

# Rational Number Project

<b>Fraction Operations and Initial Decimal Ideas Lesson 2: Overview</b>	<b>Materials</b> <ul style="list-style-type: none"><li>• Paper folding strips for students and teacher (8.5" x 1")</li><li>• Student Pages A, B, C</li><li>• Transparencies A-D</li></ul>
Students review equivalence ideas with paper folding. Students develop a symbolic rule for finding equivalent fractions.	

## Teaching Actions

### Warm Up

Order the fractions from smallest to largest. Be ready to explain your reasoning.

$$\frac{3}{4} \quad \frac{1}{20} \quad \frac{4}{10} \quad \frac{6}{12}$$

### Large Group Introduction

1. Start this lesson by asking students to discuss this question in their work groups: What does it mean for two fractions to be equivalent?
2. After providing students time to discuss this question, ask students to share their definitions. Record their ideas on the board.
3. If no students use a model to explain equivalence, ask them to use fraction circles to help explain the idea of equivalence. [example: Show 2 fractions equal to  $\frac{2}{3}$  using fraction circles].
4. If no students use context to explain equivalence, ask them to create a story problem that uses the idea of fraction equivalence. (Ex: one person eats  $\frac{1}{4}$  of a small pizza while another ate  $\frac{2}{8}$  of a small pizza. They both ate the same amount, but the pieces were cut up differently).

## Comments

Many lessons will have a warm-up question to use at the beginning of class. The question will reinforce an important idea from previous lessons. Plan on taking 5-7 minutes for the warm up to ensure you have enough class time for the major part of the lesson.

You want students to think about equivalence in more than one way. Some students may already have a rule to show that two fractions are equivalent but you also want students to be able to explain equivalence with concrete materials and meaningful contexts. If students do not use a rule, that is fine as it is developed using paper folding in this lesson.

## Teaching Actions

5. Summarize by stating that two fractions are equivalent if they represent the same amount even if the partitions differ.

6. Use the Fraction Equivalence transparencies A, B and C at this point to lead a discussion to help students see the connection between folding paper and finding equivalent fractions. The three examples on the transparency are:

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

7. For each example, record students' explanations with symbols as follows:

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

**correct notation**

This recording shows that if number of total parts doubled after folding, then the total number of parts shaded also doubles.  $\frac{6}{8}$  is the fraction amount represented by the paper folding. As the paper folding did not change the size of the unit or the amount originally shaded,  $\frac{3}{4} = \frac{6}{8}$

[Note: While the student is multiplying the numerator and denominator by 2, they are not thinking of it as multiplying the fraction by a form of 1].

## Comments

Students have used paper folding to model fractions and find equivalent fractions in the RNP Level 1 module. This lesson is a review and a step towards a symbolic rule for finding equivalent fractions.

You are looking for multiplicative language to describe how the numerator and denominator change after folding;

- If I double the total parts then the shaded parts double.
- If the total number of parts is multiplied by 3, then the shaded parts are three times as many too.
- If the number of shaded parts is multiplied by 4 then total number of parts is multiplied by 4.

At this point, we are not interpreting equivalence as multiplying the fraction by one (such as  $\frac{2}{2}$  or  $\frac{3}{3}$ , etc.).

We do this in the homework after fraction multiplication is introduced.

It may not be obvious for many students how to go from thirds to ninths. They need to fold the paper into thirds again. Students have a natural inclination to keep doubling.

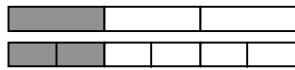
Look for this doubling error when completing the second example; ask students to explain how to go from thirds into ninths using paper folding. [Student explanation: Go back to the 3<sup>rd</sup> fold. Since  $3 \times 3 = 9$ , you fold into 3<sup>rd</sup>s].

Some students may record this idea as shown below. Correct notation encourages students to communicate they are acting on both the

## Teaching Actions

### Small Group/Partner Work

8. Students complete Student Pages A, B and C. On pages A and B students draw in the folding lines to show fraction equivalence. On page C students rely on their symbolic rule.
9. Do this problem together first.  $\frac{1}{3} = \frac{2}{6}$



Students need to partition each third into two equal parts to show 6ths.

### Wrap Up

10. Show transparency D.  
Kia said she could show  $\frac{1}{2} = \frac{1}{3}$  using these pictures.



Ask: Does this make sense?

## Comments

numerator and denominator.

$$\frac{3}{4} \times 3 = \frac{9}{12}$$

**incorrect notation**

Solving problems using pictures of paper folding is a prelude to showing fractions on the number line.

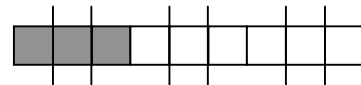
Students will likely approach the problems on Student Pages A and B from two perspectives.

Students who have not internalized the symbolic rule will partition first to find the missing value. Students who have internalized the symbolic rule will find the missing value first and then determine how to partition the picture.

This second strategy is an example of a *symbolic to picture translation*. The process of partitioning after finding the answer will reinforce for these students what this abstract rule means.

Example of student thinking:  $\frac{1}{3} = \frac{1}{9}$

Student said: that's 3 ninths so I needed 3 parts shaded. He partitions  $\frac{1}{3}$  into 3 equal parts and partitions the other un-shaded thirds into 3 equal parts each.



In this wrap up example you are making it explicit that when you compare fractions you assume that the unit is the same for both fractions.

You might ask this question too:

Rachel and William both spent  $\frac{1}{4}$  of their allowances. Is it possible that Rachel spent more than William?

## Teaching Actions

11. Ask students how they can undo their multiplication rule for equivalent fractions to find these equivalences:

$$\frac{4}{5} = \frac{12}{15} \quad \frac{3}{4} = \frac{6}{8} \quad \frac{3}{9} = \frac{1}{3} \quad \frac{8}{12} = \frac{2}{3} = \frac{4}{6}$$

## Comments

Reducing fractions is reinforced in extended practice and warm ups through out the lessons. It is important that students know common equivalences just as they know their basic whole number facts. The following equivalences are important:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

$$\frac{1}{6} = \frac{2}{12}$$

### Translations:

- Symbols to concrete to verbal to symbols
- Symbols to pictures to symbols

- Fold a paper strip into fourths and shade 3-fourths.
- Fold the paper strip to find how many 8ths  $\frac{3}{4}$  equals.

$$\frac{3}{4} = \frac{\quad}{8}$$

Questions:

1. How did you fold the paper strips to make 8ths?
2. The number of total parts increased from 4 to \_\_\_\_\_. Describe how the total parts increased.
3. The number of total parts shaded increased from 3 to \_\_\_\_\_. Describe how the shaded parts increased.

- Fold a paper strip into thirds and shade 2-thirds.
- Fold the paper strip to find how many 9ths  $\frac{2}{3}$  equals.

$$\frac{2}{3} = \frac{\quad}{9}$$

Questions:

1. How did you fold the paper strips to make 9ths?
2. The number of total parts increased from 3 to \_\_\_\_\_. Describe how the total parts increased.
3. The number of shaded parts increased from 2 to \_\_\_\_\_. Describe how the shaded parts increased.

- Fold a paper strip into thirds and shade 1-third.
- Fold the paper strip to find the missing denominator.

$$\frac{1}{3} = \frac{4}{\quad}$$

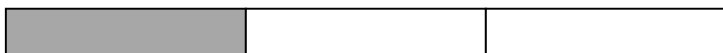
Questions:

1. How did you fold the paper strips to make 4 parts shaded?
2. The number of shaded parts increased from 1 to \_\_\_\_\_. Describe how the shaded parts increased.
3. The number of total parts increased from 3 to \_\_\_\_\_. Describe how the total parts increased.

## Final Problem

Kia said she could show that  $\frac{1}{2} = \frac{1}{3}$

Here is her picture:



What do you think?

Order the fractions from smallest to largest. Be ready to explain your reasoning.

$$\frac{3}{4} \quad \frac{1}{20} \quad \frac{4}{10} \quad \frac{6}{12}$$

**Looking for Equivalences**

Draw the needed lines on each picture of a paper-folding strip to find the equivalent fraction. The lines should show the folds if you used a paper-folding strip to model each example.



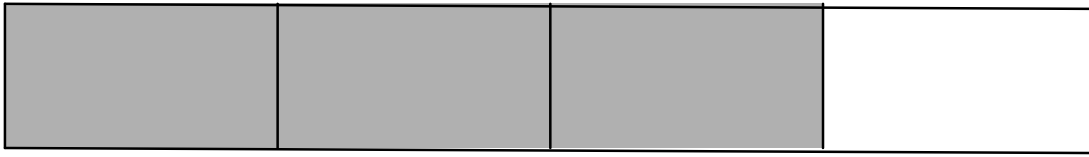
$$\frac{1}{3} = \frac{\quad}{9}$$



$$\frac{4}{6} = \frac{\quad}{12}$$



$$\frac{3}{5} = \frac{\quad}{15}$$



$$\frac{3}{4} = \frac{\quad}{8}$$



$$\frac{1}{2} = \frac{\quad}{6}$$



$$\frac{1}{2} = \frac{\quad}{8}$$

Solve using your multiplication rule:

$$\frac{1}{3} = \frac{\quad}{9}$$

$$\frac{4}{6} = \frac{\quad}{12}$$

$$\frac{1}{2} = \frac{\quad}{8}$$

**Looking for Equivalences**

Working in your groups, solve each problem. Be ready to explain your thinking when the teacher stops by your group.

$\frac{2}{\square} = \frac{1}{12}$	$\frac{4}{\square} = \frac{12}{45}$
$\frac{6}{\square} = \frac{\square}{\square}$	$\frac{3}{\square} = \frac{\square}{\square}$
$\frac{1}{\square} = \frac{\square}{8}$	$\frac{10}{\square} = \frac{\square}{4}$
Record 3 fractions equal to $\frac{2}{7}$	Record three fractions equal to $\frac{4}{9}$
Record 6 fractions equal to $\frac{1}{2}$	Record 4 fractions equal to $\frac{6}{8}$

## Post Lesson Reflection

Lesson \_\_\_\_\_

1) Number of class periods allocated to this lesson: \_\_\_\_\_

2) Student Pages used: \_\_\_\_\_

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: