Rational Number Project

Fraction Operations and Initial Decimal Ideas Lesson 3: Overview	Materials
Students use their order and equivalence skills to estimate fraction sums and differences.	• Student Pages A - D

Teaching Actions

Warm Up

Estimate the sum: $\frac{12}{13} + \frac{7}{8}$. The choices are: 1, 2, 19, or 21.

Large Group Introduction

1. Explain to the students that in this lesson they will use their understanding of order and equivalence to make sense of fraction addition.

2. Ask students what order ideas they used to do the warm up problem (both fractions are close to 1.)

3. Present this task and ask students to consider if this solution makes sense.

   $\frac{1}{3} + \frac{8}{9} = \frac{9}{12}$ Do you agree?

Comments

Explain that only 24% of eighth graders on a national test could do this correctly. Ask: What is a reasonable estimate? Why did so many students choose 19 or 21? Why is addition and subtraction of fractions so hard for so many students?

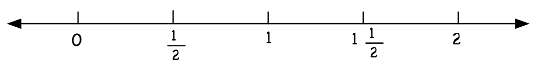
When students estimate fraction sums and differences they will rely on their mental images for fractions, their order and equivalence ideas, and their understanding of what addition and subtraction means. The order strategies used most often are: same numerator, residual, and using ½ as a benchmark.

After students share their ideas guide the discussion with these questions:

- Let's use our fraction order ideas to judge the reasonableness of this answer: Is $\frac{8}{9}$ > or < $\frac{1}{2}$? *[Using ½ as a benchmark]*
- What would you add to $\frac{8}{9}$ to equal one whole? *[Residual idea]*
- Is $\frac{1}{3}$ > or < $\frac{1}{9}$?

Teaching Actions

4. If you placed the exact answer on this number line would it be between 0 and $\frac{1}{2}$; $\frac{1}{2}$ and 1; 1 and $1\frac{1}{2}$; or $1\frac{1}{2}$ and 2?



5. Explain: We all agree that the answer will be greater than one but less than $1\frac{1}{2}$. We will find the exact answer at another time.

6. Repeat for this subtraction problem.

   Ramla had $1\frac{3}{4}$ pounds of candy hearts. She gave her sister about $1\frac{1}{6}$ of a pound. About how many pounds did she have left? Do not find the exact answer, just make a reasonable estimate and explain your thinking.

Small Group/Partner Work

7. Ask students to work with a partner to complete problems on Student Pages A and B. Explain that they should prepare to explain their answers when called on in the large group discussion.

8. Student pages C and D are extensions to be used at the teacher's discretion.

Wrap Up

9. Share thinking strategies for each problem. Each time a student uses an ordering idea, record that

Comments

Is the sum > or < 1? *[Comparing fractions with same numerator]*

- Is $\frac{9}{12} > or < 1$?

Explain to students you are interested in only the range, not the exact spot on the number line.

- Possible explanation: $\frac{1}{6}$ is less than $\frac{1}{4}$; $\frac{2}{4}$ equals $\frac{1}{2}$; take away less than $\frac{1}{4}$ from $\frac{3}{4}$ you are left with more than $\frac{1}{2}$.
- Another possible response: $\frac{3}{4}$ is 3 blues. A pink is smaller than 2 blues. If you cover 3 blues with 1 pink you will have more than 2 blues uncovered. Answer is $> \frac{1}{2}$.

When calling on students to share their thinking make sure you call on students who struggle in math. Use

Teaching Actions	Comments
example on the board. Examine the list at the end of the lesson to show the different ways students can think about fractions.	this opportunity to do an informal assessment of the strength of students' mental images for fractions. Students with strong mental images for fractions are successful estimators. But low achieving students may need help using these mental images to consider what happens when you operate on fractions. Don't be surprised if some students can use their informal ordering strategies to order fractions but do not realize that these strategies can help them estimate. Some students will still resort to whole number thinking when estimating.

Translations:

- Symbolic to verbal to symbolic
- Real life to verbal to symbolic

Additional Teacher Notes
Lesson 3

Below find examples of students' written explanations for estimating addition and subtraction problems for the homework labeled Lesson 3. Notice how students use the strategies, same numerator, residual, and ½ as a benchmark to compare fractions. These are the type of explanations you want to encourage.

- $\frac{11}{12} - \frac{1}{3} = \frac{10}{12}$ This doesn't make sense. $\frac{11}{12}$ is close to 1; so $1 - \frac{1}{2} = \frac{1}{2}$. $\frac{10}{12}$ is closer to 1 than $\frac{1}{2}$.

- $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$. Yes, I do think this is reasonable. $\frac{1}{3}$ is close to 1 and $\frac{1}{4}$ is close to $\frac{1}{3}$. If you add them you are close to 1. Also $\frac{11}{12}$ is close to 1.

- $\frac{1}{5} + \frac{2}{3} = \frac{3}{5}$. Doesn't make sense. $\frac{2}{3}$ is bigger than $\frac{2}{5}$. It would be closer to $\frac{4}{5}$.

- $\frac{2}{3} - \frac{1}{4} = \frac{1}{12}$. Doesn't make sense. $\frac{1}{4}$ is smaller than $\frac{1}{3}$. $\frac{1}{12}$ is smaller than $\frac{1}{4}$ More than $\frac{1}{3}$ is left.

- $\frac{8}{15} - \frac{1}{3} = \frac{7}{12}$. $\frac{8}{15}$ is practically the same thing as $\frac{7}{12}$; both a little larger than $\frac{1}{2}$. And you're taking away a little bit less than $\frac{1}{2}$. So, no, it doesn't make sense.

Below are examples of students' estimation errors and examples where students' explanations could be more precise. Teachers can build on these types of responses and with appropriate questions help students become better estimators.

Are the students over using the "close to 1 and 0" idea and not using other order strategies? Is this the reason for their incorrect estimates?

- $\frac{9}{10} - \frac{2}{100} = \frac{7}{10}$. This makes sense. $\frac{9}{10}$ is almost 1. $\frac{2}{100}$ is almost 0. $\frac{7}{10}$ is almost 1. 1-0=1 *[Student could be more precise. 2/100 < 1/10 so the answer must be greater than 8/10; 7/10 is really too small.]*

- $\frac{9}{10} - \frac{2}{100} = \frac{7}{10}$. This looks right because $\frac{9}{10}$ is close to one and $\frac{2}{100}$ is very close to 0. So the answer would be between 1/2 and one.

In the examples below students are considering the size of the fractions operated on but not the size of the given answer. In what ways could students be more thoughtful in their estimation?

- $\frac{1}{4} - \frac{2}{100} = \frac{1}{3}$. Doesn't make sense because $\frac{1}{4}$ is less than $\frac{1}{2}$ and $\frac{2}{100}$ is almost 0. *(Student could have commented that* $\frac{1}{3} > \frac{1}{4}$*.)*

- $\frac{1}{4} - \frac{2}{100} = \frac{1}{3}$. Wrong! $\frac{2}{100}$ is nothing. $\frac{1}{4}$ - nothing does not equal $\frac{1}{3}$. *(Student could have noted that* $\frac{1}{3} > \frac{1}{4}$*).*

- $\frac{8}{15} - \frac{1}{3} = \frac{7}{12}$. This does not make sense. $\frac{8}{15}$ is a little bit more than $\frac{1}{2}$. And $\frac{1}{3}$ is a little less than $\frac{1}{2}$. *(Student could have commented that* $\frac{7}{12} > \frac{1}{2}$*)*

Estimate the sum: $\frac{12}{13} + \frac{7}{8}$.

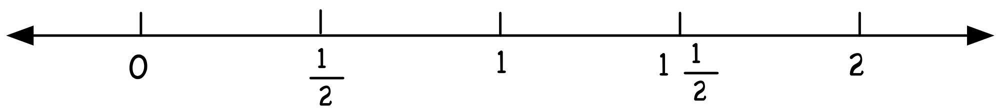
The choices are: 1, 2, 19, or 21.

Name______________________________

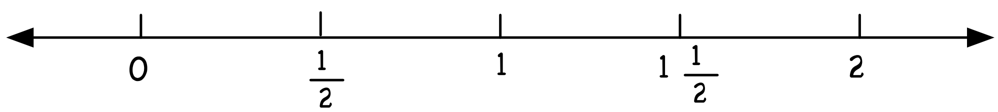
Fraction Estimation

For each problem, imagine the fractions using fraction circles. Estimate the value of each sum or difference. Put an X in the interval where you think the actual answer will be.

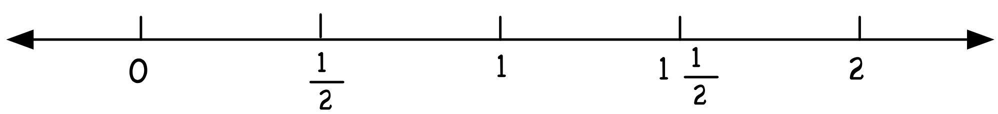
$\frac{5}{6} + \frac{11}{12}$



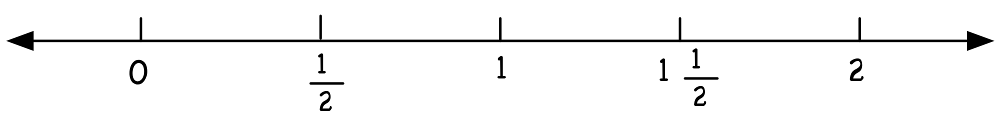
$\frac{4}{7} + \frac{6}{11}$



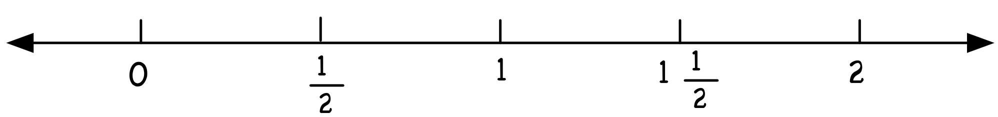
$\frac{7}{16} + \frac{5}{12}$



$\frac{1}{15} + \frac{1}{30}$

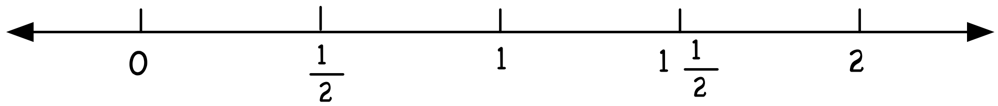


$\frac{1}{4} + \frac{1}{3}$

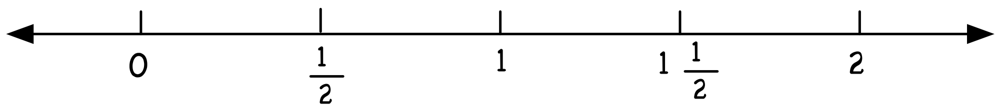


Name________________________________ Lesson 3 / Student Page B

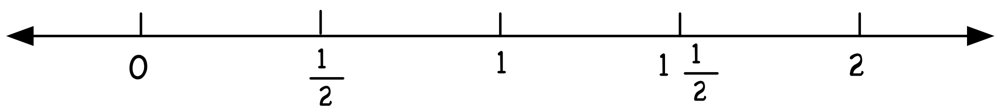
$1\frac{9}{10} - \frac{3}{4}$



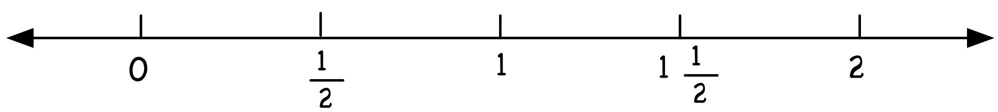
$\frac{8}{10} - \frac{1}{100}$



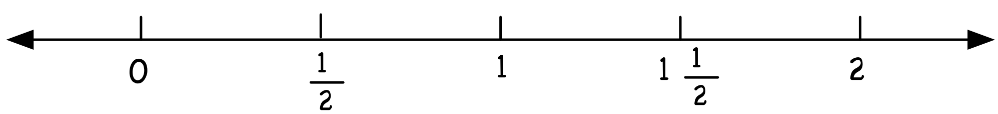
$\frac{8}{9} - \frac{7}{8}$



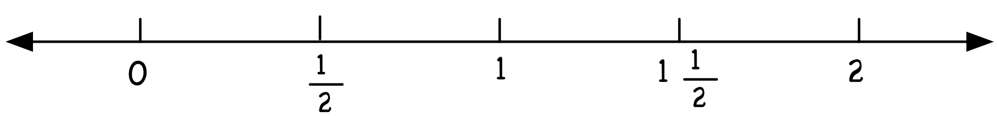
$1\frac{1}{4} - \frac{1}{10}$



$1\frac{7}{14} - \frac{9}{10}$



$\frac{10}{20} - \frac{3}{10}$



Estimation and Story Problems

1. After the party, there was $1\frac{8}{9}$ of a pizza left. Then Brenna ate an amount equal to $\frac{7}{8}$ of a whole pizza. About how much of one pizza was left?

Provide a reasonable estimate with a clear explanation of your thinking. Exact answer is not needed!

2. Joshkin ran $15\frac{3}{4}$ laps around the track. Caylee ran $14\frac{1}{5}$ laps. Approximately how many more laps did Joshkin run than Caylee?

Provide a reasonable estimate with a clear explanation of your thinking. Exact answer is not needed!

Extensions

1. Pirate Jack buried $\frac{1}{2}$ of his treasure. He gave $\frac{1}{3}$ of the remaining treasure to his trusty mate Pirate Joe. Pirate Joe received $3000 in gold. Exactly how much gold was in Pirate Jack's whole treasure? <u>Draw a picture</u> to show the solution.

2. Joshkin built a tower using blocks that linked together. I noticed that he had 27 blocks in $\frac{3}{7}$ of his tower. Exactly how many blocks were in this entire tower?

   Provide a clear description of your solution strategy.

3. The line below is $\frac{3}{4}$ as long as a ribbon I have. Draw a line the same length as my ribbon and another line that is $1\frac{1}{6}$ as long as my ribbon. Label the lines.

Post Lesson Reflection

Lesson_________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: __________________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: