Rational Number Project

Fraction Operations and Initial Decimal Ideas
Lesson 22: Overview

Students use patty paper (an area model) to multiply fractions. Students develop the algorithm for multiplying fractions by noticing patterns related to the patty paper model.

Materials

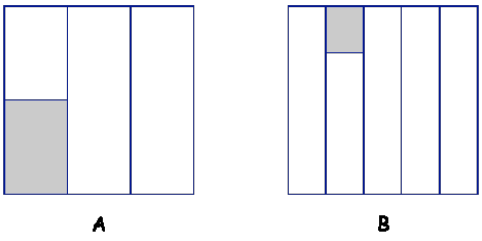
- Patty paper
- Two color pencils or markers
- Student sheets A and B
- Transparencies 1 and 2

Teaching Actions

Warm Up

Determine the fraction of the square that is colored gray.

Show A and B together then discuss. Show B and C together and discuss.

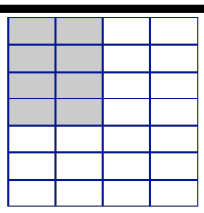


A

Fraction:

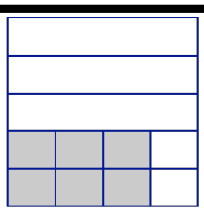
B

Fraction:



C

Fraction:



D

Fraction:

Comments

The students should be able to do the warm up problems mentally. Please give them a minute to think of the fractional amounts and have them write the fractions on a piece of scratch paper.

The purpose of squares A and B is to focus the students' attention on getting equal sized pieces. Students may suggest that you draw in lines on the transparency to show equal sized pieces when you discuss the answers.

The emphasis for Square C is on multiplication. Try to get students to explain how they calculated the fraction colored gray. Some students may notice that the total number of equal sized pieces (denominator) can be found by multiplying 7 by 4 and the number of colored in pieces can be found by multiplying 4 by 2 (numerator).

The emphasis on Square D is a combination of strategies for the previous squares. Multiplication can be used to count both the number of shaded rectangles as well as the number of total rectangles when determining the fraction of the square that is

Teaching Actions

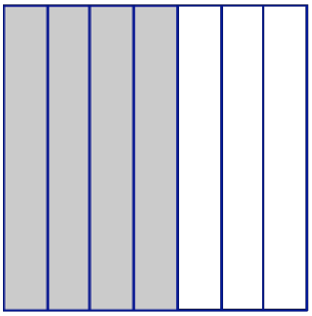
Comments

shaded. This is an important part of explaining why the algorithm for multiplication of fractions works.

Large Group Introduction

1. Explain to the students that we are going to develop a rule for multiplying fractions today. To begin this exploration, ask them to look at the picture on transparency 1. Explain that the grayed in part represents the amount of cake Kathy has left from yesterday.

Kathy's Cake



2. Ask the students to think about how much cake Kathy has left. Record the fractional amount below the cake.

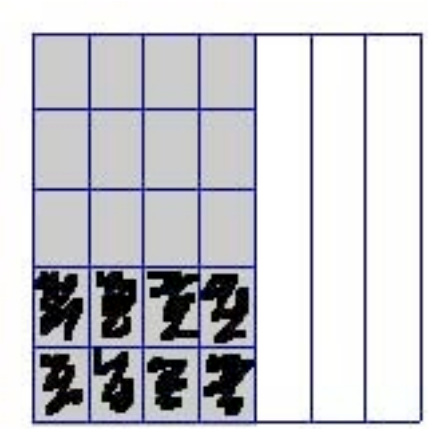
There is four-sevenths of a cake remaining.

3. Explain that Kathy wants to eat two-fifths of the remaining cake. Ask the students to think how Kathy could find two-fifths of the remaining cake.

4. Have a volunteer come up and shade in two-fifths of the three-sevenths of a cake with a dark marker. It may look something like what is shown below.

The student may or may not draw a picture as shown to the left. The sevenths and the fifths were chosen for this example so the students will draw horizontal and vertical lines to show the total number of pieces. It is easier to explain why the multiplication algorithm works if students draw one fraction vertically and the fraction of that fraction horizontally.

Teaching Actions



5. Ask the students to explain <u>what fraction of the whole cake will Kathy eat</u> ($\frac{8}{35}$). Ask a few students how they determined this fraction. [Students most likely will draw in the horizontal lines across the whole square to explain why the total number of parts is 35].

6. Ask: What multiplication sentence matches the actions we made on the picture of the patty paper?

$$\frac{2}{5} \times \frac{4}{7} = \frac{6}{35}$$

Small Group/Partner Work

7. Assign Student Pages A & B. You may want to go over the first question with the students so they know how to complete the class work.

Wrap Up

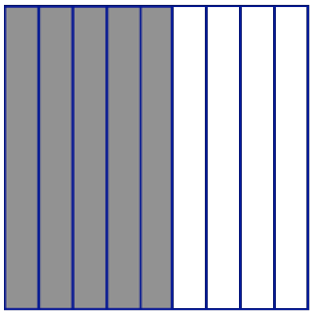
8. Review select problems from the Student Pages. To help students construct the rule for multiplying fractions, guide the discussion with these questions:
    - How did you show $\frac{2}{3} of \frac{1}{5}$?
    - How did the picture change when you did that?
    - If you extend the horizontal lines across the square, how many total parts is the square partitioned into?
    - Where is a 3 by 5 rectangle in your picture?

Comments

Teaching Actions

- Where is a 2 by 1 rectangle in your picture?

9. Ask the students to picture a piece of patty paper with $\frac{5}{9}$ colored. Show transparency 2.



$$- \times \frac{5}{9} = -$$

10. Write a 3 in the denominator of the first fraction in the multiplication sentence.

$$\frac{-}{3} \times \frac{5}{9} = -$$

Ask the students to explain why multiplying 9 by 3 will give you the denominator of the product.

Ask: What number will we write in the denominator of the result fraction? (27) Where is the 3 by 9 rectangle in the picture?

Ask: What does the 27 represent" (The number of equal-sized pieces that the unit is partitioned.) You can write three horizontal lines on the transparency to show the 27 equal sized pieces.

11. Write a 2 in the numerator of the multiplication sentence.

$$\frac{2}{3} \times \frac{5}{9} = \frac{--}{27}$$

Ask: What number should we write in the numerator of the product? (10)

Comments

Students have trouble explaining why the multiplication algorithm works. The goal of the student pages and the wrap up is to help them verbalize why the algorithm works using the patty paper model.

Students should be able to explain that each of the 9ths will be cut into 3rds. Multiplying 9 by 3 counts the number of total pieces the unit is cut into. One-third of one-ninth is one- twenty-seventh.

Teaching Actions

Ask: What does the 10 represent? (The 10 pieces represent the number of pieces that are darkly shaded.) You may want to show two-thirds of five-ninths on the transparency to show how multiplication helps you show the product. Where is the 2 by 5 rectangle in the picture?

12. Explain that the "algorithm" for multiplying fractions is to multiply the numerators and the dominators. The product of the numerators will give you the number of pieces in the answer and the product of the denominators will give you the number of pieces in the unit.

    Write the following number sentence with letters:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

multiplication algorithm

Comments

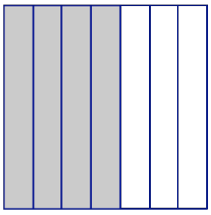
Translations:

- Picture to symbols
- Symbols to picture to symbols
- Pictures to symbols to verbal

Additional Notes to the Teacher
Lesson 22

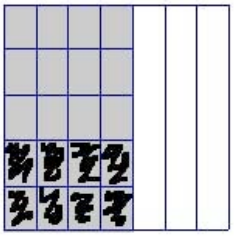
The goal for this lesson is for students to develop the rule or algorithm for multiplying two fractions.

The students will use pictures of patty paper to partition fractions of fractions. We found that students seem to be able to see multiplication ideas better if they draw one fraction using vertical lines and then finding the fraction of this fraction using horizontal lines. In the Kathy's cake example, the students are told that Kathy has three-sevenths of a cake left over as shown in the picture below.



Kathy's Cake

Kathy is then going to eat two-fifths of this amount of cake. Although you could draw in vertical lines to find two-fifths of the three-sevenths it would be difficult to see. Please model and encourage your students to draw in horizontal lines when they partition the three-sevenths into fifths. To explain the amount of the darkly shaded part of the cake, students will need to extend the horizontal lines across the whole patty paper. If students folded the patty paper first into sevenths and then into fifths, the paper would have creases showing the horizontal partitions.

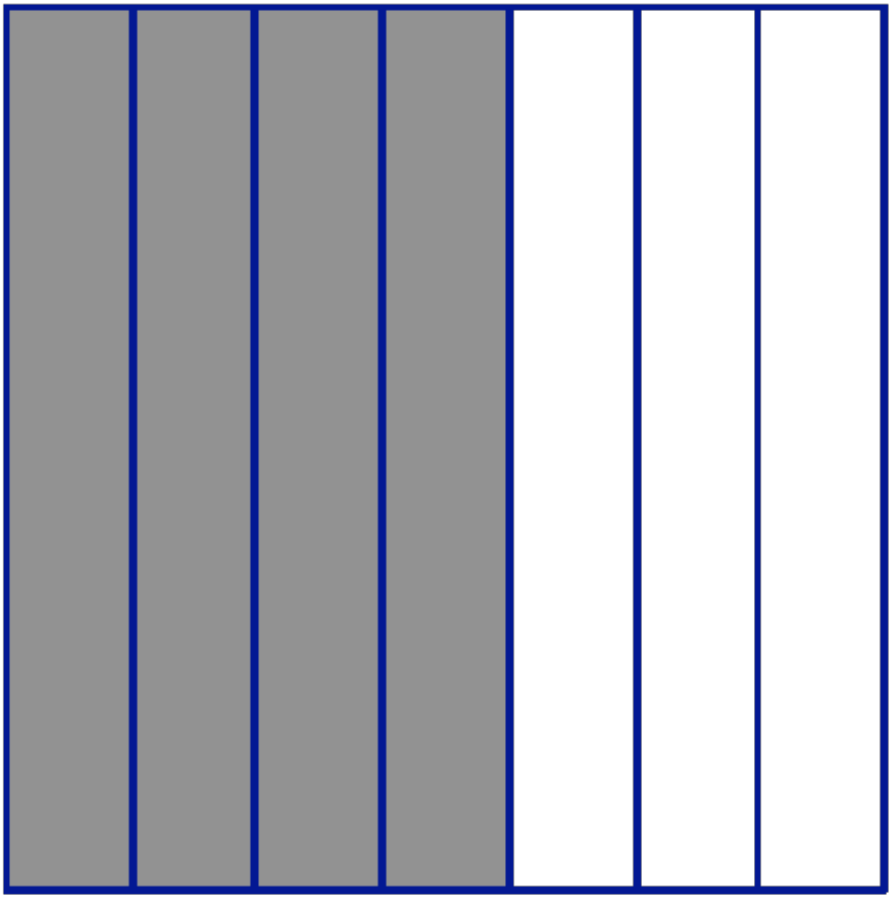


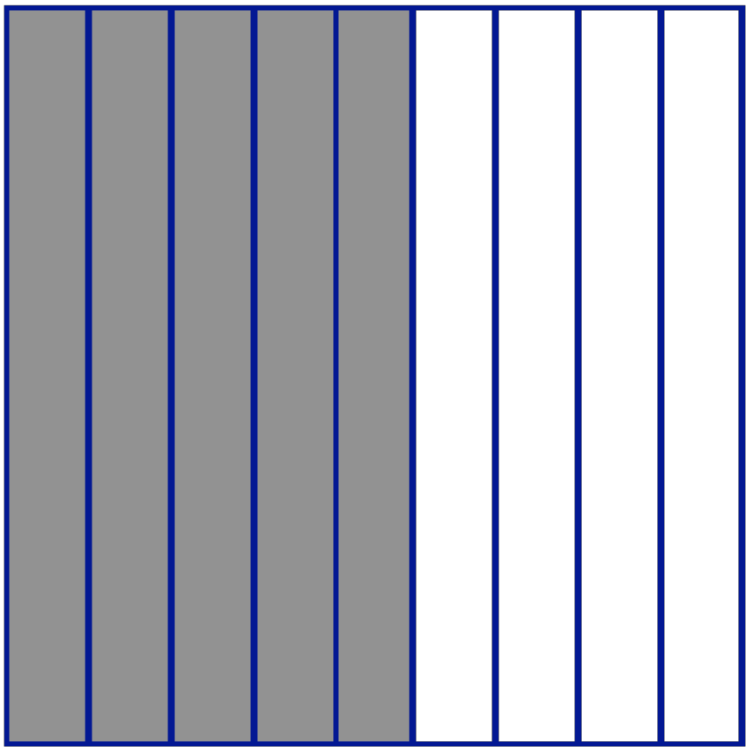
two-fifths of three-sevenths darkly shaded

By the end of the lesson you want students to be able to explain that the product of the denominators show the size of the pieces. When you take fifths of sevenths you will get pieces that are thirty-fifths of the unit. The product of the numerators is more difficult to explain. The students should be able to explain that the total number of darkly shaded pieces can be counted by finding the product of the numerators.

By the end of this lesson students should be able to reason that the product of the numerators determine the number of pieces that you are interested in and the product of the denominators is the number of pieces in which the unit is partitioned.

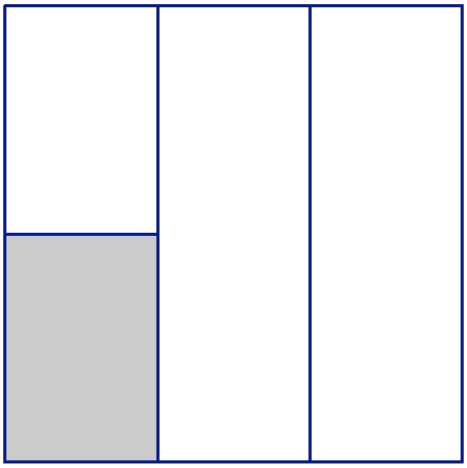
Kathy's Cake





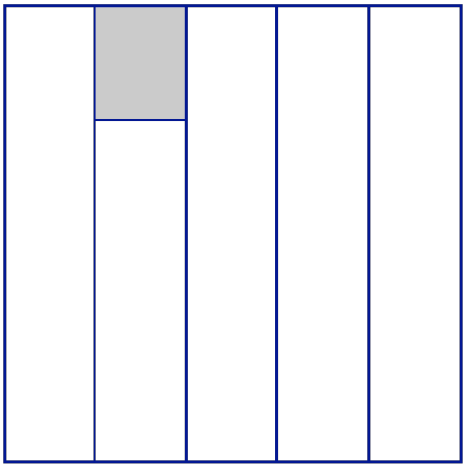
$$\frac{\quad}{\quad} \times \frac{5}{9} = \frac{\quad}{\quad}$$

Determine the fraction of the square that is colored gray.



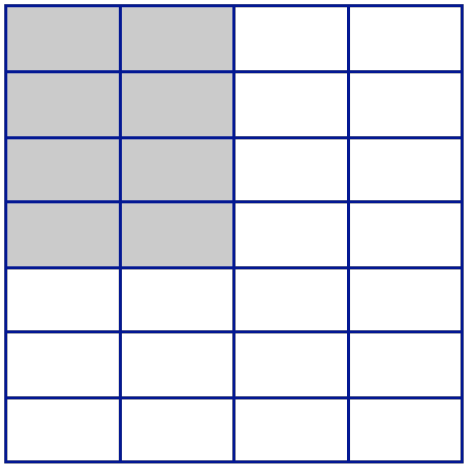
A

Fraction:



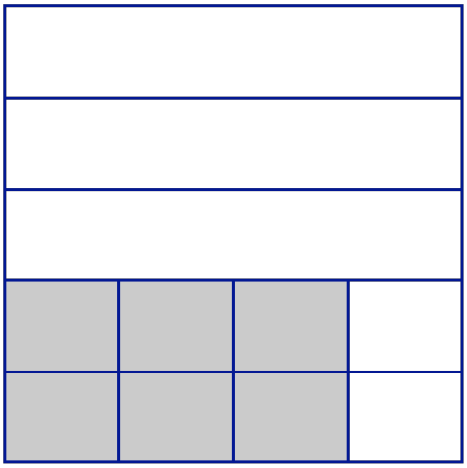
B

Fraction:



C

Fraction:



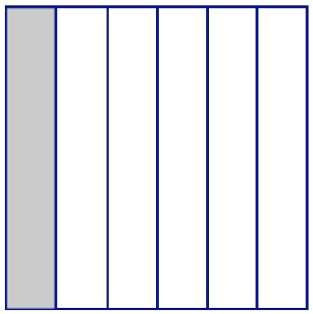
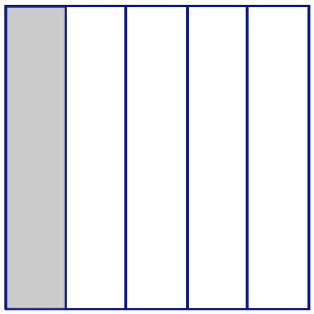
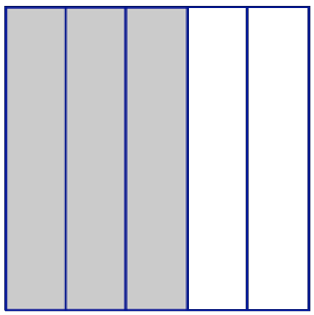
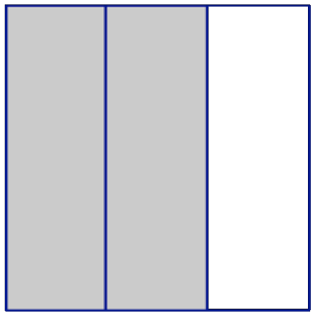
D

Fraction:

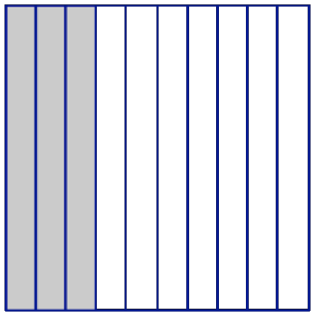
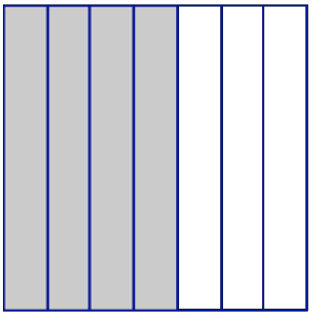
Name _____ Lesson 22 / Student Page A

Multiplying Fractions

1. Fill in the table below.

Multiplication Problem	Picture	# of darkly shaded pieces / # of total pieces in square	Fraction of the square shaded dark
$\frac{3}{4} \times \frac{1}{6}$		-------	
$\frac{2}{3} \times \frac{1}{5}$		-------	
$\frac{1}{4} \times \frac{3}{5}$		-------	
$\frac{2}{3} \times \frac{2}{3}$		-------	

Name _____ Lesson 22 / Student Page B

$\frac{2}{5} \times \frac{3}{10}$		- - - - - - -	
$\frac{3}{5} \times \frac{4}{7}$		- - - - - - -	

2. Look for patterns in each row. Explain the patterns you see.

3. Use the pattern you describe in question 2 to the answers to the problems below.

a) $\frac{5}{7} \times \frac{4}{11} =$

b) $\frac{7}{10} \times \frac{3}{17} =$

4. Rewrite the pattern you used as a rule for multiplying fractions. Explain why your rule works.

Post Lesson Reflection

Lesson_____

1) Number of class periods allocated to this lesson: _____

2) Student Pages used: _____

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: