1. You earned ____ points out of 10. Given the function \( f(x) = -2x^2 + 8x - 1 \):

   a. [8] Write the function in vertex form.

   Complete the square

   \[
   f(x) = -2x^2 + 8x - 1
   = -2(x^2 - 4x) - 1
   \]

   Complete square using \( \left( \frac{1}{2}(-4) \right)^2 = 4 \)

   \[
   = -2(x^2 - 4x + 4 - 4) - 1
   = -2(x^2 - 4x + 4) - 2(-4) - 1
   = -2(x - 2)^2 + 7
   \]

   Or, find the vertex and then put the coordinates into the vertex formula.

   \[
   x_{\text{vertex}} = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2
   \]

   \[
   y_{\text{vertex}} = f(x_{\text{vertex}}) = f(2) = -2(2^2) + 8(2) - 1 = 7
   \]

   Vertex formula is \( f(x) = a(x - h)^2 + k \)

   so we have \( a(x - 2)^2 + 7 \)

   In the original formula the coefficient of the \( x^2 \) term is \(-2\) so that is what \( a \) is:

   \[
   f(x) = -2(x - 2)^2 + 7
   \]

   Ans: \( f(x) = -2(x - 2)^2 + 7 \)

   b. [2] Find the equation of the axis of symmetry.  \( \text{Ans: Axis of symmetry: } x = 2 \)

   The axis of symmetry is the vertical line that passes through the vertex. The equation of any vertical line is \( x = k \), where \( k \) is a constant. In this case \( k \) is \( x_{\text{vertex}} \). The vertex is at \( (2, 7) \). So, the equation of the axis of symmetry is \( x = 2 \).

2. You earned ____ points out of 30. Given the function \( f(x) = \frac{x^3 - x^2 - 6x}{x^2 + 5x} \).

   a. [2] What is the domain?

   Write answer using interval notation.  \( \text{Ans: } (-\infty, -5) \cup (-5, 0) \cup (0, \infty) \)

   \[
   f(x) = \frac{x^3 - x^2 - 6x}{x^2 + 5x} = \frac{x(x^2 - x - 6)}{x(x + 5)} = \frac{x(x - 3)(x + 2)}{x(x + 5)}
   \]

   To avoid division by 0 we must exclude 0 and -5 from the domain.
b. [4] Does the graph contain any “holes”? If so, what are its coordinates?  

**Ans:** \((0, -\frac{6}{5})\)

Cancel the common factor of \(x\) to get 

\[ g(x) = \frac{(x - 3)(x + 2)}{(x + 5)} \]

So, there is a hole when \(x = 0\).

Now, plug in 0 to find the y-coordinate: 

\[ g(0) = \frac{(0 - 3)(0 + 2)}{(0 + 5)} = -\frac{6}{5} \]

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c. [2] What are the x-intercepts?  

**Ans:** \((3,0) \) and \((-2,0)\)

The x-intercepts are the values of \(x\) when \(y = 0\).

\[ g(x) = \frac{(x - 3)(x + 2)}{(x + 5)} \]

Cancel the common factor of \(x\) to get 

\[ 0 = \frac{(x - 3)(x + 2)}{(x + 5)} \]

\[ 0 = (x - 3)(x + 2) \]

\[ x = 3 \text{ or } x = -2 \]

d. [2] What are the vertical asymptotes?  

**Ans:** \(x = -5\)

The denominator of the reduced fraction has one zero at \(x = -5\). So, there is one vertical asymptote, \(x = -5\). Since the multiplicity is odd, the graph goes up on one side and down on the other side of the asymptote. Since we cancelled the common factor of \(x\) there is a hole at \(x = 0\) but no asymptote there.

e. [6] What is the horizontal or oblique asymptote?  

**Ans:** \(y = x - 6\)

If \(\text{deg num} < \text{deg denom}\), \(y = 0\) is a horizontal asymptote

If \(\text{deg num} = \text{deg denom}\), \(y = L\) is a horizontal asymptote

If \(\text{deg num} = \text{deg denom} + 1\), use long division to find oblique asymptote, \(y = mx + b\)

Since the degree of the numerator is exactly one more than the degree of the denominator there will be an oblique asymptote. We find it by doing the long division:

\[
\begin{array}{c|cccc}
 & x & -6 \\
\hline
x+5 & x^2 & -x & -6 \\
& x^2 & +5x & \\
\hline
& -6x & -6 & \\
& -(-6x & -30) & 24
\end{array}
\]

So, the equation of the oblique asymptote is \(y = x - 6\).
f. [6] Does the graph cross the horizontal or oblique asymptote? 
    If so, at what point? 
    Ans: Does not cross 
    To find the crossing point, find the value of x where the function and asymptote are equal.

    \[
    \begin{align*}
    \text{Function} &= \text{asymptote} \\
    g(x) &= y \\
    \frac{x^2 - x - 6}{x + 5} &= x - 6 \\
    x^2 - x - 6 &= (x + 5)(x - 6) \\
    x^2 - x - 6 &= x^2 - 30 \\
    -6 &= -30
    \end{align*}
    \]
    
    The equation has no solution so the graph does NOT cross the oblique asymptote.

   g. [10] Sketch the graph. Clearly show all intercepts and asymptotes.
3. You earned ____ points out of 10. Given the function \( f(x) = -2x(x + 3)^2(x - 5)^3 \):

a. [3] Find the x-intercepts. \( \text{Ans: } \) x-intercepts: \((0,0), (-3,0), (5,0)\)

The x-intercepts are the values of \( x \) when \( y = 0 \). They are the zeros of the polynomial. They are \( x = 0 \), \( x = -3 \), and \( x = 5 \).

b. [2] Find the y-intercept. \( \text{Ans: y-intercept: } (0,0) \)

The y-intercept is the value of \( y \) when \( x = 0 \).

\[
\begin{align*}
f(x) &= -2x(x + 3)^2(x - 5)^3 \\
f(0) &= -2(0)(0 + 3)^2(0 - 5)^3 \\
&= 0
\end{align*}
\]

c. [4] Determine the point(s) where the graph crosses the x-axis. \( \text{Ans: } \) crosses at \((0,0), (5,0)\)

The graph CROSSES the x-axis when the multiplicity of a zero is ODD. So, the graph crosses the x-axis when \( x = 0 \) and \( x = 5 \).

d. [4] Determine the behavior of the graph near where it touches the x-axis. That is, what function does the graph look like at that location? \( \text{Ans: } \) \( g(x) = -3072(x + 3)^2 \)

Near \((-3, 0)\), the function looks like

\[
\begin{align*}
f(x) &= -2x(x + 3)^2(x - 5)^3 \\
f(-3) &= -2(-3) \left( -3 + \frac{3}{3} \right)^2 (-3 - 5)^3 \\
g(x) &= -2(-3)(x + 3)^2(-3 - 5)^3 \\
&= -3072(x + 3)^2
\end{align*}
\]

e. [2] Determine the end behavior of the graph. That is, what power function does the graph look like for large \( |x| \)?

When \( x \) is very large numbers added to it or subtracted from it become negligible. Thus, we have the following:

\[
\begin{align*}
f(x) &= -2x(x + 3)^2(x - 5)^3 \\
\text{When } |x| \rightarrow \infty, \ f(x) \rightarrow -2x(x)^2(x^3) = -2x^6
\end{align*}
\]

This is a power function with an even exponent so it will look like a steep parabola. The negative in front means it is reflected about the x-axis. So, the end behavior will be down on both sides.
4. You earned ____ points out of 10.  

**Ans:** \((-\infty, -2] \cup (-1, 1]\)

Solve \(\frac{-2}{x+1} + x \leq 0\) Write your answer using *interval notation*.

Combine all the terms on one side into a single fraction and put 0 on the other side.

\[
\frac{-2}{x+1} + x \leq 0  \\
\frac{-2}{x+1} + \frac{x(x+1)}{x+1} \leq 0  \\
\frac{-2 + x^2 + x}{x+1} \leq 0  \\
\frac{x^2 + x - 2}{x+1} \leq 0  \\
\frac{(x-1)(x+2)}{x+1} \leq 0
\]

Now, break up the number line into intervals using the zeros of both the numerator and denominator as the breaking points (these are the places where the sign of \(f\) may change).

We have three zeros: \(x = -2\) and \(x = 1\) from the numerator and \(x = -1\) from the denominator.

**Interval:**  
\(-\infty \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad \infty\)

**Try \(x\):**  
\(-3\)  
\(-1.5\)  
\(-1\)  
\(0\)  
\(1\)  
\(2\)

**Get \(f\):**  
\((-3-1)(-3+2)\)  
\((-1.5-1)(-1.5+2)\)  
\((0-1)(0+2)\)  
\((2-1)(2+2)\)

**Simplify:**  
\(\frac{\text{neg}\cdot\text{neg}}{\text{neg}} = \text{neg}\)  
\(\frac{\text{neg}\cdot\text{pos}}{\text{neg}} = \text{pos}\)  
\(\frac{\text{neg}\cdot\text{pos}}{\text{pos}} = \text{neg}\)  
\(\frac{\text{pos}\cdot\text{pos}}{\text{pos}} = \text{pos}\)

**\(f(x) < 0\):**  
yes  
no  
no  
yes  
no

So, the solution is the interval \((-\infty, -2] \cup (-1, 1]\).  
Note that -1 is not in the solution because that would cause a division by 0.