

Rational Number Project

Fraction Operations and Initial Decimal Ideas Lesson 24: Overview	Materials <ul style="list-style-type: none">• Transparency 1• Student Pages A, B, C, and D
Students use a variety of models and the algorithm to multiply a fraction by another fraction. Students will describe connections among the number line, pictures, and the algorithm.	

Teaching Actions

Warm Up

Ask students to give an exact answer for each of the warm up problems. Encourage them to do the problems mentally but it is OK if a student chooses to write something on paper. Do each problem one at a time.

$$\frac{1}{2} \times \frac{7}{8} =$$

$$\frac{2}{3} \times \frac{1}{2} =$$

$$\frac{4}{5} \times 30 =$$

$$6 \times \frac{2}{3} =$$

Large Group Introduction

1. Ask the students to write a multiplication sentence for the following problem and use it to find the answer.

Hannah hikes along the Nature Trail at Mud Lake. The trail is $\frac{2}{3}$ of a mile. She hikes $\frac{4}{5}$ of the trail before she stops to take a picture of a

Comments

When students explain their reasoning some students may say they got their answers using the algorithm (i.e. multiply the numerators and the denominators). Other may picture a square, a number line, or clouds. Ask several students how they did the problem and probe for different strategies.

The word algorithm is used in the student pages. Please emphasize this word as students are explaining how they solved each problem.

Teaching Actions

hummingbird with her high speed Leica camera. How many miles did she hike before taking the picture?

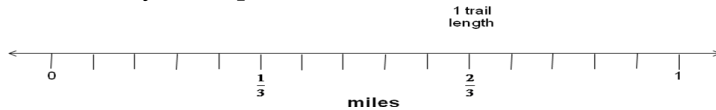
Multiplication sentence:

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

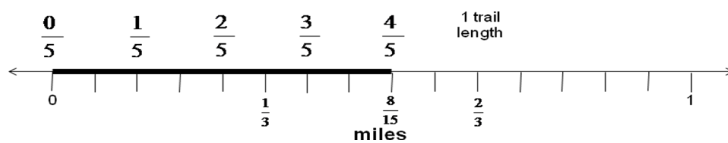
2. Ask the students to solve the same problem using a number line. The students should be able to set up a number line that looks similar to the one below without too much help from you.



Students may have trouble trying to find out how to take four-fifths of two-thirds of a mile. The algorithm tells us that the answer will be in fifteenths so one hint may be to partition the miles into fifteenths.



Some students may need to use a double number line to make sense of finding four-fifths of two-thirds. The number line below shows that one-fifth of a trail length is two-fifteenths of a mile, so four-fifths of two-thirds would be eight-fifteenths of a mile.



Ask: How does multiplying 3 by 5 in the algorithm relate to the fifteenths on the number line?

The fifteenths are created by taking fifths of thirds.

Where is the 4×2 from the algorithm shown on the number line?

Comments

Students came up with a variety of ways of showing this answer. Please look at the Teacher Notes at the end of the lesson for examples of 3 of the most common ways that students solved the problem.

Some students noticed that there were now 10 lengths between 0 and 1 trail length so four-fifths would be 8 of those lengths.

Teaching Actions

There are 4 groups of 2 fifteenths of a mile so there are 8 lengths of one-fifteenth of a mile that is counted.

3. Have the students solve the problem again using a picture of a piece of patty-paper.
4. Ask the students to solve the following problem using the algorithm, the number line, and a picture of patty-paper.

$$\frac{3}{4} \times \frac{2}{5}$$

Review the different ways the students solved the problem.

Small Group/Partner Work

5. Spend the rest of the day having students work on the problems on Student Pages A, B, C, and D in pairs.
6. As you circulate around the class and ask probing questions, observe groups of students that have different approaches to the problems. Give selected groups a blank transparency and ask them to be ready to present their solution to the class.

Wrap Up

7. Ask students to determine if the following answers are reasonable.

$$\frac{3}{4} \times 17 = 25$$

$$3 \times \frac{1}{2} = \frac{2}{3}$$

$$\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$$

Comments

The goal of the problems is for students to make connections among the different ways to multiply fractions. Work on helping students as they try to make sense of why the steps for the multiplication algorithm are related to the steps used when solving problems using either a number line model or a patty paper model.

The goal of this activity is for students to use partitioning to estimate products.

Teaching Actions**Comments**

$$\frac{3}{8} \times \frac{5}{11} = \frac{1}{2}$$

Translations:

- Symbols to verbal
- Real life to symbols
- Real life to pictures
- Real life to manipulatives to symbols

Additional Notes to the Teacher

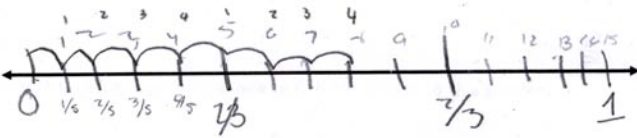
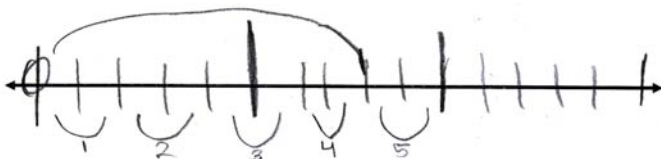
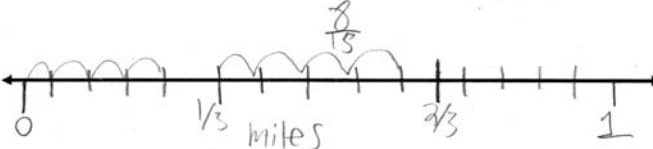
Lesson 24

The main goal for this lesson is for students to make connections among the three models for multiplying fractions introduced and the algorithm. We believe that the students will be able to explain why the algorithm works if they are able to make connections between the algorithm and a model.

The students are asked to solve the following problem using the algorithm, a picture of a piece of patty-paper, and the number line.

Hannah hikes along the Nature Trail at Mud Lake. The trail is $\frac{2}{3}$ of a mile. She hikes $\frac{4}{5}$ of the trail before she stops to take a picture of a hummingbird with her high speed Leica camera. How many miles did she hike before taking the picture?

Most students were able to solve the problem using the algorithm and patty-paper but struggled more with the number line. The students were able to use the patty-paper and algorithm results to make sense of how to solve the problem on the number line. The three approaches shown below are typical of what we found.

Work	Description
 <p style="text-align: center;">Student A</p>	<p>The bottom of the number line represents the miles. The first third is partitioned into fifths and eventually the entire number line is partitioned into 15ths. The student labels the equally spaced tick marks from 1 to 15. The numerals 1,2,3,4 written above the 1,2,3,4 seem to represent four-fifths of one-third mile. The next 1,2,3,4 above the 5,6,7,8 seem to represent four-fifths of the second third of a mile.</p>
 <p style="text-align: center;">Student B</p>	<p>This student partitioned the number line into thirds then each third is partitioned into fifths. The numbers 1,2,3,4,5 represent how two-thirds can be split into 5 equal sized pieces. The grouping of the two pieces (denoted by the U shapes above the numerals) show that four-fifths of 10 pieces is 8 pieces.</p>
 <p style="text-align: center;">Student C</p>	<p>This student partitioned each third into fifths. He then took four-fifths of the first third of a mile (getting four-fifteenths) then four-fifths of the second third of a mile (the second four-fifteenths) to get eight-fifteenths of a block.</p>

Are these answers reasonable?

$$\frac{3}{4} \times 17 = 25$$

$$3 \times \frac{1}{2} = \frac{2}{3}$$

$$\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$$

$$\frac{3}{8} \times \frac{5}{11} = \frac{1}{2}$$

Mental Mathematics

$$\frac{1}{2} \times \frac{7}{8} =$$

$$\frac{2}{3} \times \frac{1}{2} =$$

$$\frac{4}{5} \times 30 =$$

$$6 \times \frac{2}{3} =$$

Multiplying Fractions on Number Lines

(fraction \times fraction)

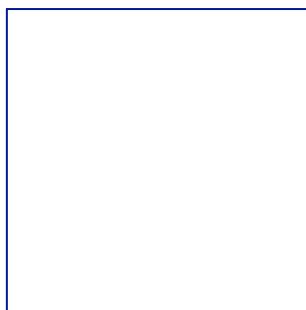
1. Max runs down 46th Street. Each block is $\frac{1}{3}$ of a mile long. He runs $\frac{2}{5}$ of a block before he gets tired and stops.
- a) Write a multiplication sentence that can be used to find the number of miles Max ran. Use the **algorithm** you developed in Lesson 22 to find the product.

- b) Use the number line to show how to find the answer to this problem.



- c) Explain why the number line is divided into fifteenths.

- d) Use the square to show how to find the answer to this problem.



- e) Explain how the picture above shows why the square is broken into fifteen equal sized pieces.

2. Xander's hose can pump 21 gallons of water every hour. How many gallons of water will he fill up if he runs the hose for $\frac{2}{3}$ of an hour?

a) Write a multiplication sentence. Use the algorithm to find the product.

b) Use the number line to determine the answer to this problem.



c) Draw a picture that would find the answer to the multiplication problem.

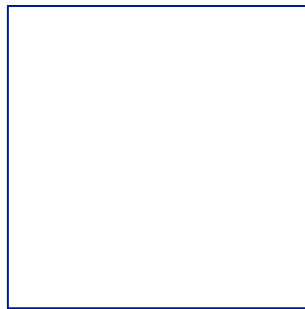
d) Explain why the algorithm answer should match the number line answer and the picture answer.

3. Hannah hikes along the Nature Trail at Mud Lake. The trail is $\frac{2}{3}$ of a mile. She hikes $\frac{4}{5}$ of the trail before she stops to take a picture of a hummingbird with her high speed Leica camera.
- a) Write a multiplication sentence that can be used to find the number of miles Hannah hiked before she stops. Use the algorithm to show how to find the product.

- b) Use the number line to show how to find the answer to this problem.



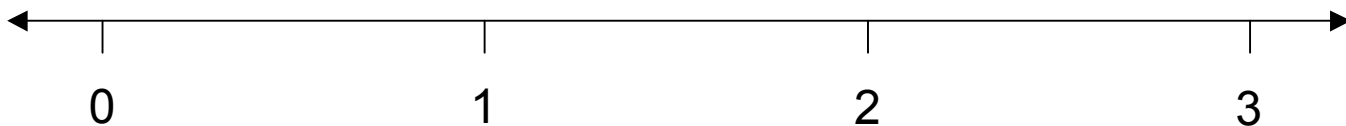
- c) Use the picture below to find the answer to the multiplication problem.



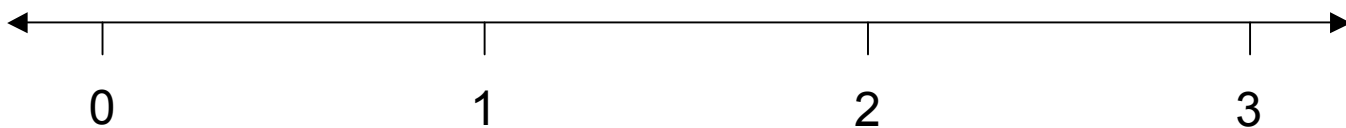
- d) Explain how multiplying 2 by 4 shows the number of 15^{ths} of a mile that were run by Hannah on both the number line and the picture.

Find the product of these multiplication problems using the algorithm and the number line.

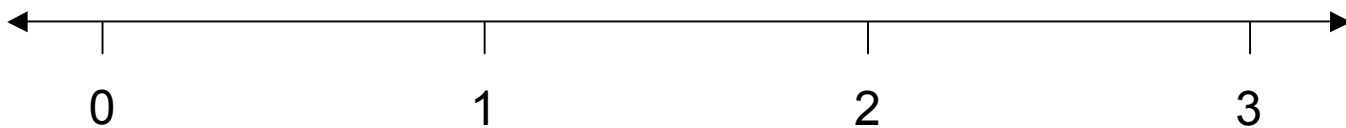
4. $\frac{3}{4} \times \frac{1}{2} = \underline{\hspace{2cm}}$



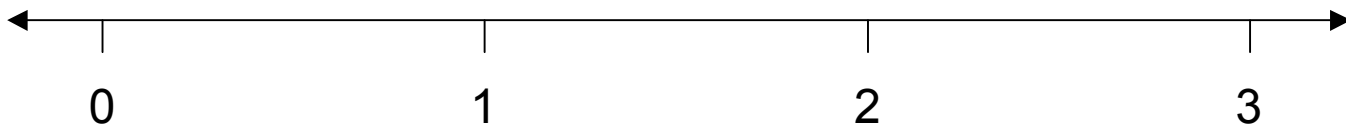
5. $3 \times \frac{2}{5} = \underline{\hspace{2cm}}$



6. $\frac{2}{3} \times \frac{3}{4} = \underline{\hspace{2cm}}$



7. $\frac{2}{5} \times \frac{4}{3} = \underline{\hspace{2cm}}$



Post Lesson Reflection

Lesson _____

1) Number of class periods allocated to this lesson: _____

2) Student Pages used: _____

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) To improve the lesson I suggest: