Pages. To help students construct the rule for multiplying fractions, guide the discussion with these questions:

- How did you show $\frac{2}{3}$ of $\frac{1}{5}$?
- How did the picture change when you did that?
- If you extend the horizontal lines across the square, how many total parts is the square partitioned into?
- Where is a 3 by 5 rectangle in your picture?
### Teaching Actions

- Where is a 2 by 1 rectangle in your picture?

9. Ask the students to picture a piece of patty paper with \( \frac{5}{9} \) colored. Show transparency 2.

![Image of patty paper with 5/9 colored]

\[-\frac{5}{9} = -\]

10. Write a 3 in the denominator of the first fraction in the multiplication sentence.

\[- \frac{5}{3} = -\]

Ask the students to explain why multiplying 9 by 3 will give you the denominator of the product.

Ask: What number will we write in the denominator of the result fraction? (27) Where is the 3 by 9 rectangle in the picture?

Ask: What does the 27 represent” (The number of equal-sized pieces that the unit is partitioned.) You can write three horizontal lines on the transparency to show the 27 equal sized pieces.

11. Write a 2 in the numerator of the multiplication sentence.

\[- \frac{2}{3} \times \frac{5}{9} = \frac{27}{27}\]

Ask: What number should we write in the numerator of the product? (10)

### Comments

Students have trouble explaining why the multiplication algorithm works. The goal of the student pages and the wrap up is to help them verbalize why the algorithm works using the patty paper model.

Students should be able to explain that each of the 9ths will be cut into 3rds. Multiplying 9 by 3 counts the number of total pieces the unit is cut into. One-third of one-ninth is one-twenty-seventh.
Teaching Actions

Ask: What does the 10 represent? (The 10 pieces represent the number of pieces that are darkly shaded.) You may want to show two-thirds of five-ninths on the transparency to show how multiplication helps you show the product. Where is the 2 by 5 rectangle in the picture?

12. Explain that the “algorithm” for multiplying fractions is to multiply the numerators and the denominators. The product of the numerators will give you the number of pieces in the answer and the product of the denominators will give you the number of pieces in the unit.

Write the following number sentence with letters:

\[
\frac{a \times c}{b \times d} = \frac{a \times c}{b \times d}
\]

**multiplication algorithm**

Translations:
- Picture to symbols
- Symbols to picture to symbols
- Pictures to symbols to verbal
Additional Notes to the Teacher
Lesson 22

The goal for this lesson is for students to develop the rule or algorithm for multiplying two fractions.

The students will use pictures of patty paper to partition fractions of fractions. We found that students seem to be able to see multiplication ideas better if they draw one fraction using vertical lines and then finding the fraction of this fraction using horizontal lines. In the Kathy’s cake example, the students are told that Kathy has three-sevenths of a cake left over as shown in the picture below.

![Kathy’s Cake](image)

Kathy’s Cake

Kathy is then going to eat two-fifths of this amount of cake. Although you could draw in vertical lines to find two-fifths of the three-sevenths it would be difficult to see. Please model and encourage your students to draw in horizontal lines when they partition the three-sevenths into fifths. To explain the amount of the darkly shaded part of the cake, students will need to extend the horizontal lines across the whole patty paper. If students folded the patty paper first into sevenths and then into fifths, the paper would have creases showing the horizontal partitions.

![Two-fifths of three-sevenths shaded](image)

two-fifths of three-sevenths darkly shaded

By the end of the lesson you want students to be able to explain that the product of the denominators show the size of the pieces. When you take fifths of sevenths you will get pieces that are thirty-fifths of the unit. The product of the numerators is more difficult to explain. The students should be able to explain that the total number of darkly shaded pieces can be counted by finding the product of the numerators.

By the end of this lesson students should be able to reason that the product of the numerators determine the number of pieces that you are interested in and the product of the denominators is the number of pieces in which the unit is partitioned.
\[ \frac{5}{9} \times \frac{5}{9} = \]
Determine the fraction of the square that is colored gray.

Fraction: \[ \frac{\text{number of gray squares}}{\text{total number of squares}} \]

A

B

C

D
## Multiplying Fractions

1. Fill in the table below.

<table>
<thead>
<tr>
<th>Multiplication Problem</th>
<th>Picture</th>
<th># of darkly shaded pieces</th>
<th>Fraction of the square shaded dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4} \times \frac{1}{6}$</td>
<td>![Picture 1]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3} \times \frac{1}{5}$</td>
<td>![Picture 2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{4} \times \frac{3}{5}$</td>
<td>![Picture 3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3} \times \frac{2}{3}$</td>
<td>![Picture 4]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Look for patterns in each row. Explain the patterns you see.

3. Use the pattern you describe in question 2 to the answers to the problems below.

   a) \[
   \frac{5}{7} \times \frac{4}{11} =
   \]

   b) \[
   \frac{7}{10} \times \frac{3}{17} =
   \]

4. Rewrite the pattern you used as a rule for multiplying fractions. Explain why your rule works.
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 23: Overview

Students multiply fractions using a number line and estimate products of fractions by mentally picturing the number line.

Materials

- Transparencies 1-5
- Student Pages A, B, and C

Teaching Actions

Warm Up

Raina was trying to find the product \( \frac{2}{5} \times \frac{3}{7} \) using patty paper. She drew the picture below and says the answer is \( \frac{2}{35} \). How did she get this answer and is she correct?

![Image of a grid with shaded parts]

After the correct answer is developed ask the students: “Does this revised answer match the algorithm we developed yesterday?”

Large Group Introduction

1. Ask the students to find the multiplication sentence that goes with the pictures below. (Transparency 1)

Comments

This is a common mistake that was made by students in Lesson 22.

Emphasize that Raina was taking two-fifths of one-seventh. She needs to take two-fifths of each of the three-sevenths. Shade in the remaining 4 rectangles.
Teaching Actions

2. Explain that they are now going to look again at number lines and show how to multiply two fractions using that model.

3. Ask the students to draw a number line as shown below. Make sure that they label the number line in miles. (Transparency 2)

![Number Line Example](image)

Ask them to solve the following problem using the number line.

Each block along Helena Avenue is one-half mile long. Duanna runs 5 blocks. Show how you can use the number line to find the total number of miles she runs. Write a multiplication sentence for her run.

Have a student come to the front of the classroom and demonstrate how they think they should use a number line to solve the problem. Other students may come to the front of the classroom if they have a different strategy.

An example of student work is shown below.

Comments

Answer: \[ \frac{1}{3} \times \frac{3}{4} = \frac{3}{12} \]

Students may suggest that you draw in the horizontal lines to show the 12ths.

Answer: \[ \frac{5}{6} \times \frac{2}{3} = \frac{10}{18} \]

Students will need to place tick marks on the number line to show the \( \frac{1}{2} \) mile lengths of the blocks.

Be sure that you ask the student to explain the meaning of each factor in the multiplication sentence. The 5 represents the number of blocks run (groups) and the one-half represents the length of each block (amount in each group). The two and one-half represents the total length of the run.
Multiplication sentence:

\[ 5 \times \frac{1}{2} = \frac{5}{2} \text{ or } 2 \frac{1}{2} \]

Ask: Does this sentence match the algorithm? Why don’t we multiply the 2 by the 5 and get \( \frac{5}{10} \) for our answer?

Five-tenths is the same as one-half and that is not a reasonable answer. Duanna runs five whole blocks. The 2 in the denominator of the fraction describes the size of the fraction and the size does not change as she runs each whole block.

4. Read the following problem to the students.
   (Transparency 3)

   Duanna is tired and stops running after \( \frac{3}{4} \) of a block. Each block is still \( \frac{1}{2} \) of a mile long.
   How many miles did she run?

Ask the students to estimate the answer first. Did Duanna run more than one-half a mile or less? Why?

Have students share their answers and procedures. If none of the students used a number line to solve this problem then show them how to solve on a number line.
Teaching Actions

miles

Ask students to put a tick mark at one-half mile to represent one block.

0  $\frac{1}{2}$  1

miles

To find three fourths of the one-half, mark the half-mile into 4 equal parts. Three-fourths of one-half is shown by the arrow below.

0  $\frac{1}{2}$  1

Ask: What value is that spot on the number line? Consider what the unit is. How do you know the answer is in 8ths? How is this similar to adding the horizontal lines to the patty paper to find the total number of equal parts the paper was partitioned into?

You may want to label the completed number line as is shown in the student work below.

blocks

Ask the students to write a multiplication sentence that describes how to find the amount that Duanna ran. Be sure to discuss the meaning of each factor and record. Also relate the answer to the algorithm.

Comments

Students should suggest that you partition the line from $\frac{1}{2}$ to 1 into fourths as well.

Note that the student partitioned to second half mile into fourths. This clearly shows each eighth of a mile. The work also shows a double number line with the miles written below the number line and the blocks written above the number line.
Teaching Actions

Multiplication Sentence:
\[
\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}
\]

5. Ask students to draw a number line as shown below to solve the problem on transparency 4. Make sure they label the 0 and the 1 on the number line and write miles below the number line as shown below.

![Number line diagram]

Chase walks down Old Shakopee Road. Each block is \(\frac{1}{4}\) of a mile. Chase stops walking after \(\frac{2}{3}\) of a block. How far did he walk?

Ask students: What is the multiplication sentence for this problem?

**Multiplication Sentence:**
\[
\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}
\]

State: “This sentence tells us that Chase walked two-twelfths or one-sixth of a mile. We are going to use the number line we drew to show how far he walked.”

Ask students: How many blocks fit in 1 mile? Have them label the miles below the number line and the blocks above the number line.

![Number line diagram with blocks]

Comments

To represent the product of two fractions, students actually rely on the algorithm to decide how to partition the number line.

If students are able to write \(\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}\) they should use the algorithm to calculate the distance walked.

You may want to ask if Chase is walking two-thirds of one-fourth mile or one-fourth of two-thirds mile. He is walking two-thirds of one-fourth mile (1 block).
Teaching Actions

State: Chase stops after walking $\frac{2}{3}$ of a block. How can you show on the number line that he walked $\frac{2}{3}$ of the one block?

The students should partition the space between 0 and $\frac{1}{4}$ into thirds, then move from 0 over two-thirds of a block.

Ask: How much of a mile does he walk before stopping? Two-twelfths of a mile.

How can the number line be partitioned further to name this amount?

Each remaining fourth mile needs to be partitioned into three equal pieces so that each tick mark is an equal length (twelfths).

How is this like adding in the horizontal lines on the patty paper?

Based on the algorithm, students know the answer should be in twelfths. That helps them see that to show that on the number line, it makes sense to partition each of the three remaining fourths into thirds. This shows why the answer is in twelfths of a mile.
Teaching Actions

Small Group/Partner Work

6. Assign Student Pages A, B, and C.

Wrap Up

7. Review select problems on the Student Pages.

8. Ask the students to estimate the following multiplication problems. (Transparency 5)

\[ \frac{2}{3} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2} \text{) } \]

\[ \frac{5}{4} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2} \text{) } \]

\[ 3 \times \frac{1}{4} \text{ (less than } \frac{1}{4} \text{ or greater than } \frac{1}{4}, \text{ less than } \frac{1}{2} \text{ or greater than } 1 \text{) } \]

Translations:
- Real life to pictures to verbal
- Real life to pictures to symbols
- Symbols to pictures

Comments

Focus on partitioning.

Use the context that each block is one-half mile and the person runs two-thirds of a block to help students create mental images. Encourage your students to determine the answer without using the algorithm by drawing a number line and labeling only 0 and \( \frac{1}{2} \); then ask them to visualize where the product would be.
The goal for this lesson is for students to develop a method for multiplying fractions using a number line. The students should be able to use the number line to help them partition a unit fraction. A unit fraction is a fraction with a numerator of 1. The second factor in each of the multiplication problems for this lesson is a unit fraction. Students will be exposed to problems where the second factor is a non-unit fraction in lesson 24.

A distance context was selected for this lesson to make connections with the number line, a model that emphasizes length, and it makes sense for students to have fractional amounts of groups (amount run) and fractional amounts in each group (length of blocks).

Many of the students naturally made double number lines as they solved the problems by labeling the miles on the bottom of the number line and the blocks on the top. This seemed to help some students as they partitioned the number line.

The number line provides students with a way to apply the algorithm they learned to multiply fractions and make further sense on why it works. The number line also provides a way for students to mentally visualize the multiplication of fractions and use this image to make estimates. For example look at the number line below and try to imagine where $\frac{2}{3} \times a$ is on the number line.

\[
\begin{array}{c}
0 \\
\hline
\hline
a
\end{array}
\]

Students should be able to visualize the distance between 0 and $a$ being partitioned into thirds and iterating over two of the thirds. They should be able to see that an estimate for this product is a number less than $a$. If they try to visualize where $3 \times a$ would be they should be able to visualize the iteration of $a$ three time. The product $\frac{5}{4} \times a$ can be visualized by partitioning the distance between 0 and $a$ into fourths then iterating this length five times which results in a product that is larger than $a$. 
Find the multiplication sentence that goes with the two pictures below.

\[ \_ \times \_ = \_ \]

\[ \_ \times \_ = \_ \]
Each block along Helena Avenue is one-half mile long. Duanna runs 5 blocks. Show how you can use the number line to find the total number of miles she runs. Write a multiplication sentence for her run.
Duanna is tired and stops running after $\frac{3}{4}$ of a block.

Each block is still $\frac{1}{2}$ of a mile long. How many miles did she run?
Chase walks down Old Shakopee Road. Each block is $\frac{1}{4}$ of a mile. Chase stops walking after $\frac{2}{3}$ of a block. How far did he walk?
\[
\frac{2}{3} \times \frac{1}{2} \quad \text{(less than} \quad \frac{1}{2} \text{ or greater than} \quad \frac{1}{2})
\]

\[
\frac{5}{4} \times \frac{1}{2} \quad \text{(less than} \quad \frac{1}{2} \text{ or greater than} \quad \frac{1}{2})
\]

\[
3 \times \frac{1}{4} \quad \text{(less than} \quad \frac{1}{4} \text{ or greater than} \quad \frac{1}{4}, \text{ less than} \quad 1 \text{ or greater than} \quad 1)
\]
Raina was trying to find the product $\frac{2}{5} \times \frac{3}{7}$ using patty paper. She drew the picture below and says the answer is $\frac{2}{35}$. How did she get this answer and is she correct?
Multiplying Fractions on Number Lines
(fraction x unit fraction)

1. Molly runs down Colfax Avenue. Each block is $\frac{1}{3}$ of a mile long. She runs 4 blocks before she gets tired and stops.

   a) Find the exact length of her run using the number line.

   ![Number Line](image)

   miles

   b) Write a multiplication sentence that shows how you can find out how far Molly ran. Explain the meaning of each factor.

2. Molly runs down Colfax Avenue again. Each block is $\frac{1}{3}$ of a mile long. She runs $\frac{1}{4}$ of a block before she gets tired and stops.

   a) Find the exact length of her run using the number line.

   ![Number Line](image)

   miles

   b) Multiplication Sentence:
3. Mark runs down Irving Avenue. Each block is $\frac{1}{3}$ of a mile long. He runs $\frac{2}{3}$ of a block before he gets tired and stops.

Find the exact length of his run using the number line and write the multiplication sentence that shows how far he runs.

![Number line with marked blocks]

Multiplication Sentence:

4. Pang determines the length of her run using the multiplication sentence $\frac{2}{3} \times \frac{1}{2} = ____$. 

a) How long is each block that she runs?

b) How many blocks does she run?

c) Find the exact length of her run using the number line and complete the multiplication sentence above that shows how many miles she runs.

![Number line with marked blocks]
Find the product of these multiplication problems using the number line. Check your answer using the algorithm.

5. \( \frac{1}{3} \times \frac{1}{2} = \) ____  

6. \( 3 \times \frac{2}{3} = \) ____  

7. \( \frac{2}{3} \times \frac{1}{5} = \) ____  

8. \( \frac{5}{4} \times \frac{1}{3} = \) ____
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
### Rational Number Project

#### Fraction Operations and Initial Decimal Ideas

**Lesson 24: Overview**

Students use a variety of models and the algorithm to multiply a fraction by another fraction. Students will describe connections among the number line, pictures, and the algorithm.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Transparency 1</td>
</tr>
<tr>
<td>- Student Pages A, B, C, and D</td>
</tr>
</tbody>
</table>

### Teaching Actions

#### Warm Up

Ask students to give an exact answer for each of the warm up problems. Encourage them to do the problems mentally but it is OK if a student chooses to write something on paper. Do each problem one at a time.

- \( \frac{1}{2} \times \frac{7}{8} = \)
- \( \frac{2}{3} \times \frac{1}{2} = \)
- \( \frac{4}{5} \times 30 = \)
- \( 6 \times \frac{2}{3} = \)

#### Large Group Introduction

1. Ask the students to write a multiplication sentence for the following problem and use it to find the answer.

   Hannah hikes along the Nature Trail at Mud Lake. The trail is \( \frac{2}{3} \) of a mile. She hikes \( \frac{4}{5} \) of the trail before she stops to take a picture of a

### Comments

When students explain their reasoning some students may say they got their answers using the algorithm (i.e. multiply the numerators and the denominators). Other may picture a square, a number line, or clouds. Ask several students how they did the problem and probe for different strategies.

The word algorithm is used in the student pages. Please emphasize this word as students are explaining how they solved each problem.
**Teaching Actions**

**hummingbird with her high speed Leica camera. How many miles did she hike before taking the picture?**

**Multiplication sentence:**

\[ \frac{4}{5} \times \frac{2}{3} = \frac{8}{15} \]

2. Ask the students to solve the same problem using a number line. The students should be able to set up a number line that looks similar to the one below without too much help from you.

![Number line for multiplication](image)

Students may have trouble trying to find out how to take four-fifths of two-thirds of a mile. The algorithm tells us that the answer will be in fifteenths so one hint may be to partition the miles into fifteenths.

![Number line for partitioning](image)

Some students may need to use a double number line to make sense of finding four-fifths of two-thirds. The number line below shows that one-fifth of a trail length is two-fifteenths of a mile, so four-fifths of two-thirds would be eight-fifteenths of a mile.

![Double number line](image)

Ask: How does multiplying 3 by 5 in the algorithm relate to the fifteenths on the number line?

**The fifteenths are created by taking fifths of thirds.**

Where is the 4x2 from the algorithm shown on the number line?

**Comments**

Students came up with a variety of ways of showing this answer. Please look at the Teacher Notes at the end of the lesson for examples of 3 of the most common ways that students solved the problem.

Some students noticed that there were now 10 lengths between 0 and 1 trail length so four-fifths would be 8 of those lengths.
### Teaching Actions

**There are 4 groups of 2 fifteenths of a mile so there are 8 lengths of one-fifteenth of a mile that is counted.**

3. Have the students solve the problem again using a picture of a piece of patty-paper.

4. Ask the students to solve the following problem using the algorithm, the number line, and a picture of patty-paper.

\[
\frac{3}{4} \times \frac{2}{5}
\]

Review the different ways the students solved the problem.

### Small Group/Partner Work

5. Spend the rest of the day having students work on the problems on Student Pages A, B, C, and D in pairs.

6. As you circulate around the class and ask probing questions, observe groups of students that have different approaches to the problems. Give selected groups a blank transparency and ask them to be ready to present their solution to the class.

### Wrap Up

7. Ask students to determine if the following answers are reasonable.

\[
\frac{3}{4} \times 17 = 25
\]

\[
3 \times \frac{1}{2} = \frac{2}{3}
\]

\[
\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}
\]
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8 x 5/11 = 1/2</td>
<td></td>
</tr>
</tbody>
</table>

**Translations:**
- Symbols to verbal
- Real life to symbols
- Real life to pictures
- Real life to manipulatives to symbols
Additional Notes to the Teacher
Lesson 24

The main goal for this lesson is for students to make connections among the three models for multiplying fractions introduced and the algorithm. We believe that the students will be able to explain why the algorithm works if they are able to make connections between the algorithm and a model.

The students are asked to solve the following problem using the algorithm, a picture of a piece of patty-paper, and the number line.

Hannah hikes along the Nature Trail at Mud Lake. The trail is \( \frac{2}{3} \) of a mile. She hikes \( \frac{4}{5} \) of the trail before she stops to take a picture of a hummingbird with her high speed Leica camera. How many miles did she hike before taking the picture?

Most students were able to solve the problem using the algorithm and patty-paper but struggled more with the number line. The students were able to use the patty-paper and algorithm results to make sense of how to solve the problem on the number line. The three approaches shown below are typical of what we found.

<table>
<thead>
<tr>
<th>Work</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.jpg" alt="Student A" /></td>
<td>The bottom of the number line represents the miles. The first third is partitioned into fifths and eventually the entire number line is partitioned into 15ths. The student labels the equally spaced tick marks from 1 to 15. The numerals 1,2,3,4 written above the 1,2,3,4 seem to represent four-fifths of one-third mile. The next 1,2,3,4 above the 5,6,7,8 seem to represent four-fifths of the second third of a mile.</td>
</tr>
<tr>
<td><img src="image2.jpg" alt="Student B" /></td>
<td>This student partitioned the number line into thirds then each third is partitioned into fifths. The numbers 1,2,3,4,5 represent how two-thirds can be split into 5 equal sized pieces. The grouping of the two pieces (denoted by the U shapes above the numerals) show that four-fifths of 10 pieces is 8 pieces.</td>
</tr>
<tr>
<td><img src="image3.jpg" alt="Student C" /></td>
<td>This student partitioned each third into fifths. He then took four-fifths of the first third of a mile (getting four-fifteenths) then four-fifths of the second third of a mile (the second four-fifteenths) to get eight-fifteenths of a block.</td>
</tr>
</tbody>
</table>
Are these answers reasonable?

\[
\frac{3}{4} \times 17 = 25
\]

\[
3 \times \frac{1}{2} = \frac{2}{3}
\]

\[
\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}
\]

\[
\frac{3}{8} \times \frac{5}{11} = \frac{1}{2}
\]
Mental Mathematics

\[
\frac{1}{2} \times \frac{7}{8} =
\]

\[
\frac{2}{3} \times \frac{1}{2} =
\]

\[
\frac{4}{5} \times 30 =
\]

\[
6 \times \frac{2}{3} =
\]
Multiplying Fractions on Number Lines
(fraction x fraction)

1. Max runs down 46th Street. Each block is \( \frac{1}{3} \) of a mile long. He runs \( \frac{2}{5} \) of a block before he gets tired and stops.

a) Write a multiplication sentence that can be used to find the number of miles Max ran. Use the algorithm you developed in Lesson 22 to find the product.

b) Use the number line to show how to find the answer to this problem.

![Number Line]

(c) Explain why the number line is divided into fifteenths.

d) Use the square to show how to find the answer to this problem.

![Square]

e) Explain how the picture above shows why the square is broken into fifteen equal sized pieces.
2. Xander’s hose can pump 21 gallons of water every hour. How many gallons of water will he fill up if he runs the hose for $\frac{2}{3}$ of an hour?

a) Write a multiplication sentence. Use the algorithm to find the product.

b) Use the number line to determine the answer to this problem.

\[ \begin{array}{c}
\quad \\
\quad \\
\end{array} \]

\[ \begin{array}{c}
\quad \\
\quad \\
\end{array} \]

c) Draw a picture that would find the answer to the multiplication problem.

d) Explain why the algorithm answer should match the number line answer and the picture answer.
3. Hannah hikes along the Nature Trail at Mud Lake. The trail is \( \frac{2}{3} \) of a mile. She hikes \( \frac{4}{5} \) of the trail before she stops to take a picture of a hummingbird with her high speed Leica camera.

a) Write a multiplication sentence that can be used to find the number of miles Hannah hiked before she stops. Use the algorithm to show how to find the product.

b) Use the number line to show how to find the answer to this problem.

```
\[\text{Number Line}\]
```

c) Use the picture below to find the answer to the multiplication problem.

```
\[\text{Picture}\]
```

d) Explain how multiplying 2 by 4 shows the number of 15\(^{th}\)s of a mile that were run by Hannah on both the number line and the picture.
Find the product of these multiplication problems using the algorithm and the number line.

4. \( \frac{3}{4} \times \frac{1}{2} = \) _____

5. \( 3 \times \frac{2}{5} = \) _____

6. \( \frac{2}{3} \times \frac{3}{4} = \) _____

7. \( \frac{2}{5} \times \frac{4}{3} = \) _____
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 25: Overview

Students draw pictures to solve measurement division story problems. Students explain their solution strategies. Story problems involve whole numbers divided by a fraction < 1; mixed numbers divided by a fraction < 1; fraction <1 divided by another fraction < 1. All answers are whole numbers.

Materials
• Student Pages A - D

Teaching Actions

Warm Up
Draw a picture of a number line and use it to solve \( \frac{2}{3} \times \frac{1}{4} \). Be ready to explain your thinking.

Large Group Work

1. Present this story problem using whole numbers:

   Addis works as a baker. She bought 50 pounds of flour. It is easier to use if she repackages the flour into smaller containers. Each container holds 5 pounds of flour. How many containers can she make with the 50 pounds of flour?

2. Ask: What do you imagine when solving this problem? What question are you answering? [How many 5’s in 50?] What number sentence represents the action in this story problem? [50 ÷ 5=10]

3. Repeat for this example and ask students what picture might be used to solve the problem.

   You have 12 pounds of peanuts. You package them into 3-pound bags. How many bags?

Comments

Consider these last 4 lessons on fraction division as a problem solving opportunity for students. Students are not directly taught how to divide fractions but are asked to solve division story problems using pictures. Students use their understanding of the part-whole model for fractions and the role of the unit to construct their own strategy for solving division tasks.

The measurement model is being used to develop fraction division. In measurement division the amount in each group is known as well as the total amount. What students are finding is the number of groups.

In the first example students should imagine constructing 5-pound containers; they are finding out “How many 5 pounds in 50 pounds? This is the question we want students to consider when dividing 3 pounds into groups of \( \frac{1}{3} \) pounds. “How many \( \frac{1}{3} \) pounds in 3 pounds?”
Teaching Actions

Small Group/Partner Work

4. Explain that each pair of students will receive 3 problems to solve. These are on Student Pages A and B. Students are to draw a picture to model each problem. (These problems are all whole numbers divided by a fraction with a whole number answer).

Comments

As students work in groups move among the groups asking students to explain how the fraction circles help them solve the problem.

Identify students to present their solutions to the whole group. Look for students who solved the problem by portioning the whole number part into fractional parts. See example below: See Teacher Notes for Lesson 25 for examples of students’ work on these problems.

Large Group Work

5. After giving students 10 – 15 minutes to solve the problems, call the group back together. Call on students to explain how to draw pictures to solve each problem.

6. When the presentations are done, look at the pictures of each solution and ask:
   • How are the solutions strategies similar?
   • What question were you answering in each one?
   • What type of number was each answer?

7. Step back and ask:
   • What number sentence matches each problem?
   • Is it addition, subtraction, multiplication or division?
   • How do you know?
   • In what way are these problems similar to the first problem of the day?
   • How are they different?

Most students will solve the problem by drawing a picture to show the amount to be partitioned first. Consider this student’s solution to a homework problem. She drew 5 rectangles to represent the 5 cups. Then she partitioned the 5 units into halves and ask: How many $\frac{1}{2}$’s go into 5?

The following student drew a different picture. He used a repeated addition idea of adding up $\frac{1}{2}$’s to reach 5. In both cases the students determined the answer to be 10 scoops. But we did notice students who drew a picture showing repeated addition often made mistakes counting up. During student presentations we did move students to use the strategy of drawing the picture of the amount to
Teaching Actions

8. Say: Let’s record the division sentence under each picture. [Also record the question: How many _ are in _?]

9. Ask students to look at the picture for problem 1. Comment on how the students partitioned the 3 whole circles into fourths to see more easily how many \( \frac{3}{4} \)'s cups of birdseed could be taken from 3 cups. Record as: \( 3 \div \frac{3}{4} = 4 \) and \( \frac{12}{4} \div \frac{3}{4} = 4 \).

10. Note that students might write the division sentence incorrectly. In the examples at the right, students wrote \( \frac{1}{2} \div 5 \) instead of \( 5 \div \frac{1}{2} \).

11. Repeat for the other problems. (See Additional Notes to the Teacher for lesson 25 for examples of students’ pictures to solve division problems).

Comments

be shared first. Notice in problem set II we encourage this strategy by providing students with the pictures.

How many \( \frac{3}{4} \)'s in 3?

\[
\begin{array}{c|c|c}
1 & 2 & 3 \\
1 & 2 & 3 \\
\end{array}
\]

\( 3 \div \frac{3}{4} = 4 \)

\( \frac{12}{4} \div \frac{3}{4} = 4 \)

At this point you are just making the observation that the picture could also be recorded in its equivalent form with same denominators.

A common error is noted in this student’s work. Instead of stating the answer as 3 the student writes \( \frac{3}{4} \).

Small Group/Partner Work

12. Send students back to work in their groups on Student Pages C and D. Direct students to use pictures to solve each problem.

Wrap Up

13. Share again in large group; draw pictures, write number sentence; rewrite the number sentence to show the whole being divided into fractional parts.

The problems on Student Pages C and D are fractions divided by fractions. The answer is still a whole number. In the next lesson, students solve problems where the answer involves a fraction.
### Teaching Actions

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. End the lesson with this problem: $1\frac{1}{2} + \frac{1}{4}$</td>
</tr>
<tr>
<td>15. Ask: What question should you think of to find the answer? [How many $\frac{1}{4}$’s are there in $1\frac{1}{2}$? What picture can you draw to solve the problem? What other number sentence can describe the picture?</td>
</tr>
</tbody>
</table>

#### Translations:
- Real life to pictures to verbal
- Real life to pictures to symbols
- Pictures to symbols to symbols
Students were creative in how they solved division story problems even though they received limited instruction initially on how to solve these problems. Students drew pictures of circles, rectangles and number lines. They intuitively understood to change the picture so they could count out fractional amounts. Examples from students’ work are shown below.

1) A scoop holds $\frac{1}{4}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3 cups of birdseed? Show how to use pictures of fraction circles to solve this problem. Draw a picture of your solution below. Explain your solution in words.

![Diagram of division problem with fraction circles and numbers indicating solution](image)

Make 3 groups of $\frac{3}{4}$ and there are 3 sets with $\frac{1}{4}$ left in each group so $3 \times \frac{1}{4} = \frac{3}{4}$ so there are 4 sets of $\frac{3}{4}$.

1) A scoop holds $\frac{1}{2}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3 cups of birdseed? Show how to use pictures of fraction circles to solve this problem. Draw a picture of your solution below. Explain your solution in words.

I made a number line and split it into three parts than I cut each of those parts into fourths. I made one forth, I kept doing this and figured out there was 4 scoops.
You have 4 cups of lemonade concentrate. If you mix 2/5 of a cup of concentrate with a gallon of water to make a pitcher of lemonade how many pitchers can you make with 4 cups of concentrate?
2) What if Kia only had \( \frac{5}{8} \) of a pound of peppermint candies and she makes small bags, each about \( \frac{1}{4} \) of a pound. How many small bags of candies can she make?
2) What if Kia only had \(\frac{6}{8}\) of a pound of peppermint candies and she makes small bags, each about \(\frac{1}{4}\) of a pound. How many small bags of candies can she make?

She can make 3 small bags of peppermint candies.

4) Show the fraction \(2 \frac{1}{3}\). How many \(\frac{1}{9}\)'s are in that amount? Draw a picture to solve this problem. Use a rectangle as the unit.

20 \(\frac{1}{9}\) are in \(2\frac{1}{3}\)
1) Kia has 2 $\frac{1}{4}$ pounds of peppermint candies. She wants to put them in small bags, each about $\frac{1}{8}$ of a pound. How many small bags of candies can she make?
Lesson 25/Warm Up

Draw a picture of a number line and use it to solve $\frac{2}{3} \times \frac{1}{4}$. Be ready to explain your thinking.
Problem Set I

1) A scoop holds $\frac{3}{4}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3 cups of birdseed? Show how to use pictures to solve this problem. Draw a picture of your solution below. Explain your solution in words.

2) You bought 4 pints of ice cream from Ben and Jerry’s for your party. You plan on serving each friend about $\frac{2}{3}$ of a pint. How many servings can you dish out? Show how to use pictures to solve this problem. Draw a picture of your solution below. Explain your solution in words.
3) You have 4 cups of lemonade concentrate. If you mix \( \frac{2}{5} \) of a cup of the concentrate with a gallon of water you make a pitcher of lemonade. How many pitchers of lemonade can you make with those 4 cups of concentrate? Show how to use pictures to solve this problem. Draw a picture of your solution below. Explain your solution in words.

Questions to ponder:

1. How are the solution strategies you used to solve each problem similar?

2. What type of number was each answer?
Problem Set II

Use the pictures to solve each problem.

1) Kia has $2 \frac{1}{4}$ pounds of peppermint candies. She wants to put them in small bags, each about $\frac{1}{8}$ of a pound. How many small bags of candies can she make?

2) What if Kia only had $\frac{6}{8}$ of a pound of peppermint candies and she makes small bags, each about $\frac{1}{4}$ of a pound. How many small bags of candies can she make?
3) You have \( \frac{4}{6} \) of a cup of lemonade concentrate. If you mix \( \frac{1}{3} \) of a cup of the concentrate with a quart of water you make a small pitcher of lemonade. How many pitchers of lemonade can you make?

4) Show the fraction \( 2\frac{1}{3} \). How many \( \frac{1}{9} \)'s are in that amount?
   Draw a picture to solve this problem. Use a rectangle as the unit.
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 26: Overview

Students solve measurement division story problems by using pictures. Students explain their solution strategies. Story problems include mixed numbers divided by a fraction <1; fraction <1 divided by another fraction <1. All answers include fractions.

Materials

- Transparencies 1-3
- Student Pages A and B

Teaching Actions

Warm Up

Use the four review problems as the day’s warm up.

Large Group Introduction

1. Present the 4 problems from the review at the overhead one at a time. Work together to draw pictures to solve the first problem. (You have \( \frac{2}{3} \) pounds of candy hearts. You put together small bags each weighing \( \frac{1}{9} \) of a pound. How many are bags did you make?)

2. State: I will use a rectangle as my unit:
   - If a rectangle is the unit, how can I show \( \frac{2}{3} \)?
   - How many \( \frac{1}{9} \)’s are in \( \frac{2}{3} \)? How can I change the picture to find this out?

3. Repeat for the other three problems. Have students come up to the overhead to draw and explain their solutions.

Comments

We found that students enjoyed presenting their solutions to the class.

How many \( \frac{1}{9} \)’s in \( \frac{2}{3} \)?

How many \( \frac{1}{9} \)’s in \( \frac{6}{9} \)?
Teaching Actions

4. Go back and translate each problem to a division sentence. For example: You have 3 yards of ribbon and cut off $\frac{1}{2}$ yard pieces, how many pieces can you cut? The division sentence would be: $3 \div \frac{1}{2} = 6$.

5. Then record showing how when they solved the problem with their pictures they change the picture so each unit was partitioned into halves. Explain how this number sentence reflects the change picture: $\frac{6}{2} + \frac{1}{2} = 6$.

6. Ask: what type of number is each answer? Is it a fraction, decimal or whole number?

7. Explain that in this lesson, they will solve fraction division problems and find out what to do when the answer is not a whole number.

8. But first ask a few questions related to flexibility of the unit. Lead this discussion using fraction circle overheads.
   - If the whole circle is the unit, what value is 1 blue?
   - If the yellow is the unit, what value is the blue?
   - If the brown is the unit, what value are 2 reds?
   - If the yellow is the unit what value are 2 reds?
   *Conclude that the value of a fractional part depends on what it is being compared to.*

9. Present this story problem from New Division Problems page.

   You have 3 pounds of fish. You are planning on serving each person $\frac{2}{3}$ pound of fish. How many full servings are possible? How can you name the amount left over?

10. Estimate first: Do you think there is at least one serving? 3 servings? Together draw pictures to solve.

Comments

In later lessons you will formalize this strategy of changing the original number sentence to one with common denominators to show one possible algorithm for solving fraction division tasks. But mastery of this algorithm is not the goal of the lessons. Expect only some students to make this translation from pictures to symbols.

A student presented her solution to this problem as follows: She used four colors to show each $\frac{2}{3}$ serving. The last third she said was $\frac{1}{2}$ of the $\frac{2}{3}$. Her final answer was $4 \frac{1}{2}$ servings.
Teaching Actions

11. Ask: We can see that you can make 4 full servings, but how can we describe the amount left over? [Draw a picture of three circles partitioned into thirds to show how to find that there are 4 full servings].

12. Describe the leftover amount: The piece that is leftover is $\frac{1}{3}$ of a whole circle. What is its value if you compare it to a $\frac{2}{3}$ amount, the size of each serving? [Draw on the board $\frac{2}{3}$ of a circle and write – size of the serving of fish underneath it.]

13. If we want to describe the fractional amount of servings (each is $\frac{2}{3}$ of a pound), is the answer $4 \frac{1}{3}$ or $4 \frac{1}{2}$?

14. Try another example: You have $2 \frac{1}{2}$ pounds of fish but now the serving size is $\frac{3}{4}$ of a pound. How many full servings? How can we describe the amount left over?

15. Conclude that when you are describing the leftover part, you need to compare it to the amount you are using as the unit.

Small Group/Partner Work

16. Student Pages A and B include 5 more examples for students to work on in their group.

Comments

Naming the remainder is the challenging part.

In this example, $\frac{2}{3}$ of a pound is being measured out of 3 pounds. From the picture you see that the leftover amount is $\frac{1}{3}$ of a pound but to name this amount it needs to be compared to the $2/3$ serving size. Therefore, the remainder is $\frac{1}{2}$.

Another student presented her solution as follows. Again, the student used different colors to highlight three different groups of $\frac{3}{4}$. To describe why the answer was $3 \frac{1}{3}$ servings, she drew a picture of the serving size $\frac{3}{4}$. She explained that the extra amount was $\frac{1}{3}$ of that amount.

For additional examples of students’ work see “Additional Notes for the Teacher for Lesson 26”.
Teaching Actions

Wrap Up

17. End the lesson by selecting students to show their picture for each problem at the board. Record the number sentences for each problem as you did in lesson one. See example below:

Original division sentence: \[3 \frac{1}{2} \div 3 \frac{3}{4} =\]
Division sentence to reflect change in the picture:
\[\frac{14}{4} \div \frac{3}{4} =\]
Final answer: \(4 \frac{2}{3}\)

The remainder is \(\frac{2}{3}\) because the two parts left over is \(\frac{2}{3}\) of a scoop.

Translations:
- Real life to manipulatives to pictures to verbal
- Real life to pictures to verbal
- Real life to pictures to symbols
- Symbols to pictures to symbols
You want students to not only draw pictures of their solutions but to be able to explain how they determined how to name the remainder. Examples from students’ work are shown below. The next step is to guide students to translate their actions on the pictures to symbols using common denominators.

**How many groups of \( \frac{1}{4} \) are there in \( 3 \frac{1}{2} \) circles?**

1) A scoop holds \( \frac{1}{2} \) cup. How many scoops of birdseed are needed to fill a bird feeder that holds \( 3 \frac{1}{2} \) cups of birdseed? Use fraction circles or the drawing below to solve this problem.
2) You bought $2 \frac{1}{6}$ pints of ice cream from Ben and Jerry's for your party. You plan on serving each friend about $\frac{2}{5}$ of a pint. How many servings can you dish out? Show how to use your fraction circles to solve this problem. Draw a picture of your solution below. Explain your solution in words.

Because each serving is $\frac{6}{2}$, so you divide the $6 \frac{1}{2}$ in the shaded parts, then figure out that the $\frac{1}{6}$ that was left over is $\frac{1}{2}$ of a serving.

I turned everything into $\frac{1}{6}$s and then counted how many two $\frac{1}{6}$s there were then I had $\frac{1}{2}$ left over and since the serving size is $\frac{2}{5}$ there is $\frac{1}{2}$ of that left over.
Review Problems

Problem A:
You have $\frac{2}{3}$ of a pound of candy hearts. You put together small bags each weighing $\frac{1}{9}$ of a pound. How many bags did you make?

Problem B:
How many $\frac{3}{8}$’s are in $\frac{3}{4}$ of a circle?
Problem C:
You have a ribbon 3 yards long. You cut off pieces \( \frac{1}{2} \) yard long. How many pieces can you cut?

Problem D:
You have \( 2 \frac{1}{2} \) cups of birdseed. How many \( \frac{1}{4} \) cups of bird seed can you scoop out?
New Division Problems

Problem A:

You have 3 pounds of fish. You are planning on serving each person \(\frac{2}{3}\) pound of fish. How many full servings are possible? How can you describe the amount left over?

Problem B:

You have \(2\frac{1}{2}\) pounds of fish but now the serving size is \(\frac{3}{4}\) of a pound. How many full servings? How can we describe the amount left over?
Dividing Fractions

1) A scoop holds $\frac{3}{4}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds $3\frac{1}{2}$ cups of birdseed? Use fraction circles or the drawing below to solve this problem.

2) You bought $2\frac{1}{6}$ pints of ice cream from Ben and Jerry’s for your party. You plan on serving each friend about $\frac{2}{6}$ of a pint. How many full servings can you dish out? What part of a serving is left? Draw a picture of your solution below. Explain your solution in words.
3) Use the pictures to solve each problem. Write a number sentence for each one.

How many $\frac{1}{2}$’s are there in $\frac{3}{4}$ of a circle?

![Circle divided into four equal parts, with one part shaded.]  

4) How many groups of $\frac{3}{4}$ are there in $3\frac{1}{2}$ circles?

![Four circles, one of whom is partially shaded.]  

5) You have $1\frac{1}{8}$ cups of m & m candies. You package them into $\frac{1}{4}$ cup baggies. How many baggies can you make? Include fraction of a package in your answer.

![Rectangular box divided into eight equal parts, with one part shaded, and a smaller rectangular box divided into four equal parts, with one part shaded.]  

Challenge: You have ribbon $\frac{1}{2}$ yard long. You want to cut the ribbon into pieces $\frac{1}{3}$ yard long. How many pieces can you cut? Is there any extra? Describe that amount?
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
**Rational Number Project**

**Fraction Operations and Initial Decimal Ideas**

**Lesson 27: Overview**

Students solve measurement division story problems by using pictures and recording solutions using division sentences with common denominators.

**Materials**

- Student Pages A-E for students
- Transparencies of student pages for teacher

---

**Teaching Actions**

**Warm Up**

Draw a picture to solve this problem.

You have \(3 \frac{1}{2}\) pounds of peanuts. If you put them into \(\frac{3}{4}\) pound bags, how many full bags can you make?

**Large Group Introduction**

1. Using a transparency of Student Page A, present this story problem. Students follow along on their copy of Student Page A.

   You have a piece of lumber that is 3 yards long. You want to cut lengths \(\frac{3}{4}\) yard long to use in a shelf you are building. How many shelves can you cut?

2. Estimate first: Do you think you can cut at least one shelf? 6 shelves?

3. Ask students to draw a picture using a rectangle as the unit.
   - First show 3 yards using 3 rectangles.
   - Partition the rectangles to show more easily how many \(\frac{3}{4}\)'s in the 3 yards.

**Comments**

The type of thinking you are encouraging would be like this: Greater than 3 because each shelf is less than 1 yard; less than 6 because each shelf is greater than \(\frac{1}{2}\) yard.
**Teaching Actions**

- Find out how many groups of the 3-fourths are in 3?
- There are 4 groups of $\frac{3}{4}$ in the 3 rectangles.

4. Record with symbols.

- $3 \times \frac{3}{4} = 4$ This is the number sentence that comes directly from the story.
- $\frac{12}{4} \div \frac{3}{4} = 4$ This is a record of how students solve the problem using pictures

5. Repeat the process with the same story using these numbers: 4 yards; $\frac{2}{3}$ of a yard length for the shelves. Estimate, draw pictures to solve problem, record in symbols. (The last problem will be done at the end of the lesson).

**Small Group/Partner Work**

6. Assign students Student Pages B – D. The last challenge problem involves a fractional answer. Observe how students solve that problem.

7. While students work, observe what difficulties they are having.

**Wrap Up**

8. Review each problem together. (See Teacher Notes for Lesson 27 for examples of students’ solutions).

9. End this section by asking the following. Imagine 6-eighths? How many groups of 2-eighths could you make from that amount? Repeat for $\frac{12}{4} \div \frac{3}{4}$;

\[
\frac{6}{18} \div \frac{1}{9}.
\]

At this point you are trying to make the connections between the picture strategy and common denominator procedure explicit. See if students can verbalize this connection.

**Comments**

Student’s work for the lumber problem.
10. Return to the last problem on Student Page A. Ask students to imagine the picture. What would you first draw? How would you partition each unit so you can easily see how many groups of $\frac{2}{5}$’s you could make?

11. Write the problem based on this image using common denominators? $\frac{10}{5} \div \frac{2}{5}$?

12. Ask: How many groups of $\frac{2}{5}$ in 10-fifths?

13. Now draw the picture they imagined to verify that the answer is 2.

14. Conduct a discussion on the use of common denominators as an easy way to help solve division problems. Use a transparency of Student Page E; students should have their own copy.

15. For each problem ask students to rewrite the symbols using common denominators. For example, for the problem $\frac{4}{6} + \frac{1}{3}$, rewrite as $\frac{4}{6} + \frac{2}{6}$. Find the solution by asking a question like: How many 2-sixths in 4-sixths? Then verify the solution using a picture.

16. The last problem has a fractional remainder. Extra attention will be needed to help students use their common denominator strategy on this problem. Verifying with a picture will help students see why the answer is $1 \frac{1}{2}$.

Translations:
- Real life to picture to verbal
- Picture to picture to verbal
Examine students’ pictures to see how they solved these division problems.

Division Story Problems Set I

You have a piece of lumber that is 4 yards long. You want to cut lengths $\frac{2}{3}$ of a yard long to use in a shelf you are building. How many shelves can you cut?

Division Story Problems Set II

1) You have 2 $\frac{1}{4}$ yards of string. You cut the string into pieces $\frac{1}{8}$ yard in length. How many $\frac{1}{8}$ yard lengths can be made from 2 $\frac{1}{4}$ yards of string. Solve using the picture below. Write two division sentences.

Division sentence:

$$\frac{12}{8} \div \frac{1}{8} = 18$$

Division sentence with common denominators:

$$\frac{16}{8} \div \frac{1}{8} = 18$$
2) You have about \( \frac{3}{8} \) of a pint of ice cream leftover in the pint container. You want to make servings of about \( \frac{1}{4} \) of a pint. How many servings can you dish out? Solve this problem using the picture below.

\[
\frac{6/8}{\frac{1}{4}} = 3
\]

Division sentence with common denominators:

\[
\frac{3}{4} - \frac{1}{4} = 3
\]

3)

\[
2 \frac{3}{4} + \frac{1}{2} = \]

Division sentence with common denominators:

\[
\frac{11}{4} = 2 \frac{3}{2} 
\]

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Challenge Problem

4) CHALLENGE: Addis is building a bookshelf. She has a piece of lumber 2 $\frac{1}{2}$ yards in length. Each shelf will be $\frac{1}{4}$ of a yard. Draw a picture to show how many lengths she can make. If there are any leftover amounts, name it as a fraction.

Picture:

Division sentence:

$$2 \frac{1}{2} \div \frac{3}{4} = 3 \frac{1}{5}$$

Division sentence with common denominators:

$$\frac{10}{4} \div \frac{3}{4} = 3 \frac{1}{3}$$
Draw a picture to solve this problem.

You have $3 \frac{1}{2}$ pounds of peanuts. If you put them into $\frac{3}{4}$ pound bags, how many full bags can you make?
## Division Story Problems Set I

<table>
<thead>
<tr>
<th>Problem</th>
<th>Estimate</th>
<th>Picture</th>
<th>Number Sentence</th>
<th>Number Sentence with Common Denominators</th>
</tr>
</thead>
<tbody>
<tr>
<td>You have a piece of lumber that is 3 yards long. You want to cut lengths ( \frac{3}{4} ) of a yard long to use in a shelf you are building. How many shelves can you cut?</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>You have a piece of lumber that is 4 yards long. You want to cut lengths ( \frac{2}{3} ) of a yard long to use in a shelf you are building. How many shelves can you cut?</td>
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</tr>
<tr>
<td>You have a piece of lumber that is 2 yards long. You want to cut lengths ( \frac{2}{5} ) of a yard long to use in a shelf you are building. How many shelves can you cut?</td>
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</tbody>
</table>
Division Story Problems Set II

1) You have $2 \frac{1}{4}$ yards of string. You cut the string into pieces $\frac{1}{8}$ yard in length. How many $\frac{1}{8}$ yard lengths can be made from $2 \frac{1}{4}$ yards of string? Solve using the picture below.

Write two division sentences.

Division sentence:

Division sentence with common denominators:

2) You have about $\frac{6}{8}$ of a pint of ice cream leftover in the pint container. You want to make servings of about $\frac{1}{4}$ of a pint. How many servings can you dish out? Solve this problem using the picture below.

Division sentence:

Division sentence with common denominators:

3) Use the pictures to solve each problem. Write a division sentence with common
Division sentence with common denominators:

\[
\frac{3}{4} + \frac{1}{8}
\]

\[
\begin{array}{c}
\hline
\text{Smaller fraction}
\end{array}
\]

\[
\begin{array}{c}
\hline
\text{Bigger fraction}
\end{array}
\]

Division sentence with common denominators:

\[
2 \frac{3}{4} + \frac{1}{2}
\]

\[
\begin{array}{c}
\hline
\text{Smaller fraction}
\end{array}
\]

\[
\begin{array}{c}
\hline
\text{Bigger fraction}
\end{array}
\]

Division sentence with common denominators:

\[
2 \frac{1}{2} + \frac{3}{4}
\]

\[
\begin{array}{c}
\hline
\text{Smaller fraction}
\end{array}
\]

\[
\begin{array}{c}
\hline
\text{Bigger fraction}
\end{array}
\]

Division sentence with common denominators:
4) **CHALLENGE:** Addis is building a bookshelf. She has a piece of lumber $2\frac{1}{2}$ yards in length. Each shelf will be $\frac{3}{4}$ of a yard. Draw a picture to show how many lengths she can make. If there are any leftover amounts, name it as a fraction.

Picture:

Division sentence:

Division sentence with common denominators:
<table>
<thead>
<tr>
<th>Problem</th>
<th>Number Sentence with Common Denominators</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4}{6} \div \frac{1}{3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{7}{4} \div \frac{2}{8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2} \div \frac{1}{3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 28: Overview

Students build on their experiences with pictures, contexts, and benchmarks of \( \frac{1}{2} \) and 1 to estimate quotients to fraction division problems.

Materials

- Student Pages A, B, and C
- Overhead of student pages for teacher
- Overhead of Student Page C for teacher (optional for students)

Teaching Actions

Warm Up

Imagine drawing a picture to solve this problem. From that picture in your mind, find the actual answer.

\[ 3\frac{1}{2} \div \frac{1}{4} \]

Large Group Introduction

1. Begin the class with these estimations problems. Students should have a copy of Student Pages A and B with these problems.

2. Ask students to imagine the picture they would draw to solve the first problem.

   *You have 4 cups of flour. The recipe you are making calls for \( \frac{2}{3} \) cup of flour. About how many full recipes can you make?*

   Ask: If the recipe called for \( \frac{1}{2} \) cup of flour, how many recipes can you make? But the recipe calls for \( \frac{2}{3} \) cup of flour; will you have more or less than 8 recipes? Do you think you will have more than 4 recipes? Explain your ideas.

3. Repeat for problem 2 on Student Page A.

Comments

Encourage students to explain their estimates based on order, equivalence ideas, and mental images students have for each problem.

For example, to estimate \( 4 \div \frac{2}{3} \), a student could reason that the answer must be greater than 4 because \( \frac{2}{3} < 1 \). The answer will be less than 8 because \( \frac{2}{3} > \frac{1}{2} \). If it was \( 4 \div \frac{1}{2} \), the answer is clearly 8.

Using 1 and \( \frac{1}{2} \) as benchmarks will help them in their estimation.
Teaching Actions

You have 2 1/2 yards of ribbon. You cut it into pieces 3/4 in length? Estimate: About how many full pieces can you cut? At least 2? At least 5?

Guiding Questions: If you cut a length of 1 yard, how many pieces? If you cut a length ½ of a yard, how many pieces? 3/4 is greater than ½. Is the total number of pieces more or less than 5?

4. Repeat for problem 3 on Student Page A

Estimate by finding the whole number closest to the exact answer: 3 2/3 + 1/4.

Guiding Questions: How many fourths in three? Is 1/4 > or < 2/3? Can you get two more fourths out of the 2/3?

Small Group/Partner Work

5. Assign the rest of Student Pages A and 2B to students to work with a partner. Guiding questions embedded in the problems lead students to their estimates.

Wrap Up

6. Use Teacher Page C to guide students to find the exact answer to three problems from the work completed in this lesson.

7. Ask for their estimate first. Then ask them to imagine the picture they would draw to solve the problem. Ask: How would they partition that picture to make solving the problem easy?

Ask: What is the number sentence with common denominators for this problem? Find the exact answer. Possible estimation strategies follows:
### Teaching Actions

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.  ( \frac{2}{5} \div \frac{2}{5} ). If the problem was (2 \div \frac{1}{2}) then the answer would be 4. ( \frac{2}{5} &lt; \frac{1}{2} ) so the answer will be more than 4 but not much more as ( \frac{2}{5} ) is close to ( \frac{1}{2} ). ( \frac{10}{5} \div \frac{2}{5} = 5 ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Translations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Story problem to picture to verbal</td>
</tr>
<tr>
<td>• Symbol to symbol to verbal</td>
</tr>
<tr>
<td>• Symbol to symbol to picture</td>
</tr>
</tbody>
</table>

| b.  \( 3\frac{1}{2} \div \frac{3}{4} \). If the problem was \(3\frac{1}{2} \div \frac{1}{2}\) the answer would be 7. If it was \(3\frac{1}{2} \div 1\), the answer would be \(3\frac{1}{2}\). \( \frac{3}{4} \) is greater than \( \frac{1}{2} \) but less than one so the answer is between 3 and 7. \( \frac{14}{4} \div \frac{3}{4} = 4 \frac{2}{3} \) |

| c.  \( 3\frac{1}{2} \div \frac{1}{8} \). It would be greater than 24 because there are 24-eighths in 3. But it is less than 32 because there are 32-eighths in 4. \( \frac{28}{8} \div \frac{1}{8} = 28 \). |
Imagine drawing a picture to solve this problem. From that picture in your mind, find the actual answer.

\[ 3 \frac{1}{2} \div \frac{1}{4} \]
Division Estimation Problems

(1) You have 4 cups of flour. The recipe you are making calls for \( \frac{2}{3} \) cup of flour.
   Estimate: About how many full recipes can you make? At least 4? At least 8?

(2) You have \( 2 \frac{1}{2} \) yards of ribbon. You cut it into pieces \( \frac{3}{4} \) in length?
    Estimate: About how many full pieces can you cut? At least 2? At least 5?

(3) Estimate by finding the whole number closest to the exact answer:
    \[ \frac{3}{2} \div \frac{1}{4} = \]
(4) You know that $2 \div \frac{1}{2} = 4$. About how much would $2 \div \frac{2}{5}$ be? Is it more or less than 4?

(5) You know that $3 \div \frac{1}{2} = 6$. About how much would $3 \div \frac{2}{3}$ be? Is it more or less than 6?

(6) You know that $4 \div \frac{1}{2} = 8$. About how much would $4 \div \frac{3}{4}$ be? Is it more or less than 8?

(7) You know that $2 \div \frac{1}{2} = 4$. About how much would $2 \div \frac{1}{3}$ be? Is it more or less than 8?

(8) You know that $3\frac{1}{2} \div \frac{1}{2} = 7$. About how much would $3\frac{1}{2} \div \frac{3}{4}$ be? Is it more or less than 7?

(9) You know that $3\frac{3}{4} \div \frac{3}{4} = 7$. About how much would $3\frac{3}{4} \div \frac{1}{8}$ be? Is it more or less than 7?
<table>
<thead>
<tr>
<th>Problem</th>
<th>Estimate</th>
<th>Number sentence with common denominators</th>
<th>Picture to verify answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 \div \frac{2}{5})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3 \frac{1}{2} \div \frac{3}{4})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3 \frac{1}{2} \div \frac{1}{8})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ______________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest:
Homework sets have been developed for many of the lessons to allow students to have extra practice and to further develop their understandings. Many of the homework sets have problems relevant to the lesson, as well as review items from previous work. While each worksheet designates a lesson that students should have had prior to doing the homework, it does not have to be given the exact day the lesson is used in the classroom. It was not the intent of the writers that students do all homework sets, the choice of when to assign homework is left to the discretion of the teacher.
Order the fractions in each set from smallest to largest.

| 4/5 | 1/10 | 7/18 | 4/8 |

Write a list of directions on how to order each set.

| 3/100 | 9/10 | 1/4 | 3/50 |

Using pictures to explain, answer the following questions.

Mary shared a large pizza with 3 other friends and received her fair share. Jose shared a large pizza (same size as Mary’s pizza) with 2 other friends and received his fair share? Who ate more? Why? (Use pictures in your explanation)
Mohammad ran \(\frac{4}{5}\) of the distance around the track. Marisela ran \(\frac{8}{9}\) of the distance around the track. Who ran more? Explain. (Use pictures in your explanation)

Bryce spent \(\frac{4}{5}\) of his allowance on a new DVD. Ada spent \(\frac{4}{5}\) of her allowance on a new calculator. Is it possible that Ada spent more than Bryce? Explain in detail.

Estimate the amount shaded:

19

35
**Fraction Estimation**

The problems below were answered by another group of 6th graders. Using your estimation skills, determine if the answer is reasonable or not. Do not find the exact answer. Make your decision based on estimation. Describe your reason. The first problem is done for you.

\[ \frac{4}{6} + \frac{4}{8} = \frac{8}{14} \]

This doesn’t make sense. The answer must be greater than one since \( \frac{4}{6} \) is greater than \( \frac{1}{2} \) and \( \frac{4}{8} \) equals \( \frac{1}{2} \). \( \frac{8}{14} \) is less than one.

\[ \frac{2}{3} + \frac{1}{4} = \frac{11}{12} \]

\[ \frac{11}{12} - \frac{1}{2} = \frac{10}{12} \]

\[ \frac{1}{5} + \frac{2}{3} = \frac{3}{5} \]
\[
\frac{1}{4} - \frac{2}{100} = \frac{1}{3}
\]

\[
\frac{2}{3} - \frac{1}{4} = \frac{1}{12}
\]

\[
\frac{11}{12} - \frac{1}{4} = \frac{10}{8}
\]

\[
\frac{4}{6} - \frac{3}{8} = \frac{1}{2}
\]
Adding and Subtracting Fractions

Directions: Read the story. Then read the questions. Decide if it is asking you to add or subtract. Then answer each of the questions.

Story:
Erik played basketball for $\frac{5}{6}$ of an hour on Monday. He played basketball for $\frac{1}{4}$ of an hour on Tuesday.

1. How much time did Erik spend playing basketball altogether?

   Circle one: Add  Subtract

   List fractions equal to $\frac{5}{6}$:

   List fractions equal to $\frac{1}{4}$:

2. How much longer did Erik play basketball on Monday than Tuesday?

   Circle one: Add  Subtract

   Write the sentence to solve:
Story:
On Thursday and Friday Scout ran a total of $\frac{7}{8}$ of a mile. On Thursday she ran $\frac{1}{2}$ of a mile.

<table>
<thead>
<tr>
<th>3. How far did Scout run on Friday?</th>
<th>4. How much further would she have to run if her goal is to run 4 miles this week?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle one: Add Subtract</td>
<td>Circle one: Add Subtract</td>
</tr>
<tr>
<td>List fractions equal to $\frac{7}{8}$:</td>
<td>Write the sentence to solve:</td>
</tr>
<tr>
<td>List fractions equal to $\frac{1}{2}$:</td>
<td>Write the sentence to solve:</td>
</tr>
</tbody>
</table>
# Adding and Subtracting Fractions

Directions: Read the story. Then read the questions. Decide if it is asking you to add or subtract. Then answer each of the questions.

**Story:**

You ride your bike \( \frac{7}{8} \) of a mile.

Your younger brother only rides \( \frac{3}{4} \) of a mile.

<table>
<thead>
<tr>
<th>1. How much further did you bike than your brother?</th>
<th>2. How much do you and your brother ride altogether?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle one: Add \ Subtract \</td>
<td>Circle one: Add \ Subtract \</td>
</tr>
</tbody>
</table>

List fractions equal to \( \frac{7}{8} \): 

List fractions equal to \( \frac{3}{4} \): 

Write the number sentence to solve:

Write the number sentence to solve:
**Story:**
Hamida and Michael made a pie. Hamida ate $\frac{1}{3}$ of the pie and Michael ate $\frac{1}{6}$ of the pie.

<table>
<thead>
<tr>
<th>3. How much of the total pie did Hamida and Michael eat?</th>
<th>4. How much more did Hamida eat than Michael?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle one: Add Subtract</td>
<td>Circle one: Add Subtract</td>
</tr>
</tbody>
</table>

List fractions equal to $\frac{1}{3}$:

List fractions equal to $\frac{1}{6}$:

Write the number sentence to solve:

Write the number sentence to solve:
## Adding and Subtracting Fractions

Directions: Read the story. Then read the questions. Decide if it is asking you to add or subtract. Then answer each of the questions.

**Story:**
Samira has a bag of candy that weighs \(2 \frac{1}{4}\) pounds. Taryn has a bag of candy that weighs \(1 \frac{7}{8}\) pounds.

<table>
<thead>
<tr>
<th>1. If Samira and Taryn put their candy together, how much will it weigh?</th>
<th>2. Who has more candy, Samira or Taryn? How much more candy does she have?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle one: Add Subtract</td>
<td>Circle one: Add Subtract</td>
</tr>
<tr>
<td>Write the number sentence to solve:</td>
<td>Write the number sentence to solve:</td>
</tr>
</tbody>
</table>

Estimate the answer. Place an X for your estimate on the number line.

3. \(\frac{6}{7} + \frac{1}{8} =\)

4. \(\frac{7}{9} - \frac{5}{7} =\)
Liam and Grace are both practicing their parts for the school musical. Liam practiced for \(3 \frac{7}{12}\) hours and Grace practiced for \(2 \frac{1}{6}\) hours.

<table>
<thead>
<tr>
<th>5. How much did they practice total?</th>
<th>6. How much more would they have to practice if their goal was to reach a total of 6 hours?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle one: Add Subtract</td>
<td>Circle one: Add Subtract</td>
</tr>
<tr>
<td>Write the sentence to solve:</td>
<td>Write the sentence to solve:</td>
</tr>
</tbody>
</table>

Find the sum or difference:

7. \(\frac{4}{5} + \frac{3}{8} = \)

8. \(\frac{7}{9} - \frac{1}{2} = \)
Record each amount in different ways as shown in the first example (picture, words, fraction symbols, decimal symbols).

<table>
<thead>
<tr>
<th>Picture</th>
<th>Describe 2 ways</th>
<th>Fraction Symbols</th>
<th>Decimal Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>3-tenths and 2-hundredths or 32-hundredths</td>
<td>$\frac{3}{10} + \frac{2}{100}$</td>
<td>.32</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>49-hundredths or 4-tenths and 9-hundredths</td>
<td>$\frac{32}{100}$</td>
<td>.08</td>
</tr>
</tbody>
</table>
Solve the review problems:

1. Add the fractions:
\[
\frac{1}{3} + \frac{1}{6} + \frac{1}{9}
\]

2. Explain which fraction is larger and why:
\[
\frac{7}{10} \quad \text{or} \quad \frac{65}{100}
\]

Partition the blank strip to show a fraction equivalent to the fraction shown on the strip above it.

3. 
\[
\frac{9}{12}
\]

4. 
\[
\frac{6}{10}
\]

\[
\frac{4}{5}
\]
Comparing Decimals

1. Shade in 0.45 of the grid

2. Shade in 0.035 of the grid

Estimate the following sums or differences. Place an X on the number line where your estimate lies.

1.  5.03 – 4.97

2.  0.87 + 1.25

3.  1 \frac{4}{5} - \frac{8}{9}
### Directions: Read the story. Then read the questions. Decide if it is asking you to add or subtract. Then answer each of the questions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Danielle is watching a movie that is (2\frac{3}{4}) hours long. She has been watching it for (1\frac{1}{3}) hours. How much more time is left in the movie?</td>
<td>2. Zach got a new puppy that weighs (4\frac{1}{2}) pounds and his sister got a new kitten that weighs (2\frac{1}{8}) pounds. What is the total weight of the animals?</td>
</tr>
<tr>
<td>3. When Laurel started listening to her sister’s iPod there was (\frac{2}{3}) of the power left in the battery. When she finished listening there was about (\frac{1}{6}) of the battery power remaining. How much of the battery’s power did Laurel use?</td>
<td>4. What fraction of an hour has passed between 11:25 and 11:55?</td>
</tr>
</tbody>
</table>
## Adding and Subtracting Decimals

Use the Addition-Subtraction boards to solve #1 and 2, then use only numbers and symbols to write the problem and solution in the third column.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Picture</th>
<th>Symbolic Solution</th>
</tr>
</thead>
</table>
| 1. Add:  
0.32 + 0.09 | ![Image] | ![Symbolic Solution] |
| 2. Subtract:  
0.83 − 0.54 | ![Image] | ![Symbolic Solution] |
In each problem below, circle the larger number, then explain how you know it is larger.

<table>
<thead>
<tr>
<th>Circle the larger number</th>
<th>Explain how you know it is larger.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/8 or 5/6</td>
<td></td>
</tr>
<tr>
<td>0.63 or 0.309</td>
<td></td>
</tr>
<tr>
<td>0.54 or 3/7</td>
<td></td>
</tr>
</tbody>
</table>

Using the given number, record the different ways to show it.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Describe 2 ways</th>
<th>Fraction Symbols</th>
<th>Decimal Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td></td>
<td></td>
<td>0.71</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td>3/100</td>
<td></td>
</tr>
</tbody>
</table>
Decimals with Grids and Number Lines
Using the grids and number lines below, show the representations of the given decimal.

1. 0.675

2. 0.8

3. Which decimal is larger, 0.675 or 0.8? Explain how you know.
Given the fraction, write an equivalent fraction.

4. \( \frac{3}{12} = \frac{4}{x} \)
Draw a picture of each fraction:

5. \( \frac{3}{8} = \frac{4}{x} \)
Draw a picture of each fraction:

6. \( \frac{34}{40} = \frac{17}{x} \)

7. \( \frac{8}{16} = \frac{1}{2} \)

8. \( \frac{1}{15} = \frac{1}{3} \)

9. \( \frac{4}{20} = \frac{3}{5} \)

Solve the following problems. Show your work.

10. Tate is selling magazines for a school fundraiser, his class has set a goal. So far Tate has sold enough magazines to help his class get \( \frac{2}{5} \) of the way to their goal. A classmate, Mariama has sold enough to help the class get \( \frac{1}{6} \) of the way to their goal. How much of the class goal remains to be met?

11. Miguel’s older brother tried to stump him on a math problem. He asked Miguel, “What should be added to \( \frac{3}{4} \) to get \( \frac{15}{16} \)?” Help Miguel solve the problem.
Number Lines and Decimals

Label each decimal on the number line.

.93

.05

.49

.52

.03

.75

.25

.33
Show each problem solution on the number line. Then check your solution by solving using symbols only.

\[0.93 - 0.05 = \]

\[0.49 + 0.25 = \]

\[0.33 - 0.03 = \]

\[0.52 + 0.48 = \]
Adding Fractions on the Number Line
Show each problem on the best number line. Record your answer below each problem, showing the equivalent fractions you used.

a) \( \frac{5}{6} + \frac{1}{12} \)
   \( = \frac{3}{5} + \frac{1}{10} \)

b) \( \frac{3}{2} + \frac{1}{3} \)
   \( = \frac{1}{2} + \frac{3}{4} \)
Show each fraction on the number line provided.

\[ \frac{2}{3} \]

\[ \frac{4}{5} \]

\[ \frac{4}{6} \]

\[ \frac{2}{6} \]

\[ \frac{5}{6} \]

\[ \frac{4}{15} \]
Multiply Fractions

Write a multiplication sentence for each picture shown below. Each cloud contains a group.

1. Unit is
   
   Words:  
   ____ groups of ____
   
   Multiplication Sentence:  
   ____ x ____ = ____

2. Unit is
   
   Words:  
   
   Multiplication Sentence:  
   

Draw a picture for each multiplication sentence then complete the sentence.

3.  
   \[3 \times \frac{1}{2} = \]

4.  
   \[4 \times \frac{3}{4} = \]
Show how to find a solution by drawing a picture, writing out in words, and writing a multiplication sentence.

| 5. Paola reads for $\frac{3}{8}$ of an hour each day for 5 days. How many total hours does she read? | 6. Molly puts $\frac{2}{3}$ a teaspoon of sugar in her cereal each morning for 7 days. How much sugar does she use in all? |
| Picture: | Picture: |
| Words: | Words: |
| Multiplication Sentence: | Multiplication Sentence: |

**Review**

7. Show the steps to find the sum:

$$\frac{2}{3} + \frac{7}{12}$$

8. Show the steps to find the difference:

$$\frac{3}{5} - \frac{1}{4}$$

9. Show how to use a number line to find the answer to the problem:

$$\frac{1}{2} + \frac{1}{3}$$
# Multiplying Fractions

*(whole number x fraction)*

Draw a picture and write a multiplication sentence for the given problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Picture</th>
<th>Multiplication Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 4 groups of ( \frac{2}{5} )</td>
<td><img src="image1.png" alt="Picture" /></td>
<td>( 4 \times \frac{2}{5} = \frac{8}{5} )</td>
</tr>
<tr>
<td>2. 3 groups of ( \frac{1}{2} )</td>
<td><img src="image2.png" alt="Picture" /></td>
<td>( 3 \times \frac{1}{2} = \frac{3}{2} )</td>
</tr>
</tbody>
</table>

Write a multiplication sentence for each picture shown below. Each cloud contains a group.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Picture</th>
<th>Multiplication Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Unit is ( \square )</td>
<td><img src="image3.png" alt="Picture" /></td>
<td>( ? \times ? = ? )</td>
</tr>
<tr>
<td>4. Unit is ( \square )</td>
<td><img src="image4.png" alt="Picture" /></td>
<td>( ? \times ? = ? )</td>
</tr>
</tbody>
</table>
Write a story problem to represent each sentence below then draw a picture to find the answer and complete the sentence.

5. \(2 \times 6 = \) ___

6. \(5 \times \frac{2}{3} = \) ___

Review

7. a) Estimate the difference:
\[ \frac{2}{3} - \frac{3}{4} = \]

b) Show the steps to find the exact difference:
\[ \frac{2}{3} - \frac{3}{4} = \]

8. a) Estimate the sum:
\[ \frac{4}{9} + \frac{2}{3} = \]

b) Show the steps to find the exact sum:
\[ \frac{4}{9} + \frac{2}{3} = \]
### Multiplying Fractions

1. Wayne earns $24 an hour. How much will he make if he works $\frac{5}{6}$ of an hour?

   **Multiplication Sentence:**
   
   \[
   \text{# of hours worked} \times \frac{\text{amount earned per hour}}{\text{amount of money earned}} = \text{amount of money earned}
   \]

2. Rachael earns $8 an hour selling popcorn. How much will she earn if she works for $\frac{3}{4}$ of an hour?

   **Multiplication Sentence:**
   
   \[
   \text{# of hours worked} \times \frac{\text{amount earned per hour}}{\text{amount of money earned}} = \text{amount of money earned}
   \]

3. There are 24 students that went camping. $\frac{3}{4}$ of the students are girls. How many girls went camping?

   **Multiplication Sentence:**
   
   \[
   \text{# of students} \times \frac{\text{percentage}}{\text{total}} = \text{number of girls}
   \]

4. Daniel earns $40 an hour building Legos. How much will he earn if he works $\frac{7}{4}$ hours?

   **Multiplication Sentence:**
   
   \[
   \text{# of hours worked} \times \frac{\text{amount earned per hour}}{\text{amount of money earned}} = \text{amount of money earned}
   \]
5. Write a multiplication sentence for the picture shown below. Each cloud contains a group. Unit is

Words:

Multiplication Sentence:

6. Show how to find a solution by drawing a picture, writing out in words, and writing a multiplication sentence.

Winona puts $\frac{3}{4}$ a teaspoon of sugar in her cereal each morning for 5 days. How much sugar does she use in all?

Picture:

Words:

Multiplication Sentence:

Draw a picture for each multiplication sentence then complete the sentence.

7. $15 \times \frac{4}{5} =$

8. $\frac{3}{4} \times 8 =$

9. a) Place an x on the number line to estimate the sum:

$$\frac{3}{4} + \frac{7}{8} =$$

b) Show steps to find the exact sum:

$$\frac{3}{4} + \frac{7}{8} =$$
# Multiplying Fractions on Number Lines

Use a double number line and write a multiplication sentence to answer each of the questions below.

<table>
<thead>
<tr>
<th>Question</th>
<th>Number Line</th>
<th>Multiplication Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. For the track fundraiser, runners earn $6 for every mile they run. How much money will Sara earn if she runs $2 \frac{2}{3}$ miles?</td>
<td><img src="image1" alt="Number Line" /></td>
<td>( \text{amount of money earned ($)} = \times ) ( \frac{\text{# of miles run}}{\text{amount earned per mile}} \times \frac{\text{amount of money earned}}{\text{miles run}} )</td>
</tr>
<tr>
<td>2. JD travels 16 miles for every hour he rides his bike. How far will JD travel if he rides for $1 \frac{3}{4}$ hours?</td>
<td><img src="image2" alt="Number Line" /></td>
<td>( \text{amount of miles traveled} = \times ) ( \frac{\text{# of hours riding}}{\text{miles traveled per hour}} \times \frac{\text{amount of miles traveled}}{\text{hours}} )</td>
</tr>
<tr>
<td>3. There were 27 pizzas ordered for the class party. If $\frac{2}{9}$ of the pizzas were vegetarian, how many vegetarian pizzas were ordered?</td>
<td><img src="image3" alt="Number Line" /></td>
<td>( \text{Multiplication Sentence:} )</td>
</tr>
<tr>
<td>4. It costs $8 each hour to ice skate at the rink. If Manuel skates for $1 \frac{3}{4}$ hours, how much will it cost?</td>
<td><img src="image4" alt="Number Line" /></td>
<td>( \text{Multiplication Sentence:} )</td>
</tr>
</tbody>
</table>
### Review.

<table>
<thead>
<tr>
<th>Multiplication Sentence</th>
<th>Words</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3 \times \frac{2}{3} = ___ )</td>
<td>5 groups of ( \frac{1}{4} )</td>
<td></td>
</tr>
<tr>
<td>( 4 \times \frac{3}{2} = ___ )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Partition the blank strip to show a fraction equivalent to the fraction shown on the strip above it. Think in terms of paper folding. How did the strip look before the last fold?

\[ \frac{4}{12} \]

\[ \frac{1}{3} \]
Multiplying Fractions

1. Shade in the fractional amounts of patty-paper pictures shown below.

2. Max finds \( \frac{1}{3} \) of a square cake out on the counter of his kitchen. He eats \( \frac{7}{8} \) of this piece. What fraction of the entire cake did Max eat? Show how you can find the answer by shading in the square below then writing a multiplication sentence to show solution.

   Multiplication Sentence:

3. Write the multiplication sentences as word phrases and draw pictures to show answers.

   a) \( \frac{1}{3} \times \frac{1}{4} = \)

   b) \( \frac{2}{3} \times \frac{1}{3} = \)

   c) \( \frac{3}{4} \times \frac{3}{5} = \)

   ___ group of \( \frac{1}{4} \)

   ___ groups of ___

   ___ groups of ___
Solve on the number line and write a multiplication sentence that shows how to find the answer.

4. Xander earns $40 an hour. How much will he make if he works \( \frac{3}{8} \) of an hour?

\[
\begin{array}{c|c|c|c|c|c|c|c}
0 & 40 & 80 \\
\end{array}
\]

(amount earned ($))

Multiplication Sentence:

5. Sophie earns $32 an hour selling popcorn. How much will she earn if she works for \( \frac{7}{8} \) of an hour?

\[
\begin{array}{c|c|c|c|c|c|c|c}
0 & 40 & 80 \\
\end{array}
\]

(amount earned ($))

Multiplication Sentence:

6. There are 24 students that went camping. \( \frac{5}{12} \) of the students are girls. How many girls went camping?

\[
\begin{array}{c|c|c|c|c|c|c|c}
0 & & & & & & \\
\end{array}
\]

Multiplication Sentence:

7. Taurean earns $28 an hour building Legos. How much will he earn if he works \( \frac{7}{4} \) hours?

\[
\begin{array}{c|c|c|c|c|c|c|c}
0 & & & & & & \\
\end{array}
\]

Multiplication Sentence:

Draw a picture for each multiplication sentence then complete the sentence.

8. \[
\frac{1}{2} \times \frac{1}{4} = 
\]

9. \[
\frac{3}{7} \times 21 = 
\]
## Multiplying Fractions

1. Fill in the table below.

<table>
<thead>
<tr>
<th>Multiplication Problem</th>
<th>Picture</th>
<th># of darkly shaded pieces</th>
<th># of total pieces in square</th>
<th>Fraction of square shaded dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{5} \times \frac{1}{4} )</td>
<td>![Fraction Picture]</td>
<td>3</td>
<td>1</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>( \frac{2}{7} \times \frac{1}{3} )</td>
<td>![Fraction Picture]</td>
<td>2</td>
<td>3</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>( \frac{5}{6} \times \frac{1}{2} )</td>
<td>![Fraction Picture]</td>
<td>5</td>
<td>6</td>
<td>( \frac{5}{12} )</td>
</tr>
<tr>
<td>( \frac{3}{4} \times \frac{3}{4} )</td>
<td>![Fraction Picture]</td>
<td>9</td>
<td>16</td>
<td>( \frac{9}{16} )</td>
</tr>
</tbody>
</table>
2. Use the patterns from the table to find a shortcut to do these problems.

   a) \( \frac{2}{8} \times \frac{3}{5} = \)
   
   b) \( \frac{6}{7} \times \frac{3}{4} = \)

**Review**

3. Shadow ate \( \frac{3}{4} \) of a brownie. A whole brownie has 120 calories. How many calories did Shadow eat?

4. Ginger uses \( \frac{1}{3} \) cup of flour for a cookie recipe. She uses \( \frac{3}{8} \) cups of flour for her pancake recipe. How much flour does she use in all?

5. Find the sum:

   \( 0.36 + 0.40 = \)

6. Order the numbers from smallest to largest.

   \( 0.07, \frac{1}{100}, 0.1 \)

7. In the long jump, Malik jumped \( 5\frac{1}{6} \) feet and Austin jumped \( 4\frac{2}{3} \) feet. How much further did Malik jump than Austin?

8. When asked to estimate \( 4\frac{3}{4} - 2\frac{3}{8} \), Michelle thought the answer would be greater than 2 and Alicia thought it would be less than 2. Who is correct, and why?
Multiplying Fractions

Find the exact length of the runs below using the number line and write the multiplication sentence that shows how far each person runs.

1. Winona runs down Oakland Avenue. Each block is $\frac{1}{3}$ of a mile long. She runs $\frac{1}{2}$ of a block before she gets tired and stops. How far in miles does she run?

Multiplication Sentence:

2. Rachael runs along 46th street. Each block is $\frac{1}{4}$ of a mile long. She runs $\frac{2}{3}$ of a block before stopping. How far in miles does she run?

Multiplication Sentence:

3. Bryce runs along Minnehaha Parkway. Each block is $\frac{1}{2}$ a mile. He runs $\frac{2}{3}$ of a block before stopping. How many miles does he run?

Multiplication Sentence:

4. Ramla sprints $\frac{1}{3}$ of a $\frac{1}{4}$ mile track. How much of a mile does she run?

Multiplication Sentence:
5. Write a multiplication sentence for the picture shown below. Each cloud contains a group. Unit is

Words:
____ groups of ____

Multiplication Sentence:

6. Laura earns $8 an hour. How much does she earn if she works \(\frac{1}{4}\) of an hour? Show your answer on the number line and write a multiplication sentence.

Multiplication Sentence:

Multiply the following fractions by drawing on the squares.

<table>
<thead>
<tr>
<th>number</th>
<th>symbols</th>
<th>picture</th>
<th>use algorithm to find answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>(\frac{2}{5} \times \frac{1}{3})</td>
<td>![Picture 1]</td>
<td>(\frac{2}{5} \times \frac{1}{3} = \frac{2}{15})</td>
</tr>
<tr>
<td>8.</td>
<td>(\frac{4}{5} \times \frac{3}{7})</td>
<td>![Picture 2]</td>
<td>(\frac{4}{5} \times \frac{3}{7} = \frac{12}{35})</td>
</tr>
</tbody>
</table>
Multiplying Fractions
Show how to solve the following problems using patty paper, the number line and your rule.

1. Chloe has been hoping for a lot of snow. The weather report said that \( \frac{4}{5} \) of an inch of snow is falling every hour. Suppose it only snows for \( \frac{3}{4} \) of an hour. How much snow will fall?

Patty Paper: Number Line: Rule:

2. Brynn’s new puppy is gaining \( \frac{5}{6} \) of a pound every month. She wants to know how much weight her puppy will gain in \( \frac{2}{3} \) of a month.

Patty Paper: Number Line: Rule:

3. In the last problem, why does it make sense that the denominator in the answer is eighteenths?

Match each patty paper problem with its multiplication sentence.

4. \((1/4) \cdot (\frac{3}{4})\) a. \(\frac{5}{6} \times \frac{4}{5}\)
5. \( \left( \frac{2}{3} \right) \cdot \left( \frac{3}{8} \right) \)

b. \( \frac{1}{4} \cdot \frac{3}{4} \)

6. \( \left( \frac{5}{6} \right) \cdot \left( \frac{4}{5} \right) \)

c. \( \frac{2}{3} \cdot \left( \frac{3}{2} \right) \)

7. \( \left( \frac{2}{3} \right) \cdot \left( \frac{3}{2} \right) \)

d. \( \frac{2}{3} \cdot \frac{3}{8} \)

8. Find the answers to the multiplication sentences in #4-7.

\[
\begin{align*}
\frac{1}{4} \cdot \frac{3}{4} &= \\
\frac{2}{3} \cdot \frac{3}{8} &= \\
\frac{5}{6} \cdot \frac{4}{5} &= \\
\frac{2}{3} \cdot \left( \frac{3}{2} \right) &= 
\end{align*}
\]
Dividing Fractions

Solve the following problems by drawing a picture.

1. If it takes $\frac{2}{3}$ of an hour to play a board game. How many games can Michael play in 2 hours?

2. There are $4\frac{1}{2}$ ounces of tea leaves in Salma’s house. If $\frac{1}{4}$ of an ounce is needed to make a serving, how many servings of tea can Salma make?

3. How many $\frac{1}{4}$’s are in $\frac{6}{8}$?

4. Explain how you figured out your answer to #3.
Use a picture to solve each problem.
5. Nina has to practice piano for 3 hours this week. She likes to practice for $\frac{3}{4}$ of an hour at a time. How many times will Nina have to practice to make sure she practices for 3 hours?

6. Jabarti has been watching television for $1\frac{1}{4}$ hours. If each television show is $\frac{1}{2}$ an hour long, how many shows has he watched?

7. It takes Tate about $\frac{3}{5}$ of an hour to put together a puzzle. How many puzzles can he put together in 2 hours?

8. Explain how you figured out #7.
Dividing with Fractions
Solve each problem by drawing a picture or number line. Then write the division sentence and the division sentence with common denominators.

1. Kameron and Debbie want to make 3 batches of cookies to give to their friends. If each batch takes them about \( \frac{3}{4} \) of an hour to make, how many hours will it take them to bake all 3 batches?

   Picture:                                                            Division Sentence:

   Division Sentence with Common Denominators:

2. Asad has \( \frac{5}{6} \) of the battery power left on his portable dvd player. Each movie uses about \( \frac{1}{3} \) of the battery power. How many movies can Asad watch before there is no more power left in the battery?

   Picture:                                                            Division Sentence:

   Division Sentence with Common Denominators:

3. Sophea wants to make as many pans of brownies as she can. Each batch calls for \( \frac{3}{4} \) cup of chocolate chips. If Sophea has 2 cups of chocolate chips, how many pans of brownies can she make?

   Picture:                                                            Division Sentence:

   Division Sentence with Common Denominators:
For each problem give an estimate, then answer the question using a division sentence. Then draw a picture to justify.

4. Helena goes to the arcade with $4. Her favorite game costs $\frac{3}{4}$ of a dollar to play. How many times can she play her favorite game?

Estimate: 
Number Sentence: 
Picture: 

5. How many $\frac{5}{6}$’s are there in $3\frac{2}{3}$?

Estimate: 
Number Sentence: 
Picture: 

6. Katie wants to make costumes for the school play. She has $\frac{7}{8}$ of a yard of one of the fabrics that she will need. If each costume calls for $\frac{3}{4}$ of a yard of this fabric, for how many costumes can she use this fabric?

Estimate: 
Number Sentence: 
Picture: 
Appendices

Fraction Circles

Quizzes

End of Module Assessments

Student Interviews
Pictures of the Fraction Circles

Black
2 pieces

Yellow
3 pieces

Brown
4 pieces

Blue

Orange
5 pieces

Pink
6 pieces

Light Blue
7 pieces

Gray
8 pieces

White
9 pieces

Purple
10 pieces

Red
12 pieces

Green
15 pieces
YELLOW
PINK
WHITE
PURPLE
1. Rachel ate $\frac{3}{8}$ of a small pizza for dinner. The next day she ate $\frac{1}{4}$ of a small pizza. How much of a small pizza did she eat in all?

2. Galen lives $\frac{4}{6}$ of a mile from school. After going $\frac{1}{10}$ of a mile his bike broke down and he had to walk the rest of the way to school. How far did he walk to school?

   o Estimate: Did he walk more or less than $\frac{1}{2}$ mile? Explain your thinking.

   o Find the actual distance that he walked.
3. Show how to do this problem on the number line: \[ \frac{1}{3} + \frac{5}{6} = \]

4. Explain your choice of a number line to solve the problem.
1. Show this problem on the Decimal +/- board.  \[0.55 - 0.3 =\]

2. Final answer is: 

\[\square\]
3. What number is shown by the arrow?

4. Show that number on this grid:

5. Imagine 0.75 on a 10 x 10 grid. Add 0.03 to it. Estimate: Is the sum greater or less than one? Explain your thinking.
1. Jessie earns $24 an hour working at Starbucks. How much will she earn if she works \( \frac{3}{4} \) of an hour? Please show how you can determine this answer using a number line.

2. Show how to use the square picture below to solve the following problem.

Tasty-Bars are sold in square pans. There is \( \frac{2}{3} \) of a pan of bars in the display case at the bakery. Jessica buys \( \frac{3}{5} \) of what was left in this pan. How much of a whole pan of Tasty-Bars did she purchase?
3. Randy runs along Hollywood Boulevard every Tuesday evening. He ran $\frac{2}{3}$ of a block before getting tired. Each block along Hollywood Boulevard is $\frac{1}{4}$ of a mile long. How far does he run before he gets tired? Show how to solve the problem using the number line below. Circle your final answer.
Name_______________  Date _______________

Fraction Addition and Subtraction Interview

The goal of the student interview is to better understanding student’s thinking. Does the student have strong mental images to support their ability to order fractions and estimate reasonable sums and differences? Can the student use models to add and subtract fractions and connect those models to the symbolic procedure? Does the student see the need for common denominators?

Order:

1. Sort these fractions from smallest to largest. Explain your thinking. (If student uses common denominator approach, then ask him/her to sort based on their images of fraction circles)

\[
\frac{2}{3} \quad \frac{1}{19} \quad \frac{11}{13} \quad \frac{10}{20} \quad \frac{5}{12}
\]

Order these fractions. Explain your thinking. What pictures do you imagine that help you order these fractions?

2. \(\frac{8}{9} \quad \frac{4}{5}\)

3. \(\frac{4}{11} \quad \frac{6}{8}\)
Addition and Subtraction - Estimation:

Estimate the answer to this problem by deciding about where on the number line the answer would be. For example, if you think the answer is between 0 and $\frac{1}{2}$, then put an x here. (Show this on a sample number line.) **Tell me what you are picturing in your mind as you estimate.**

4. $\frac{3}{4} + \frac{1}{5}$

5. $\frac{14}{15} - \frac{5}{7}$

6. $1\frac{3}{5} - \frac{3}{8}$
Addition and Subtraction - exact answer

7. Find the actual answer to this problem using fraction circles. Explain how you did this.

\[
\frac{3}{8} + \frac{1}{4}
\]

8. Now show how to solve that problem with symbols. Explain how this represents what you did with the fraction circles.

9. Hamdi lives \(1 \frac{2}{3}\) mile from school. She rode her bike \(\frac{1}{4}\) of a mile when the tire went flat. She had to walk the rest of the way. How far did she walk? Solve this problem using symbols.

10. Look at your answer. Use your estimation skills to determine if your answer is reasonable. Explain your thinking,
Fraction Interview Addition and Subtraction
Materials for Interview

Cards for Question 1 (cut apart):

\[
\begin{array}{cccc}
\frac{2}{3} & \frac{1}{19} & \frac{11}{13} & \frac{10}{20} \\
\frac{5}{12} & \frac{8}{9} & \frac{4}{5} & \\
\frac{4}{11} & \frac{6}{8} \\
\end{array}
\]
\[ \frac{3}{4} + \frac{1}{5} \]

\[ \frac{14}{15} - \frac{5}{7} \]

\[ 1\frac{3}{5} - \frac{3}{8} \]
Now show how to solve that problem with symbols. Explain how this represents what you did with the fraction circles.

Hamdi lives $1 \ 2 \frac{2}{3}$ mile from school. She rode her bike $\frac{1}{4}$ of a mile when the tire went flat. She had to walk the rest of the way. How far did she walk? Solve this problem using symbols.

Look at your answer. Use your estimation skills to determine if your answer is reasonable. Explain your thinking,
Decimal Interview

The goal of the student interview is to better understand student’s thinking. Does the student have strong mental images to support their ability to order decimals and to estimate reasonable sums and differences? Can the student use the models to add and subtract decimals and connect those models to the symbolic procedure?

1. Sort these decimals from smallest to largest. Explain your thinking. (If student uses a procedural approach, then ask him/her to sort based on their images of the 10 x 10 grid)

0.245  0.025  0.249  0.3

For problems 2 – 5:

Name each decimal. Describe the picture you have in your mind when you think of this decimal. Which decimal is larger or are they equal? Explain your thinking.

(2)  0.9  0.009

(3)  0.11  0.110

(4)  0.75  0.9
For problems 6 – 8

Imagine the decimal +- board. Use that image to solve each problem below. Do not use paper and pencil. Describe what you are thinking.

(6) \( .37 + .4 = \)

(7) \( .55 - 0.3 = \)

(8) \( 2.3 - .05 = \)

(9) Picture 57-hundredths on the decimal +- board. If you took away 9-thousandths would the amount left shaded be more than \( \frac{1}{2} \) or less than \( \frac{1}{2} \)? Explain without finding the exact answer.
(10) Picture 28-hundredths on the decimal +- board. If you added 6-hundredths more, would the amount shaded be more than \( \frac{1}{2} \) or less than \( \frac{1}{2} \)? Explain without finding the exact answer.

(11) Show the same problem using the Decimal + - board. (Show the symbol card here)

(12) Show how to solve this problem using symbols. Explain what you are doing. Do you get the same answer as with the Decimal +- board? If not, which answer is correct?

(13) Show how to subtract .67 - .5 using the Decimal + - board. Explain your steps. Now record those actions with symbols. Are the answers the same?

(14) Now show 1.85 - .4 on the number line. Explain how to do this. How is solving the problem on the number line similar to solving with symbols?
Decimal Interview - Materials for Interview

Card for question 1

0.245  0.025  0.249  0.3

Cards for questions 2 – 5

0.9  0.009
0.055  0.5

0.11  0.110
0.75  0.9

Cards for questions 6 – 8

0.37 + .4 =
0.55 − 0.3 =
2.3 - .05 =
0.57

0.28 + .06 =

Decimal + - board for question 11
Decimal + - board for question 13
1.85 - .4 =

Number line for question 14
Fraction Multiplication and Division Interview

Directions for Questions 1 – 3

Estimate this product by putting an X on the number line where you think the exact will be. For example, if you think the answer is between 0 and 5 put an X here (Point to the space between 0 and 5 on the number line). Tell me what you are picturing in your mind as you estimate.

1. 10 groups of \( \frac{3}{7} \) = ?

2. \( \frac{4}{3} \times 10 \) = ?

3. \( 3 \frac{3}{4} \div \frac{1}{2} \) = ?
4. Draw pictures to show how to solve the problem. Explain what you are doing.

Michael ate \(\frac{2}{3}\) of a candy bar each day for 4 days. How many candy bars did he eat in all?

5. Show how to use the patty paper to solve this problem. Explain each step and how to determine the final answer from folding the patty paper.

Torie’s family had a pie for dessert last night. They didn’t like it so there was \(\frac{3}{4}\) of the pie left. In the morning Torie ate \(\frac{1}{4}\) of the left over pie. How much of the whole pie did she eat in the morning?

6. Draw a number line to solve this problem. Explain each step and how to determine the final answer from the number line.

Joshkin earns $12.00 an hour mowing lawns. If he works \(\frac{5}{6}\) of an hour how much money did he earn?
7. Show how to solve this problem on the number line. Explain each step.

Lauren lives 1/2 of a mile from school. She rode her bike 2/3 of the way when the tire went flat. How much of a mile did Lauren ride her bike before the tire went flat?

8. Draw a picture to solve the problem. Explain each step.

A bird feeder holds about $1 \frac{1}{4}$ cups of seed. The scoop you use to fill the bird feeder holds about $\frac{3}{8}$ of a cup. How many scoops does it take to fill the feeder?

9. Draw a picture to solve the problem. Explain each step.

You have a piece of ribbon 4 yards long. How many pieces of ribbon 4/5 of a yard long can you cut from this ribbon? If the answer is not a whole number, what name can you give to the remainder?
Fraction Multiplication and Division Interview
Materials for Interview

Card for Question 1

10 groups of \( \frac{3}{7} \)

Card for Question 2

\( \frac{4}{3} \times 10 \)

Card for Question 3

\( 3 \frac{3}{4} \div \frac{1}{2} \)
Michael ate $\frac{2}{3}$ of a candy bar each day for 4 days. How many candy bars did he eat in all?

Torie’s family had a pie for dessert last night. They didn’t like it so there was $\frac{3}{4}$ of the pie left. In the morning Torie ate $\frac{1}{4}$ of the left over pie. How much of the whole pie did she eat in the morning?

Joshkin earns $12.00 an hour mowing lawns. If he works $\frac{5}{6}$ of an hour how much money did he earn?

Lauren lives $\frac{1}{2}$ of a mile from school. She rode her bike $\frac{2}{3}$ of the way when the tire went flat. How much of a mile did Lauren ride her bike before the tire went flat?
A bird feeder holds about \(\frac{1}{4}\) cups of seed. The scoop you use to fill the bird feeder holds about \(\frac{3}{8}\) of a cup. How many scoops does it take to fill the feeder?

You have a piece of ribbon 4 yards long. How many pieces of ribbon \(\frac{4}{5}\) of a yard long can you cut from this ribbon? If the answer is not a whole number, what name can you give to the remainder?
Thank you for doing your very best on this assessment.

Your teacher will first show you 3 problems on the overhead. ESTIMATE the answer to each problem by writing the whole number you think the actual answer is closest to. Write the number in the appropriate box found below.

(1) 

(2) 

(3)
Now your teacher will show you two more problems. Estimate by placing an X on the number line to show about how big the actual answer is. For example, if you think the actual answer is between 0 and 5 put an x anywhere on the number line between 0 and 5.

(4)

(5)

Continue to work on the rest of the test. Show all your work so we know how you solved the problems. Thank you.
Circle the larger of each pair or both if they are equal. Explain how you solved each problem.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(6)</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{2}{3}$</td>
<td>(7)</td>
</tr>
<tr>
<td>(8)</td>
<td>$\frac{6}{14}$</td>
<td>$\frac{9}{16}$</td>
<td>(9)</td>
</tr>
<tr>
<td>(10)</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{6}{8}$</td>
<td>(11)</td>
</tr>
<tr>
<td>(12)</td>
<td>$\frac{4}{9}$</td>
<td>$\frac{4}{15}$</td>
<td>(13)</td>
</tr>
<tr>
<td>(14)</td>
<td>.35</td>
<td>.9</td>
<td>(15)</td>
</tr>
<tr>
<td>(16)</td>
<td>.044</td>
<td>.05</td>
<td>(17)</td>
</tr>
<tr>
<td>(18)</td>
<td>0.12</td>
<td>0.120</td>
<td>(19)</td>
</tr>
</tbody>
</table>
(20) Rewrite the fractions so they are from smallest to largest.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Put answer here:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{9}{10} )</td>
<td>smallest</td>
</tr>
<tr>
<td>( \frac{5}{9} )</td>
<td>largest</td>
</tr>
<tr>
<td>( \frac{1}{6} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{5}{12} )</td>
<td></td>
</tr>
</tbody>
</table>

(21) Rewrite the decimals so they are from smallest to largest.

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Put answer here:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.345</td>
<td>smallest</td>
</tr>
<tr>
<td>0.035</td>
<td>largest</td>
</tr>
<tr>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>0.357</td>
<td></td>
</tr>
<tr>
<td>0.038</td>
<td></td>
</tr>
</tbody>
</table>
For these last problems use your **estimation** skills. Don’t find the actual answer.

---

**Imagine the decimal .54 on a 10 x 10 grid. Take off 9-thousandths.**

(22) Is your answer greater than \( \frac{1}{2} \) or less than \( \frac{1}{2} \)?

(23) Explain your thinking

---

**Joshua ran 3.7 kilometers. Bryce ran 3.09 kilometers.**

(24) Who ran the furthest?

(25) Explain your thinking
Note to teacher: These are the first 5 test questions. Duplicate on an overhead transparency. Show one problem at a time for about 30 seconds.

(1) \[\frac{7}{8} + \frac{12}{13}\]

(2) \[\frac{3}{8} + \frac{5}{12}\]

(3) \[\frac{8}{9} - \frac{7}{8}\]

(4) \[20 \times \frac{2}{3}\]

(5) \[5 \div \frac{3}{4}\]
**Addition and Subtraction Problems**
Show how to do these problems on the Decimal +- board

<table>
<thead>
<tr>
<th>(26)</th>
<th>(0.27 + 0.4 = )</th>
<th>(28)</th>
<th>(0.6 - 0.05 = )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Grid" /></td>
<td><img src="image2.png" alt="Grid" /></td>
<td><img src="image3.png" alt="Grid" /></td>
<td><img src="image4.png" alt="Grid" /></td>
</tr>
<tr>
<td>(27) Final answer:</td>
<td>(29) Final answer:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(30) Chase’s plant was .25 units on Monday. It grew .3 units in two weeks. How tall was his plant at the end of the two weeks? Show how to solve this problem on the number line. Record the final answer in the box.

(31) Final answer is:

(32) \[0.83 + 0.9 =\]

(33) \[3.45 - 1.7 =\]
(34) Molly rode her bike $3\frac{1}{2}$ miles on Monday. On Wednesday, she rode $2\frac{1}{4}$ miles. On Saturday she rode $5\frac{1}{8}$ miles. How many miles did Molly ride in the 3 days. Show all your work below.

(35) Use the information in the table to answer this question: *How much taller is Michael than Paola?*. Show your work below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Height in Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michael</td>
<td>$67\frac{5}{6}$</td>
</tr>
<tr>
<td>Paola</td>
<td>$61\frac{1}{4}$</td>
</tr>
</tbody>
</table>
Addis reads $\frac{3}{4}$ of an hour each day. How many hours does she read in 5 days?

(36) Draw a picture that shows how to find the answer for this problem.

(37) Write a number sentence with the answer that shows how to solve this problem.
Laura notices that she has $\frac{2}{3}$ of a pan of birthday cake in her refrigerator. She eats $\frac{3}{4}$ of the leftover cake. How much of one pan of cake does she eat?

(38) Draw a picture that shows how to find the answer for this problem.

![Pan of cake](image)

(39) Write a number sentence with the answer that shows how to solve this problem.
Emily runs down Grand Avenue. Each block is \( \frac{1}{4} \) of a mile long. She runs \( \frac{2}{3} \) of a block before she gets tired and stops. How much of a mile did she run?

(40) Solve the problem using the number line below. Be sure to show all your work.

![Number line](image)

(41) Write a mathematical sentence with the answer that shows how to solve this problem.

Bryce makes $20 an hour selling ice cream at the Lake Nokomis concession stand during the summer. How much will Bryce make if he works \( \frac{4}{5} \) of an hour?

(42) Solve the problem using the number line below. Be sure to show all your work.

![Number line](image)

(43) Write a number sentence with the answer that shows how to solve this problem.
(44) A scoop holds $\frac{1}{5}$ kg of flour. How many scoops of flour are needed to fill a bag with 6 kg of flour? Draw a picture to solve this problem. Circle the final answer.

(45) You have 3 $\frac{1}{2}$ pounds of fish. Each serving is $\frac{3}{4}$ of a pound. How many full servings are possible? If there is an amount left over, what fraction of a serving is that?

Show how to solve the problem using the picture below. Circle your final answer.