

Rational Number Project

Fraction Operations and Initial Decimal Ideas Lesson 23: Overview

Students multiply fractions using a number line and estimate products of fractions by mentally picturing the number line.

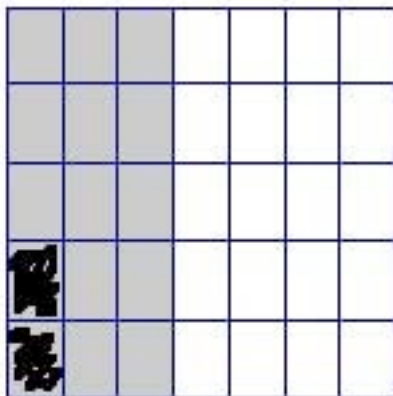
Materials

- Transparencies 1-5
- Student Pages A, B, and C

Teaching Actions

Warm Up

Raina was trying to find the product $\frac{2}{5} \times \frac{3}{7}$ using patty paper. She drew the picture below and says the answer is $\frac{2}{35}$. How did she get this answer and is she correct?



After the correct answer is developed ask the students: "Does this revised answer match the algorithm we developed yesterday?"

Large Group Introduction

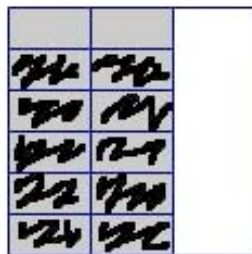
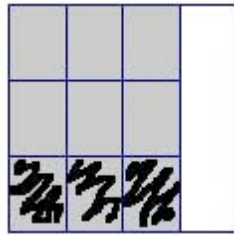
1. Ask the students to find the multiplication sentence that goes with the pictures below. (Transparency 1)

Comments

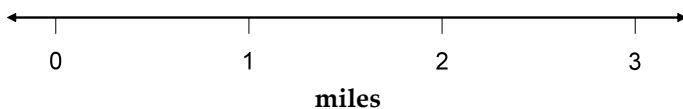
This is a common mistake that was made by students in Lesson 22.

Emphasize that Raina was taking two-fifths of one-seventh. She needs to take two-fifths of each of the three-sevenths. Shade in the remaining 4 rectangles.

Teaching Actions



2. Explain that they are now going to look again at number lines and show how to multiply two fractions using that model.
3. Ask the students to draw a number line as shown below. Make sure that they label the number line in miles. (Transparency 2)



Ask them to solve the following problem using the number line.

Each block along Helena Avenue is one-half mile long. Duanna runs 5 blocks. Show how you can use the number line to find the total number of miles she runs. Write a multiplication sentence for her run.

Have a student come to the front of the classroom and demonstrate how they think they should use a number line to solve the problem. Other students may come to the front of the classroom if they have a different strategy.

An example of student work is shown below.

Comments

Answer: $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$

Students may suggest that you draw in the horizontal lines to show the 12ths.

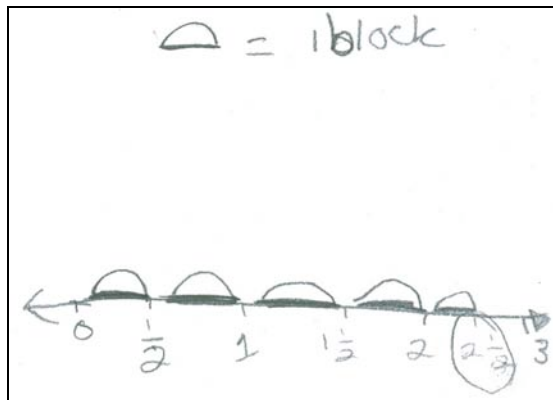
Answer: $\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$

Students will need to place tick marks on the number line to show the $\frac{1}{2}$ mile lengths of the blocks.

Be sure that you ask the student to explain the meaning of each factor in the multiplication sentence. The 5 represents the number of blocks run (groups) and the one-half represents the length of each block (amount in each group). The two and one-half represents the total length of the run.

Teaching Actions

Comments



Multiplication sentence:

$$5 \times \frac{1}{2} = \frac{5}{2} \text{ or } 2\frac{1}{2}$$

Ask: Does this sentence match the algorithm? Why don't we multiply the 2 by the 5 and get $\frac{5}{10}$ for our answer?

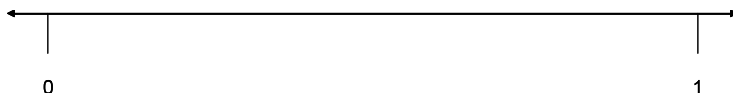
Five-tenths is the same as one-half and that is not a reasonable answer. Duanna runs five whole blocks. The 2 in the denominator of the fraction describes the size of the fraction and the size does not change as she runs each whole block.

4. Read the following problem to the students.
(Transparency 3)

Duanna is tired and stops running after $\frac{3}{4}$ of a block. Each block is still $\frac{1}{2}$ of a mile long. How many miles did she run?

Ask the students to estimate the answer first. Did Duanna run more than one-half a mile or less? Why?

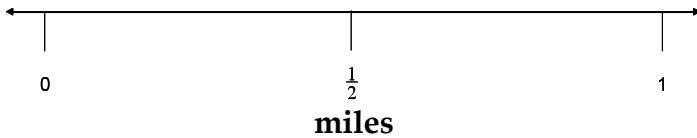
Have students share their answers and procedures. If none of the students used a number line to solve this problem then show them how to solve on a number line.



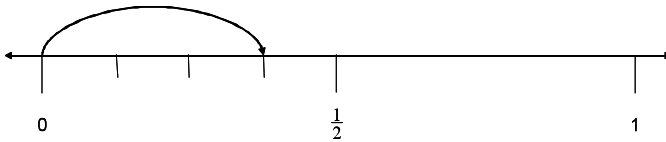
Teaching Actions

miles

Ask students to put a tick mark at one-half mile to represent one block.

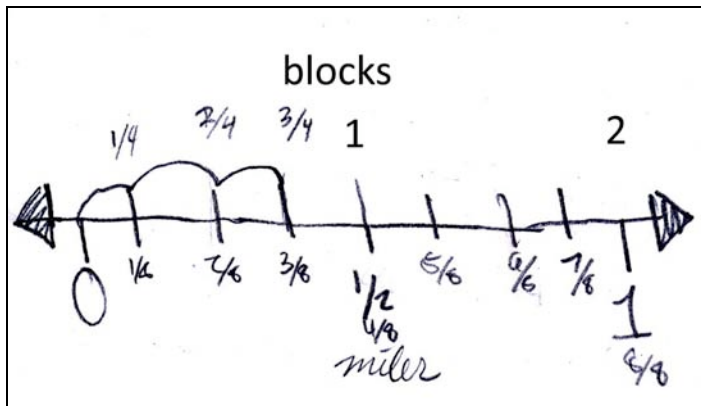


To find three fourths of the one-half, mark the half-mile into 4 equal parts. Three-fourths of one-half is shown by the arrow below.



Ask: What value is that spot on the number line? Consider what the unit is. How do you know the answer is in 8ths? How is this similar to adding the horizontal lines to the patty paper to find the total number of equal parts the paper was partitioned into?

You may want to label the completed number line as is shown in the student work below.



Ask the students to write a multiplication sentence that describes how to find the amount that Duanna ran. Be sure to discuss the meaning of each factor and record. Also relate the answer to the algorithm.

Comments

Students should suggest that you partition the line from $\frac{1}{2}$ to 1 into fourths as well.

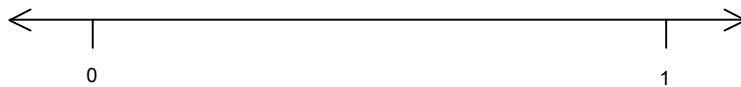
Note that the student partitioned to second half mile into fourths. This clearly shows each eighth of a mile. The work also shows a double number line with the miles written below the number line and the blocks written above the number line.

Teaching Actions

Multiplication Sentence:

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

5. Ask students to draw a number line as shown below to solve the problem on transparency 4. Make sure they label the 0 and the 1 on the number line and write miles below the number line as shown below.



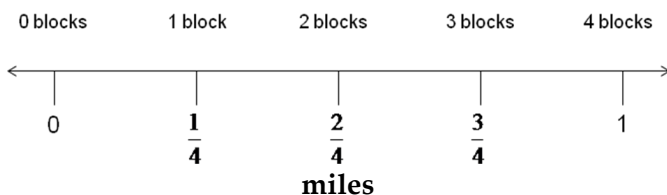
miles
Chase walks down Old Shakopee Road. Each block is $\frac{1}{4}$ of a mile. Chase stops walking after $\frac{2}{3}$ of a block. How far did he walk?

Ask students: What is the multiplication sentence for this problem?

Multiplication Sentence: $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$

State: "This sentence tells us that Chase walked two-twelfths or one-sixth of a mile. We are going to use the number line we drew to show how far he walked."

Ask students: How many blocks fit in 1 mile? Have them label the miles below the number line and the blocks above the number line.



Comments

To represent the product of two fractions, students actually rely on the algorithm to decide how to partition the number line.

If students are able to write $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$ they should use the algorithm to calculate the distance walked.

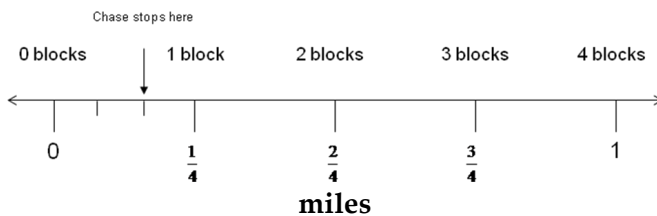
You may want to ask if Chase is walking two-thirds of one-fourth mile or one-fourth of two-thirds mile. He is walking two-thirds of one-fourth mile (1 block).

Teaching Actions

State: Chase stops after walking $\frac{2}{3}$ of a block.

How can you show on the number line that he walked $\frac{2}{3}$ of the one block?

The students should partition the space between 0 and $\frac{1}{4}$ into thirds, then move from 0 over two-thirds of a block.

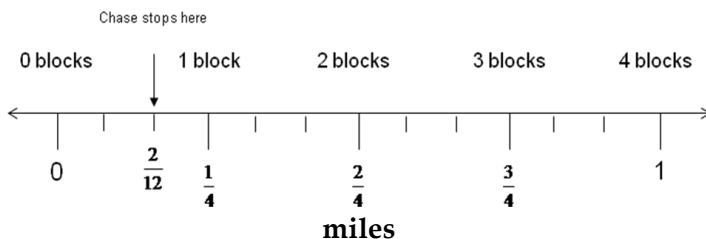


Ask: How much of a mile does he walk before stopping?

Two-twelfths of a mile.

How can the number line be partitioned further to name this amount?

Each remaining fourth mile needs to be partitioned into three equal pieces so that each tick mark is an equal length (twelfths).



How is this like adding in the horizontal lines on the patty paper?

We added lines on the patty paper to partition the unit into equally sized rectangles. On the number line we add tick marks to partition the number of equally spaced lengths that make up 1 mile (the unit).

Comments

Based on the algorithm, students know the answer should be in twelfths. That helps them see that to show that on the number line, it makes sense to partition each of the three remaining fourths into thirds. This shows why the answer is in twelfths of a mile.

Teaching Actions

Comments

Small Group/Partner Work

6. Assign Student Pages A, B, and C.

Wrap Up

7. Review select problems on the Student Pages.
8. Ask the students to estimate the following multiplication problems. (Transparency 5)

$$\frac{2}{3} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2} \text{)}$$

$$\frac{5}{4} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2} \text{)}$$

$$3 \times \frac{1}{4} \text{ (less than } \frac{1}{4} \text{ or greater than } \frac{1}{4}, \text{ less than } 1 \text{ or greater than } 1)$$

Focus on partitioning.

Use the context that each block is one-half mile and the person runs two-thirds of a block to help students create mental images. Encourage your students to determine the answer without using the algorithm by drawing a number line and labeling only 0 and $\frac{1}{2}$; then ask them to visualize where the product would be.

Translations:

- Real life to pictures to verbal
- Real life to pictures to symbols
- Symbols to pictures

Additional Notes to the Teacher

Lesson 23

The goal for this lesson is for students to develop a method for multiplying fractions using a number line. The students should be able to use the number line to help them partition a unit fraction. A unit fraction is a fraction with a numerator of 1. The second factor in each of the multiplication problems for this lesson is a unit fraction. Students will be exposed to problems where the second factor is a non-unit fraction in lesson 24.

A distance context was selected for this lesson to make connections with the number line, a model that emphasizes length, and it makes sense for students to have fractional amounts of groups (amount run) and fractional amounts in each group (length of blocks).

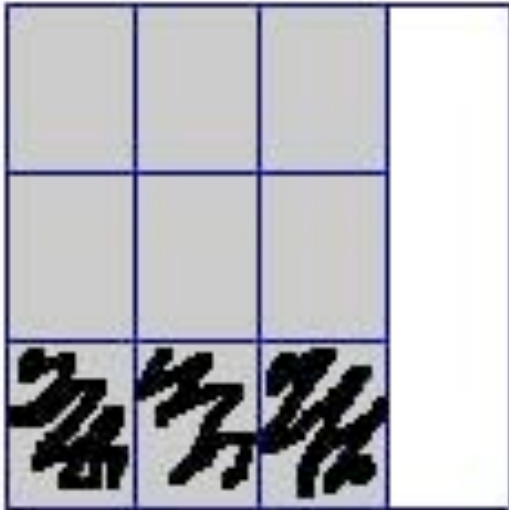
Many of the students naturally made double number lines as they solved the problems by labeling the miles on the bottom of the number line and the blocks on the top. This seemed to help some students as they partitioned the number line.

The number line provides students with a way to apply the algorithm they learned to multiply fractions and make further sense on why it works. The number line also provides a way for students to mentally visualize the multiplication of fractions and use this image to make estimates. For example look at the number line below and try to imagine where $\frac{2}{3} \times a$ is on the number line.

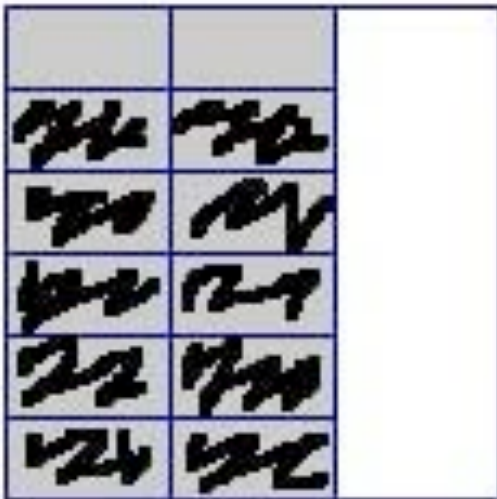


Students should be able to visualize the distance between 0 and a being partitioned into thirds and iterating over two of the thirds. They should be able to see that an estimate for this product is a number less than a . If they try to visualize where $3 \times a$ would be they should be able to visualize the iteration of a three times. The product $\frac{5}{4} \times a$ can be visualized by partitioning the distance between 0 and a into fourths then iterating this length five times which results in a product that is larger than a .

Find the multiplication sentence that goes with the two pictures below.



$$_ \times _ = _$$



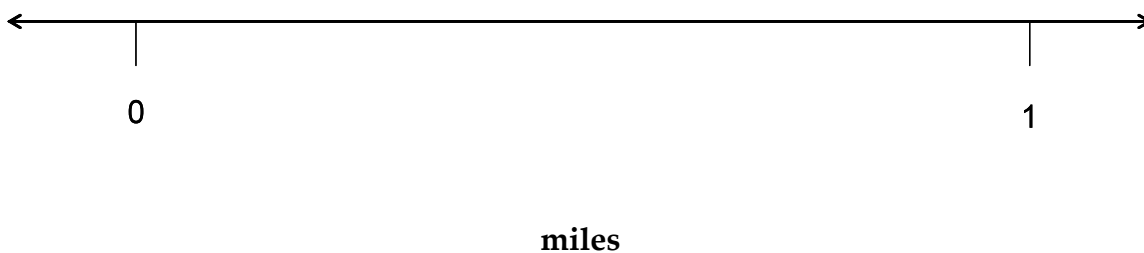
$$_ \times _ = _$$

Each block along Helena Avenue is one-half mile long. Duanna runs 5 blocks. Show how you can use the number line to find the total number of miles she runs. Write a multiplication sentence for her run.

Duanna is tired and stops running after $\frac{3}{4}$ of a block.

Each block is still $\frac{1}{2}$ of a mile long. How many miles did she run?

Chase walks down Old Shakopee Road. Each block is $\frac{1}{4}$ of a mile. Chase stops walking after $\frac{2}{3}$ of a block. How far did he walk?

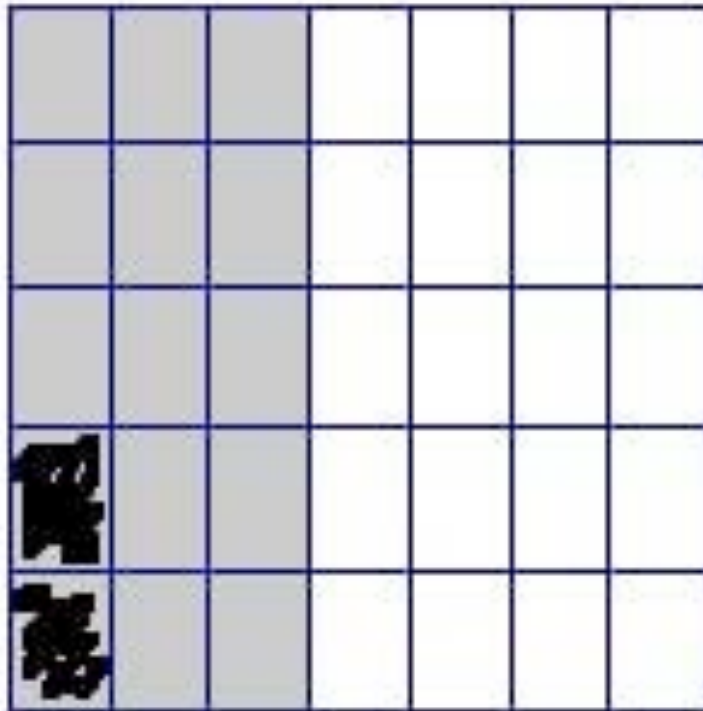


$$\frac{2}{3} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2}\text{)}$$

$$\frac{5}{4} \times \frac{1}{2} \text{ (less than } \frac{1}{2} \text{ or greater than } \frac{1}{2}\text{)}$$

$$3 \times \frac{1}{4} \text{ (less than } \frac{1}{4} \text{ or greater than } \frac{1}{4}, \text{ less than 1 or greater than 1)}$$

Raina was trying to find the product $\frac{2}{5} \times \frac{3}{7}$ using patty paper. She drew the picture below and says the answer is $\frac{2}{35}$. How did she get this answer and is she correct?

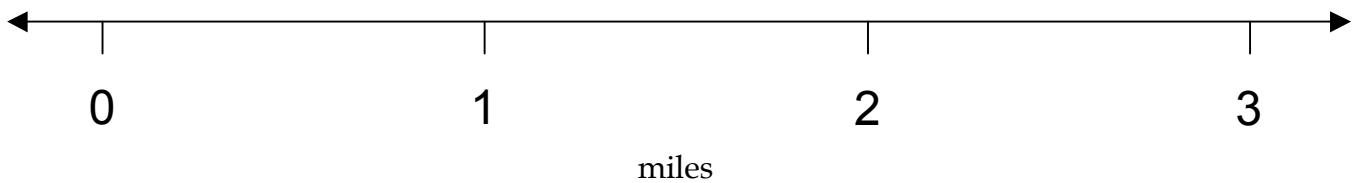


Multiplying Fractions on Number Lines

(fraction \times unit fraction)

1. Molly runs down Colfax Avenue. Each block is $\frac{1}{3}$ of a mile long. She runs 4 blocks before she gets tired and stops.

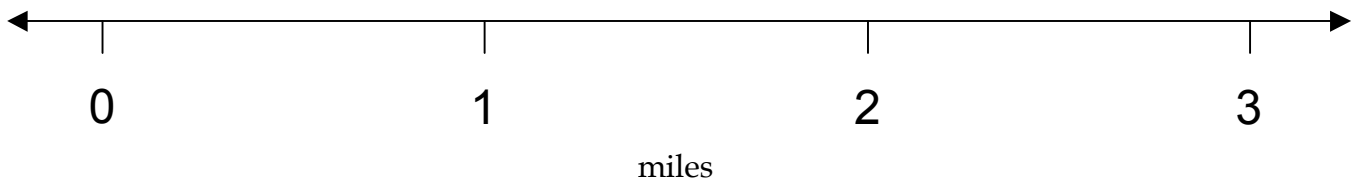
a) Find the exact length of her run using the number line.



b) Write a multiplication sentence that shows how you can find out how far Molly ran. Explain the meaning of each factor.

2. Molly runs down Colfax Avenue again. Each block is $\frac{1}{3}$ of a mile long. She runs $\frac{1}{4}$ of a block before she gets tired and stops.

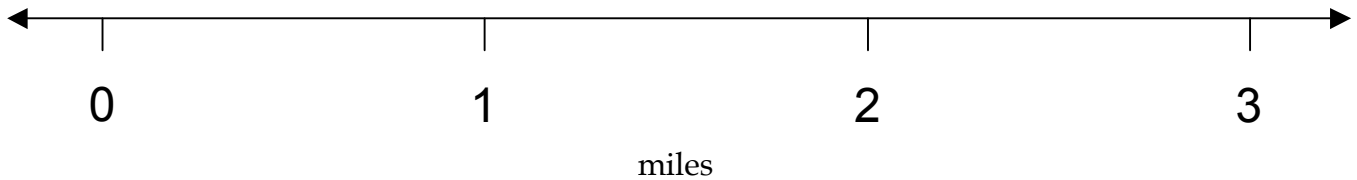
a) Find the exact length of her run using the number line.



b) Multiplication Sentence:

3. Mark runs down Irving Avenue. Each block is $\frac{1}{3}$ of a mile long. He runs $\frac{2}{3}$ of a block before he gets tired and stops.

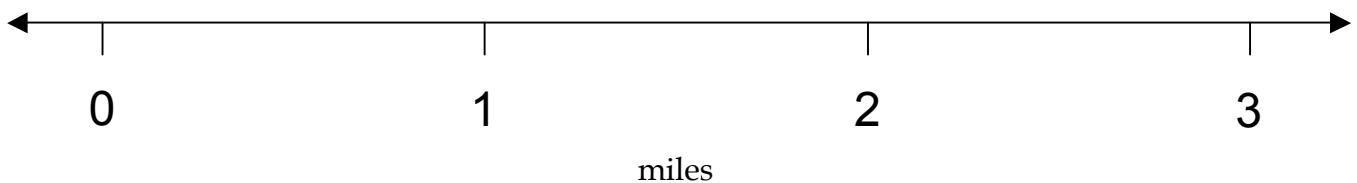
Find the exact length of his run using the number line and write the multiplication sentence that shows how far he runs.



Multiplication Sentence:

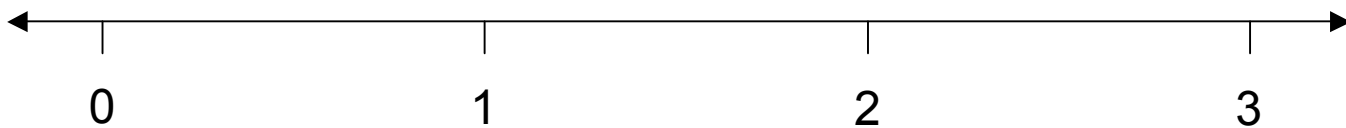
4. Pang determines the length of her run using the multiplication sentence $\frac{2}{3} \times \frac{1}{2} = \underline{\hspace{2cm}}$.

- a) How long is each block that she runs?
- b) How many blocks does she run?
- c) Find the exact length of her run using the number line and complete the multiplication sentence above that shows how many miles she runs.

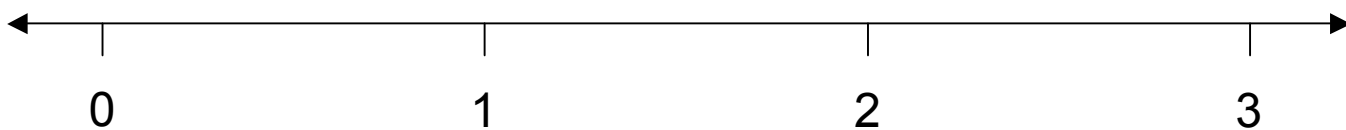


Find the product of these multiplication problems using the number line. Check your answer using the algorithm.

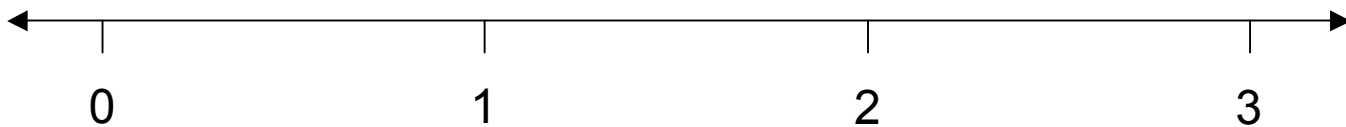
5. $\frac{1}{3} \times \frac{1}{2} = \underline{\hspace{2cm}}$



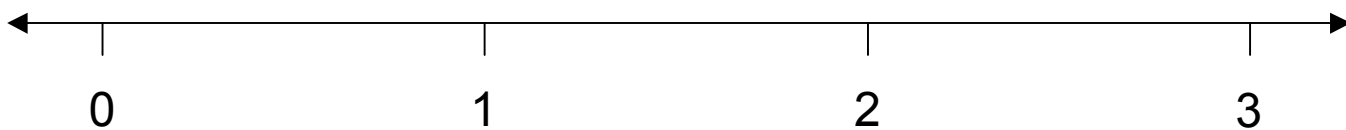
6. $3 \times \frac{2}{3} = \underline{\hspace{2cm}}$



7. $\frac{2}{3} \times \frac{1}{5} = \underline{\hspace{2cm}}$



8. $\frac{5}{4} \times \frac{1}{3} = \underline{\hspace{2cm}}$



Post Lesson Reflection

Lesson _____

1) Number of class periods allocated to this lesson: _____

2) Student Pages used: _____

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: