

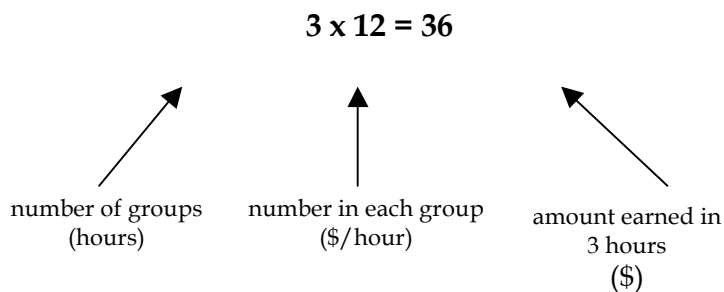
Rational Number Project

Fraction Operations and Initial Decimal Ideas Lesson 19: Overview	Materials <ul style="list-style-type: none">• Student Pages A, B, and C• Transparencies 1 and 2
Students use number lines to multiply a whole number by a fraction.	

Teaching Actions

Warm Up

Faisal makes \$12 an hour. How much does he make in 3 hours of work? Write a multiplication sentence that describes how to find the answer.



Ask the students the following questions:

- **What does the 3 represent in the multiplication sentence?** The 3 represents the number of hours Faisal worked.
- **What does the 12 represent in the multiplication sentence?** The dollars earned per hour.
- **What does the 36 represent in the multiplication sentence?** The amount of money earned in 3 hours.
- **Does the multiplication sentence represent 3 groups of 12 or 12 groups of 3? Why?**

Comments

In the previous two lessons we tried to develop meaning for the factors in a multiplication sentence. The first factor is typically the number of groups. The second factor is the number of objects in each group. Many real world situations make it difficult to have a fractional number of groups.

When we work with rate multiplication situations as shown in the first problem, the first factor is the amount of time (# of groups), the second factor is the rate (amount in each groups), and the result represents the amount of money earned. It is easier to make either factor a fraction in a rate situation.

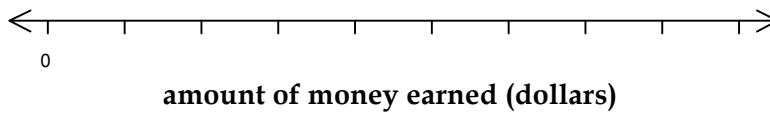
Teaching Actions

Comments

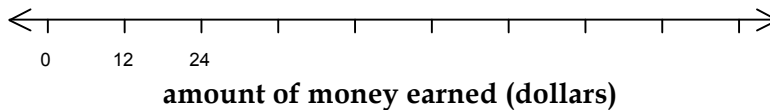
Large Group Introduction

1. Show the students how they can solve the word problem from the warm-up using a number line.

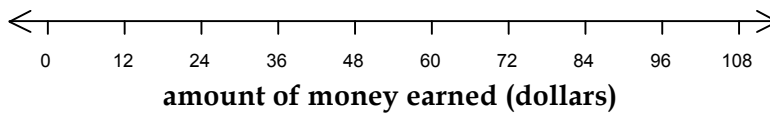
Explain that a number line can be used to show the amount of money that Faisel earns when he works a certain number of hours. Take out transparency 1 and give students Student Page A.



Fill in the amounts for the first two hours with the students. Explain that a jump from 0 to the first tick mark represents one hour of work and the \$12 represents the amount Faisel will earn in one hour. The \$24 represents how much Faisel will earn in two hours of work.



Ask the students to fill in the numbers below the tick marks of the number line on their paper.

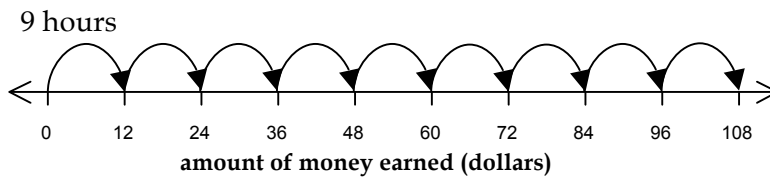
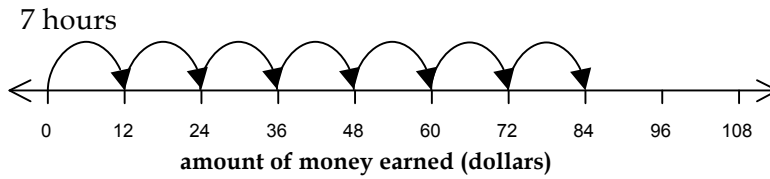
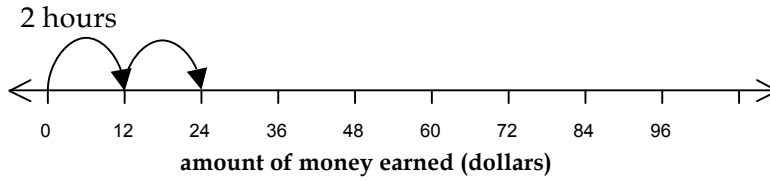


Ask the students to explain how the number line can be used to find the amount of money Faisel will earn if he works 3 hours.

2. Ask individual students how the number line can be used to show how much Faisel will earn if he works 2 hours. 7 hours. 9 hours. Have the students use their pencils or fingers to trace along the same number line they have been using. Have them start tracing at 0 and make jumps for each hour worked.

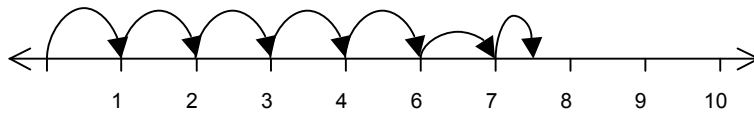
Students typically have a hard time using a number line when representing numbers. Emphasize the length characteristics of the number line by starting at 0 and showing the jumps of 12.

Teaching Actions



3. Ask the students to use number line I on Student Page A to find the amount of money Faisel will earn if he works 5 hours. Encourage them to draw in the jumps on the number line and then write a multiplication sentence that solves the problem. The multiplication sentence should be $5 \times 12 = 60$.

4. Ask the students to use the number line II to find the exact amount of money that Faisel will earn if he works $6\frac{1}{2}$ hours.



Have a student or two come to the front of the classroom and explain how they solved the problem using the number line.

Ask them to write a multiplication sentence for this problem and explain the meaning of each factor:

$$6\frac{1}{2} \times 12 = 78$$

Comments

The number line also demonstrates how multiplication can be demonstrated by repeated addition:

7 jumps of 12 is the same as 7×12 or $12+12+12+12+12+12+12$.

Ask a student to explain the meaning of each factor in the multiplication sentence.

The purpose of this question is to emphasize that this is a multiplicative situation.

Students should notice that each jump of \$12 on the number line is one hour of work. They should reason that Faisel earns \$72 for 6 hours of work and an additional \$6 for the extra half-hour of work for a total of \$78. One goal of this lesson is to help students see how to partition the number line.

Teaching Actions**Comments**

- the number line? (1 hour of work)
- How do you jump three-fourths of an hour? (Partition the number line into fifths and jump 3 of these)
 - How could you calculate $\frac{3}{5} \times 20$ without using the number line? (Divide 20 by 5 and multiply by 3)

Translations:

- Real world to verbal
- Real world to picture to verbal
- Symbol to picture

Additional Notes to the Teacher

Lesson 19

Partitioning

The main goal of this lesson is for students to use partitioning to find a fractional part of a whole number. For example, when looking at $\frac{5}{6} \times 18$ the students will learn to find the product by dividing 18 into 6^{ths} to get 3 and multiplying this result by 5 to get 15. This partitioning idea is an essential part of what it means to multiply fractions and will be emphasized throughout the next several lessons.

Number Line for Multiplication

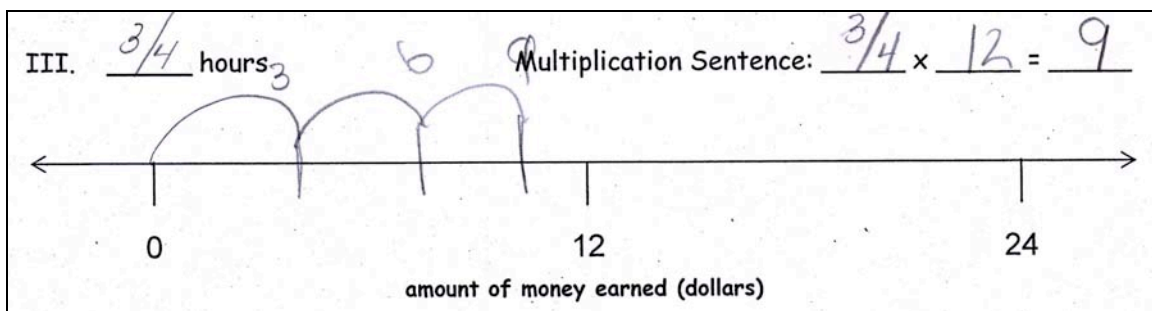
Another goal of the lesson is for the students to learn how the number line can be used to do multiplication of whole numbers. This is emphasized at the beginning of the lesson. The number line is a representation that makes sense to partition. Once students understand how multiplication can be modeled using the number line with whole numbers it makes it easier for them to make sense of expressions like $\frac{3}{4} \times 20$. As students begin to understand that 3×20 can be modeled on the number line as 3 jumps of 20 they will have a mental image of multiplication that allows them to think about $\frac{3}{4} \times 20$ in a meaningful way. Some students may struggle with relating the symbolic representation with the number line once fractions are introduced.

Labeling the Number Line

We found that students labeled the number lines in two different ways when multiplying a fraction and a whole number. Although the differences between the methods are subtle, the two interpretations of the number line are important to highlight. Both methods involve making tick marks on the number line but the students labeled the amounts in two different ways. Some students labeled the tick marks (Method I) with the amounts while others labeled the intervals (Method II) with the amounts earned.

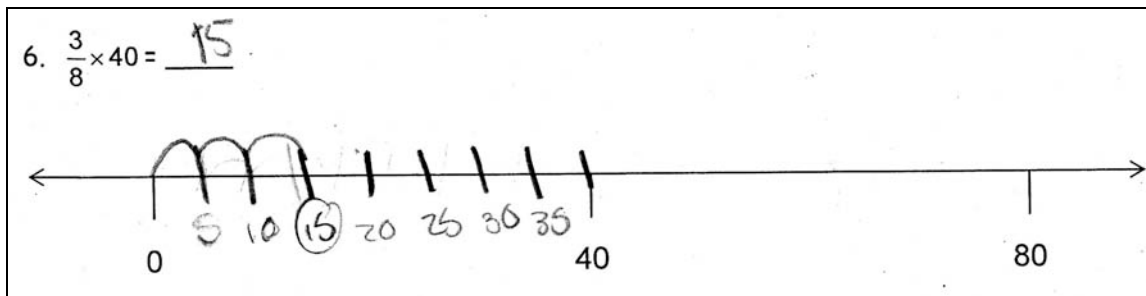
Method I: Points on a Number Line

The tick marks on the number line represent the amounts of money earned at specific times. The student's work shown below shows this approach.



Student work using Method I (with context)

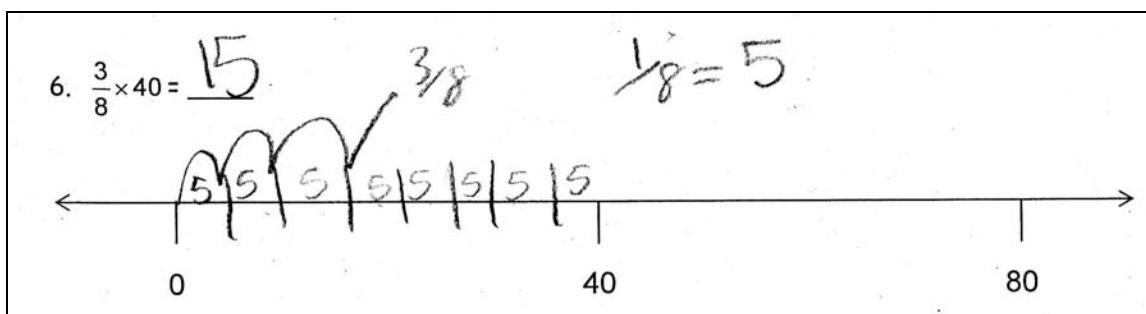
The tick mark labeled 3 represents a passage of time of $\frac{1}{4}$ of an hour. This tick mark is a specific point on the number line and has meaning when referenced to time 0.



Student work using Method I (no context)

Method II: Value of Interval

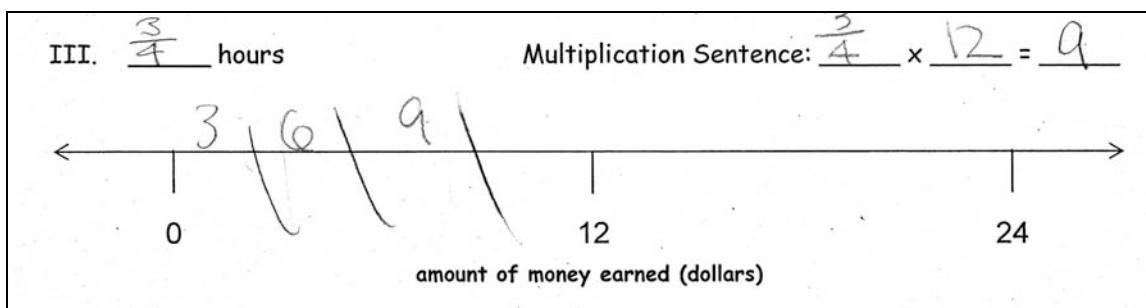
The student whose work is shown below labeled the intervals with the values.



Student work using Method II (no context)

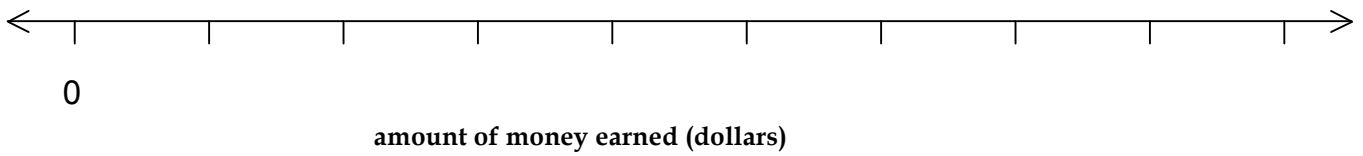
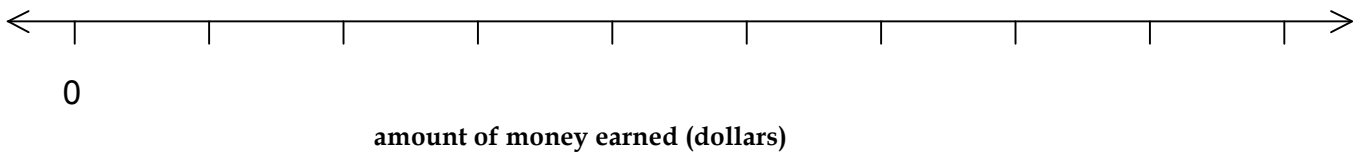
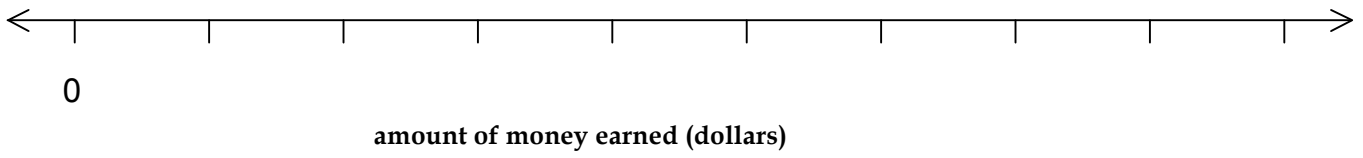
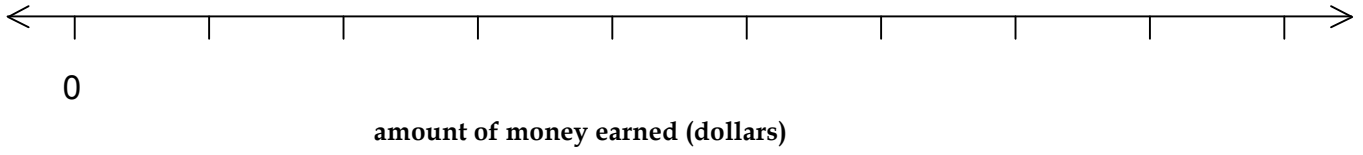
Notice that this student labeled each interval as being worth 5. This labeling system highlights the value of the interval between two tick marks and does not necessarily reference 0 as above.

The student work below still emphasizes the value of the interval. The interval labeled 3 represents the first $\frac{1}{4}$ of an hour worked. This student seems to show that if Faisal works the first interval worth of time he will earn \$3. The second interval that is labeled 6 might be more accurately labeled 3 since the second interval of time is one-quarter hour that is worth \$3. This interval seems to represent a running total.

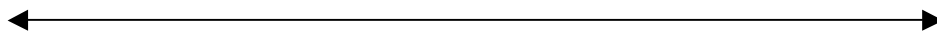


Student work using Method II (in context)

Although this recording scheme is somewhat misleading (i.e. the value of the interval labeled 6 is \$3) it still represents correct thinking and does not need to be corrected. It is important that children share thinking and recording strategies with each other during class discussions.



Tyanna earns \$20 an hour growing flowers. How much does she earn if she works $\frac{3}{5}$ of an hour?



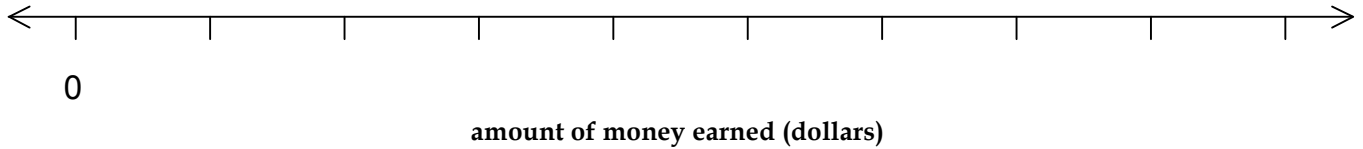
amount of money earned (dollars)

- What does a jump from 0 to 20 mean on the number line?
- How do you jump three-fifths of an hour?
- How could you calculate $\frac{3}{5} \times 20$ without using the number line?

Faisel makes \$12 an hour. How much does he make in 3 hours of work? Write a multiplication sentence that describes how to find the answer.

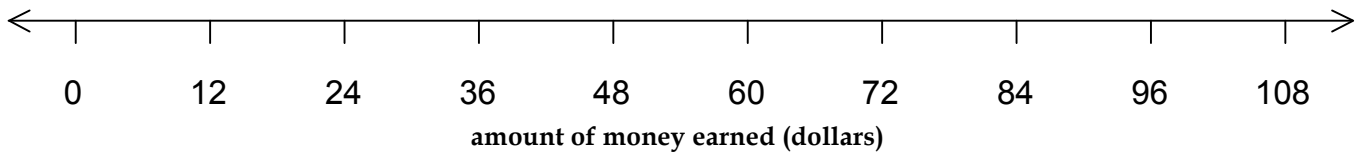
Multiplying Fractions on Number Lines

Faisel makes \$12 an hour.



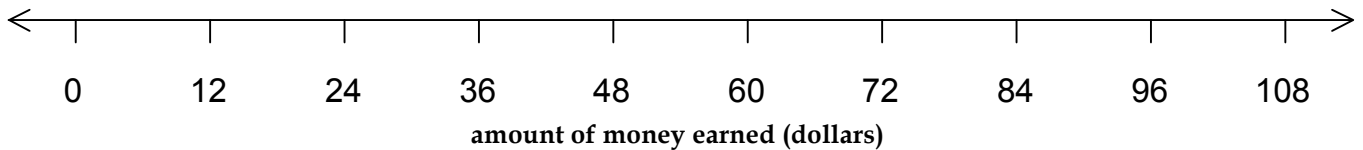
I. 5 hours

Multiplication Sentence: _____ x _____ = _____



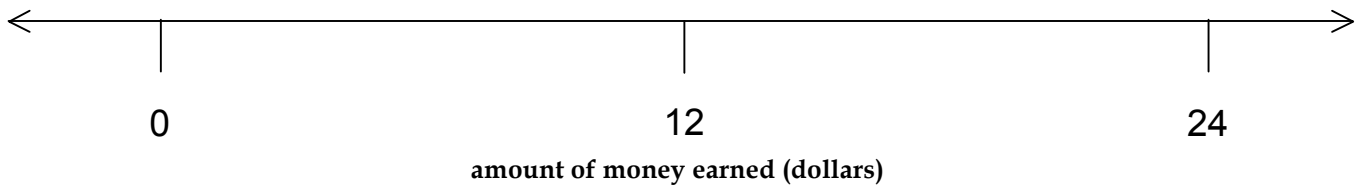
II. _____ hours

Multiplication Sentence: _____ x _____ = _____



III. _____ hours

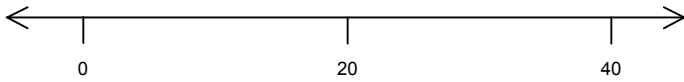
Multiplication Sentence: _____ x _____ = _____



Multiplying Fractions on Number Lines

Use a number line and write a multiplication sentence to answer each of the questions below.

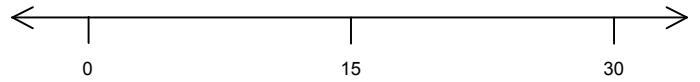
1. Nicki earns \$20 an hour fixing cars. Use the number line to determine how much she earns if she works $\frac{1}{2}$ an hour.



Multiplication Sentence:

$$\begin{array}{c} \text{_____} \\ \# \text{ of} \\ \text{hours} \\ \text{worked} \end{array} \times \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{earned} \\ \text{per hour} \end{array} = \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{of money} \\ \text{earned} \end{array}$$

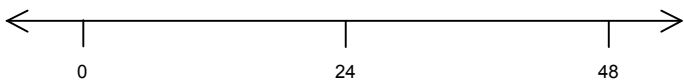
2. Joshkin earns \$15 an hour cleaning bird cages. How much will he earn if he works $\frac{2}{3}$ of an hour?



Multiplication Sentence:

$$\begin{array}{c} \text{_____} \\ \# \text{ of} \\ \text{hours} \\ \text{worked} \end{array} \times \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{earned} \\ \text{per hour} \end{array} = \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{of money} \\ \text{earned} \end{array}$$

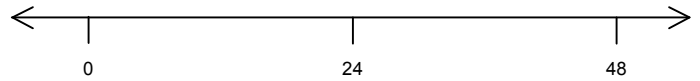
3. Emily makes \$24 an hour. How much will she make if she works $\frac{1}{8}$ of an hour?



Multiplication Sentence:

$$\begin{array}{c} \text{_____} \\ \# \text{ of} \\ \text{hours} \\ \text{worked} \end{array} \times \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{earned} \\ \text{per hour} \end{array} = \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{of money} \\ \text{earned} \end{array}$$

4. Ty earns \$24 an hour mowing lawns? How much will he earn if he works $\frac{3}{4}$ of an hour?

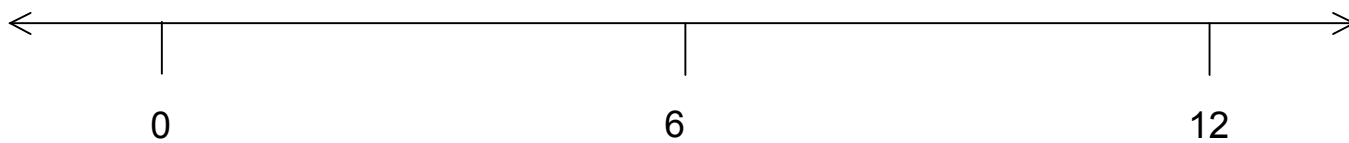


Multiplication Sentence:

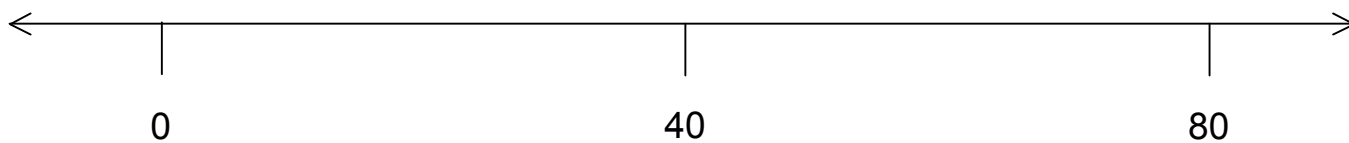
$$\begin{array}{c} \text{_____} \\ \# \text{ of} \\ \text{hours} \\ \text{worked} \end{array} \times \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{earned} \\ \text{per hour} \end{array} = \begin{array}{c} \text{_____} \\ \text{amount} \\ \text{of money} \\ \text{earned} \end{array}$$

Use the number line to find the answer to the following problems.

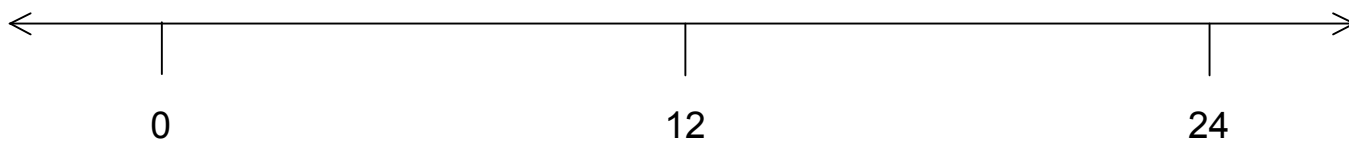
5. $\frac{1}{2} \times 6 = \underline{\quad}$



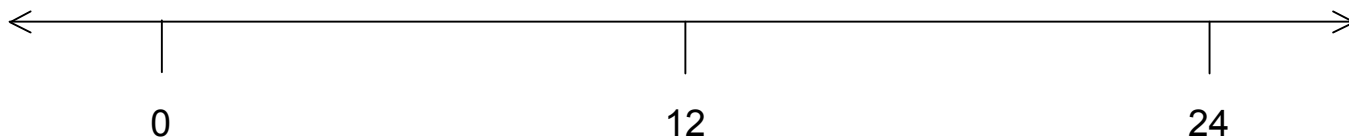
6. $\frac{3}{8} \times 40 = \underline{\quad}$



7. $\frac{2}{3} \times 12 = \underline{\quad}$



8. $\frac{5}{4} \times 12 = \underline{\quad}$



Post Lesson Reflection

Lesson _____

1) Number of class periods allocated to this lesson: _____

2) Student Pages used: _____

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: