Final Exam

Name: ............................................

I.D.: ............................................

Section: ........................................

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Good Luck!
READ AND FOLLOW THE FOLLOWING INSTRUCTIONS!

This booklet contains 12 pages, including the cover page and this instruction page. Check to see if any are missing. **FILL IN** the cover page, and sign your name. Print your name in the space provided on the top of each page in case the pages get separated. A simple ‘scientific’ calculator may be used. Do your work in the blank spaces or on the backs of the pages in this booklet. **SHOW ALL YOUR WORK**!

The examination consists of 10 multiple-choice questions (worth 3 points each), and 7 hand-graded problems (worth 10 points each).

**Instructions for the Multiple-Choice Part (Questions M1–M10):** You **MUST** use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and **DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET**. When you have decided on a correct answer for a question, **CIRCLE** that answer in this booklet and **BLACKEN COMPLETELY** the corresponding oval on the answer sheet. If you erase something, do so completely.

Each question has one correct answer. If you give two answers, that question will be marked wrong. There is no penalty for guessing. If you skip a question, skip the corresponding line on the answer sheet. Go on to the next question.

**Instructions for the Hand-Graded Part (Questions 1–7):** **SHOW ALL WORK** in the hand-graded part! Unsupported results will receive little credit. Also show all your work on the multiple-choice questions. Such work will be used to make judgments if we decide to regrade an exam paper.

After you have finished both parts of the examination place the answer sheet between two pages of this booklet (make a sandwich) with the side marked ‘General Purpose Answer Sheet’ facing **down**. Have your ID card in your hand when you hand in your paper.
M1. Which of the following lines is perpendicular to the line $y = -3x + 2$ and passes through the point $(3, 2)$?
   
   a) $y = -3x - \frac{1}{2}$.
   
   b) $y = 3x + 2$.
   
   c) $y = -\frac{1}{3}x + \frac{1}{2}$.
   
   d) $y = \frac{1}{3}x + 1$.
   
   e) $y = x - 1$.

M2. Consider the circle given by the equation $x^2 + y^2 + y = 0$. The radius of this circle is

   a) 4.
   
   b) 2.
   
   c) 1.
   
   d) $\frac{1}{2}$.
   
   e) $\frac{1}{4}$.

M3. Consider the function $f(x) = 3x^4 + 2x^2 - 1$. The graph of $f(x)$

   a) is symmetric with respect to the $x$-axis.
   
   b) is symmetric with respect to the $y$-axis.
   
   c) is symmetric with respect to the origin.
   
   d) has no $y$-intercept.
   
   e) has an $x$-intercept of $\sqrt{\frac{1}{2}}$.

M4. Which of the following polynomials has a degree of 16 and touches the $x$-axis exactly 3 times?

   a) $f(x) = -3x^2(x - 3)^2(x - 1)^3(x + 1)^5(x + 4)^4$.
   
   b) $f(x) = -3(x - 3)(x - 1)^6(x + 1)^5(x + 4)^4$.
   
   c) $f(x) = 3x^4(x - 3)^3(x - 1)^3(x + 1)^5(x + 4)^2$.
   
   d) $f(x) = 3(x - 3)^4(x - 1)^2(x + 1)^5(x + 4)^4$.
   
   e) None of the above.
M5. Which of the following rational functions has the oblique asymptote \( y = 3x + 1 \) and a \( y \)-intercept of 2?

a) \( f(x) = \frac{3x}{x^2 - 3} \).
b) \( f(x) = \frac{6x+2}{2x+1} \).
c) \( f(x) = \frac{3x^2+4x+2}{x+1} \).
d) \( f(x) = \frac{3x^2+x}{x} \).
e) \( f(x) = \frac{3x^3+1}{x} \).

M6. The following is a table of values of two one-to-one functions \( f(x) \) and \( g(x) \):

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<tr>
<th>( x )</th>
<th>( f(x) )</th>
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<td>-2</td>
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Which of the following statements is true?

a) \( f \circ g(-2) = -1 \).
b) \( g \circ f(-2) = -1 \).
c) \( f^{-1}(-2) = -2 \).
d) \( g^{-1}(-2) = -2 \).
e) None of the above statements is true.

M7. The domain of the function \( g(x) = \sqrt{\frac{x}{1-x}} \) is given by

a) \([0, \infty)\).
b) \((0, 1)\).
c) \((0, 1)\).
d) \([0, 1)\).
e) all real numbers.

M8. The inverse function of \( f(x) = x + 2 \) is given by

a) \( f^{-1}(x) = \frac{1}{x} + 2 \).
b) \( f^{-1}(x) = x + \frac{1}{2} \).
c) \( f^{-1}(x) = \frac{1}{x+2} \).
d) \( f^{-1}(x) = x - 2 \).
e) \( f^{-1}(x) = -x - 2 \).
M9. The algebraic expression $\log_2(x) + \log_4(x)$ equals

a) $\log_6(x)$.
b) $\log_8(x)$.
c) $3 \log_2(x)$.
d) $\frac{3}{2} \log_2(x)$.
e) $\frac{1}{2} \log_4(x)$.

M10. The value of $e^{-2\ln(w)}$ is

a) $w$.
b) $\sqrt{w}$.
c) $w^2$.
d) $\frac{1}{w^2}$.
e) $\sqrt{\frac{1}{w}}$. 
1. A circle $C$ is centered at $(4, 5)$ and touches the $x$-axis.

   a) Find the equation of the circle $C$.
   b) Find the $y$-intercept(s) of the circle $C$.
   c) What is the radius of a circle whose area is four times the area of the circle $C$?
2. A line $L$ passes through the points $(0, 4)$ and $(8, 0)$.

![Graph of line L passing through points](image)

a) Find the equation of the line $L$.

b) A rectangle is bounded by the $x$- and $y$-axes and by the graph of the line $L$. Find the area $A$ of the rectangle as a function of $x$.

c) What is the domain of the function $A(x)$ found in b)?

d) For which value of $x$ is the area $A(x)$ from b) a maximum? Find the value of this maximum.
3. Consider the function \( f(x) = -|x - 2| + 1 \).

a) Graph the function \( f(x) \) using transformations, starting with the graph of \( y = |x| \). Clearly list all the transformations that you use!

b) Using the graph of \( f(x) \) obtained in a), answer the following questions:
   
   i) For which value(s) of \( x \) does the graph of \( f(x) \) have a local maximum/minimum?
   
   ii) List the interval(s) on which the graph of \( f(x) \) is decreasing, if any.
   
   iii) Find all intercepts of \( f(x) \), if any.
4. Solve the following equations:
   
a) \( e^{2x+3} = \frac{1}{e^{2x+4}} \).
   
b) \( \log_2 \left( \log_2(x + 3) \right) = 1 \).
5. Graph the rational function

\[ R(x) = \frac{3(x + 1)(x - 2)}{(x + 2)(x - 3)}. \]

To do so, first analyze the function \( R(x) \); in particular, find the domain of \( R(x) \), locate any \( x \)- and \( y \)-intercepts, find all vertical and horizontal or oblique asymptotes and construct a table (intervals, location of the graph with respect to the \( x \)-axis, etc.) as presented in class.
6. Consider the function 

\[ f(x) = \frac{2x - 1}{x + 3}. \]

Answer the following questions about this function:

a) Find the y-intercept of the graph of \( f(x) \), if any.
b) Find the x-intercept(s) of the graph of \( f(x) \), if any.
c) Find \( f^{-1}(x) \) or explain why the function does not have an inverse.
d) Find the domain of \( f(x) \).
e) Find the range of \( f(x) \).
7. The temperature $F$, in degrees Fahrenheit, of a dessert placed in a freezer for $t$ hours is given by the rational function

$$F(t) = \frac{60}{t^2 + 2t + 1}, \quad t \geq 0.$$

a) Find the temperature of the dessert after it has been in the freezer for 4 hours.

b) After how many hours in the freezer does the dessert have a temperature of $15^\circ F$?

c) What temperature will the dessert approach as $t \to \infty$?

d) What is the average rate of change of the temperature of the dessert during the first 4 hours in the freezer?