

Rational Number Project

Fraction Operations & Initial
Decimal Ideas

2009

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This material is based upon work supported by the National Science Foundation under
Grant No. 0628005

Fraction Operations & Initial Decimal Ideas Curriculum Module

Companion Module to -
*RNP: Initial Fraction Ideas**

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2009

* Originally published under the title: RNP: Fraction Lessons for the Middle Grades Level 1

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RNP Website: <http://cehd.umn.edu/rationalnumberproject/>

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The Rational Number Project (RNP)

The National Science Foundation initially funded the Rational Number Project in 1979. It started out as a cooperative project among three universities. The original members were Merlyn Behr, Northern Illinois University, Thomas Post, University of Minnesota, and Richard Lesh, Indiana University. Project personnel expanded over the years. Kathleen Cramer became involved in the RNP in 1980 and she along with Merlyn Behr developed *RNP: Fraction Lessons for the Middle Grades Level 1* module. We have updated that module and renamed it to better reflect its content: *RNP: Initial Fraction Ideas*. Terry Wyberg, University of Minnesota and Seth Leavitt, Middle School Mathematics Teacher from Minneapolis joined the RNP project with this latest funding cycle. The lessons in this module were created with NSF support from October 2006 through June 2009. They were created to be a companion module to the original one mentioned above.

The early work done by Rational Number Project personnel involved investigating how best to teach fourth and fifth graders initial fraction ideas. (Behr, Wachsmuth, Post, & Lesh, 1984; Bezuk & Cramer, 1989; Post). The curriculum mentioned above, *RNP: Fraction Lessons for the Middle Grades Level* (renamed as *RNP: Initial Fraction Ideas*) is the culminating product of this early work on teaching initial fraction ideas. Initial fraction learning includes developing meaning for fractions using a part-whole model; constructing informal ordering strategies based on mental representations for fractions; creating meaning for equivalence concretely; and adding and subtracting fractions using concrete models. Initial fraction understanding does not

include formal algorithms, and instruction with formal algorithms was not part of this earlier RNP curriculum module. The curriculum module, *RNP: Initial Fraction Ideas*, is available at the RNP website.

Studies have consistently shown that students using RNP lessons outperform students using traditional curricula on fraction assessments. Project personnel strongly believe that one reason the RNP fraction curriculum has been so successful is that the lessons emphasize translations within and between modes of representation. This idea is explained in more detail in the teacher guide.

Another factor is the extended amounts of time students spend with concrete models for fractions (Cramer & Henry, 2002). In addition, the curriculum invests more instructional time developing concepts, order, and equivalence ideas before working with the operations. Out of the 23 lessons in the first RNP module only five dealt explicitly with fraction addition and subtraction. Despite the limited time spent on fraction operations, assessments showed that RNP students performed as well as students using conventional texts on fraction addition and subtraction symbolic tasks while they consistently outperformed other students on estimation tasks involving fraction addition and subtraction (Cramer, Post, & delMas, 2002).

RNP: Fraction Operations and Initial Decimals Ideas Curriculum Module is a companion to this original set of lessons. This new module builds on experiences in the first set of lessons to help students build conceptual and procedural understanding for fraction operations. These lessons move into decimals and provide students

opportunities to develop meaning for decimals, order and equivalence ideas related to decimals, and ends with decimal addition and subtraction.

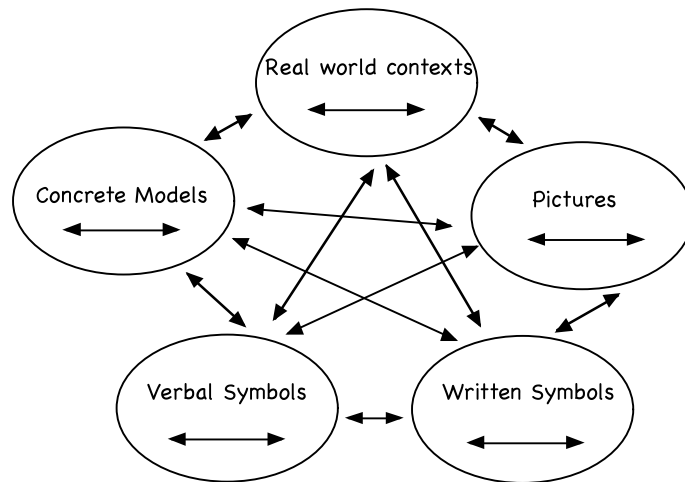
Both RNP curriculum modules were created using *Teaching Experiments*. In a teaching experiment, researchers go into the classroom as teachers and teach a classroom of students using an initial draft of the lessons based on a well thought-out theoretical framework. During the teaching experiment researchers study children's learning as they progress through the lessons. This is done with classroom observations, student interviews, and written assessments. The lessons are then revised based on what is learned about children's thinking. The *RNP: Fraction Operations and Initial Decimal Ideas* module went through two cycles of teaching experiments each 6 weeks in length. Then 8 sixth grade teachers in the same district implemented these lessons. Student interviews and written tests evaluated students' learning. Our analysis of these data and input from the eight teachers led to another round of revisions.

We are thankful to the teachers and students who partnered with us on this project. With their help we were able to create a curriculum that helps students develop a deep understanding of fraction operations and decimals.

Theoretical Framework

Children using the RNP lessons use a variety of manipulative models. They work in small groups talking about mathematical ideas and interact with their teacher in large group settings. They draw pictures to record actions with different models. They solve story problems using manipulatives to model actions in the stories.

This model for teaching and learning reflects the theoretical framework suggested by Jean Piaget, Jerome Bruner, and Zoltan Dienes. Richard Lesh, a founding member of the RNP group, built on the work of these three theorists and created a model to guide curriculum developers. Lesh suggested an instructional model that clearly shows how to organize instruction so children are actively involved in their learning. Consider the picture below. Lesh suggests in this model that mathematical ideas can be represented in the five ways shown. Children learn by having opportunities to explore ideas in these different ways and by making connections or translations between the different representations.



In RNP lessons, for example, students will solve story problems for fraction multiplication using pictures. They record their actions with pictures as number sentences. This is an example of a real world to picture to symbol translation. We believe that students need many opportunities exploring fractions and decimals using multiple modes of representation and translating between different representations. In

this curriculum, the major translations for each lesson are identified at the end of each lesson.

Lesson Format

The lesson format reflects a classroom organization that values the important role teachers play in student learning as well as the need for students to work cooperatively, talk about ideas, and use manipulative or pictorial models to represent fractions and decimals. Each lesson includes an overview of the mathematical ideas developed. Materials needed by the teacher and students are noted. The lesson begins with a class **Warm Up**. Warm Ups are used to review ideas developed in previous lessons and should take only 5-10 minutes of class time. There is a **Large Group Introduction** section in each lesson. The teacher's lesson plans provide problems and questions to generate discussion and target the exploration. **Small Group/Partner Work** is included in each lesson where students together continue the exploration of ideas introduced in the large group. The class ends with a **Wrap Up**. A final activity is presented to bring closure to the lesson. At times this will be a presentation by students of select problems from the group work. We found that students like to share their thinking. At other times the Wrap Up will be another problem to solve as a group.

Homework is provided in a separate section at the end of all 28 lessons.

Homework sets have been developed for many of the lessons to allow students to have extra practice and to further develop their understandings. Many of the homework sets have problems relevant to the lesson designated on the worksheets, as well as review items from previous work. While each homework worksheet designates a lesson that

students should have had prior to doing the homework, it does not have to be given the exact day the lesson is used in the classroom. It was not the intent of the writers that students do all homework sets, the choice of when to assign homework is left to the discretion of the teacher.

Manipulative Materials

Fraction circles are an important manipulative model used in the RNP lessons. Our work over 20 years has shown that of all the manipulatives available for teaching about fractions, fraction circles are the most effective for building mental images for fractions. These mental images for fractions support students' understanding of order, equivalence and fraction operations.

Students are introduced to other models in these lessons: paper folding for fraction equivalence, patty paper for fraction multiplication, 10 x 10 grid for decimals, and the number line for fractions and decimals. An important goal in creating this curriculum module was to determine effective models for teaching fractions and decimals and how to sequence the models. The models mentioned above proved effective in helping students build meaning for fraction and decimal concepts, order and equivalence, and operations with fractions and decimals.

Special Notes on Children's Thinking Related to Fractions

We want students to develop number sense for fractions and decimals. Underlying number sense for fractions is the ability to judge the relative size of fractions using what the RNP group has labeled as "Students Informal Ordering Strategies". Students using RNP materials have consistently constructed for themselves

powerful ordering strategies that are not based on common denominators. These ordering strategies are: same numerator, same denominator, transitive and residual.

When comparing two fractions like $\frac{3}{8}$ and $\frac{2}{8}$ (fractions with the same denominator) students using fraction circles will comment that both fractions use the same size pieces. Students will then reflect that 3 of the same-sized pieces are bigger than 2 of the same-sized pieces. This type of thinking is different than stating a rule given by the teacher; when students use fraction circles they construct the rule for themselves and use language closely tied to the manipulative model.

When comparing $\frac{2}{3}$ and $\frac{2}{6}$ (fractions with the same numerator) students can conclude that $\frac{2}{3}$ is larger because thirds are larger than sixths. Two of the larger pieces must be larger than two of the smaller pieces. This strategy involves understanding that an inverse relationship exists between the number of parts the unit is partitioned into and the size of the parts. This strategy also reflects the role of the numerator. An ordering decision based on the denominator only works if the numerators are the same. It is not enough for students to just say the denominator is smaller so the fraction is bigger. For example, when ordering $\frac{3}{4}$ and $\frac{1}{2}$ a student who only considered the denominators might conclude that $\frac{1}{2}$ is bigger because halves are bigger than fourths. This type of error is common among students who do not construct the same numerator strategy for themselves but uses a rule the teacher provides.

When comparing $\frac{3}{7}$ and $\frac{5}{9}$ a student can determine that $\frac{5}{9}$ is larger by using $\frac{1}{2}$ as a benchmark. $\frac{5}{9}$ is bigger than $\frac{1}{2}$ while $\frac{3}{7}$ is less than $\frac{1}{2}$. We found students

using fraction circles often used $\frac{1}{2}$ as a benchmark to judge the relative size of other fractions. This ordering strategy is termed the transitive strategy.

Students using fraction circles often construct one other strategy called the residual strategy. When comparing $\frac{3}{4}$ and $\frac{5}{6}$ students considered the amount away from one whole each fraction was to determine the larger fraction. $\frac{3}{4}$ is a fourth away from the whole while $\frac{5}{6}$ is a sixth away from the whole. Using the same numerator strategy students conclude that $\frac{1}{6}$ is less than $\frac{1}{4}$. $\frac{5}{6}$ is larger because it is closer to one whole. You should expect students in your class to construct these strategies. Students' construction of these strategies is one indicator of number sense.

Goals

The table below lists goals for the curriculum module:

| Fraction- Addition and Subtraction | Fraction- Multiplication and Division | Decimals |
|--|--|---|
| Estimate sums and differences building on mental images for fractions and operations on them. | Use fraction circles, patty paper, number line models, and pictures to multiply fractions. Be able to explain the process. | Use 10 × 10 grids to develop meaning for decimals based on fractions and place value idea. |
| Use fraction circles and number line models to add and subtract fractions. | Record process for multiplying fractions with the patty paper model using symbols to develop meaning for algorithm. | Develop order strategies based on mental images of concrete models to judge the relative size of decimals. |
| Record process with concrete models using symbols to develop meaning for the common denominator algorithm. | Estimate products building on mental images for fractions and what happens when they are multiplied. | Use 10 × 10 grids to understand decimal equivalence. |
| Use common denominator algorithm to add and subtract fractions and be able to explain the process. | Use multiplication algorithm to make sense of multiplying two fractions on a number line. | Use Decimal +- board to develop strategies for adding and subtracting decimals. |
| Use common denominator algorithm to make sense of addition and subtraction on a number line. | Use pictures to solve fraction division problems based on measurement interpretation and be able to explain the process. | Represent decimals and addition and subtraction of decimals on a number line. |
| | Record the division process with pictures using symbols showing the same denominator algorithm for fraction division. | Solve decimal addition and subtraction problems symbolically and be able to judge reasonableness of results using estimation. |

Our goals for fraction operations include being able to add and subtract fractions with like and unlike denominators using the standard algorithm. Students are considered to have this understanding if they can provide reasonable estimates to fraction addition and subtraction problems, if they can use fraction circles and the number line to add and subtract fractions, and if they can explain how the actions with those models corresponded to adding and subtracting fractions with symbols. We

believed that students' prior experience with the RNP fraction lessons for initial fraction ideas provides the foundation upon which these new skills can be built.

The initial goals for fraction multiplication and division included students being able to model both operations embedded in story problems with concrete models and pictures and to be able to explain the process of multiplying and dividing fractions with concrete models and pictures.

We built on students' understanding of whole number multiplication and extended it to whole number times a fraction, fraction times a whole number, and fraction times a fraction. We used fraction circles to help students construct understanding of whole number times a fraction; a number line to build meaning for fraction of a whole number; an area model (patty paper) to develop meaning for fraction times a fraction as well as the algorithm. We discovered that a number line model for fraction times a fraction is too complex a model for initially developing meaning and the algorithm for fraction times a fraction tasks. But we did find that once students learned the algorithm for fraction times a fraction, they were able to use that knowledge to make sense of fraction times a fraction on a number line. We believe this added translation to the number line makes the algorithm more meaningful to the students. We also learned that as students constructed meaning behind fraction multiplication, their understanding of what a fraction is evolved from a part-whole model or measurement construct to considering a fraction as an operator. This came naturally to students as they solve fraction multiplication tasks. For example when students multiplied $\frac{3}{4} \times 12$, they partitioned 12 into 4 parts and then multiplied this

result by three. When multiplying $\frac{3}{4} \times \frac{1}{2}$ on number line they would first show $\frac{1}{2}$ on then number line (measurement idea). They would partition this amount into 4 equal parts and iterate 3 of those parts; this is the operator idea

Story problems for division reflect a measurement model. While students should know that there are other interpretations for fraction division, we believe that the measurement context provides students the support they need to build meaning to fraction division using a common denominator approach. By translating from story problems to pictures to language students develop mental images that support their number sense for fraction division as well as meaningful symbolic work with fractions division.

The development of decimals follows a similar trajectory as the one constructed for teaching fractions found in the original RNP fraction lessons. Time is invested in developing meaning for decimals using multiple models. We use a 10×10 grid model and a number line for decimals. Students develop order and equivalence ideas using these models. The 10×10 grid in particular helped students developed strong mental images for decimals that in turn supported their work with decimal operations.

Addition and subtraction with decimals are developed using what we call a Decimal +-board based on the 10×10 grid. As you read through the table of contents you will see that fraction addition and subtraction procedure is developed first using fraction circle model. Decimals are introduced next using 10×10 grid and the number line. We return to fraction addition and subtraction using a number line after the work with decimals as we found students understood the number line model more easily with

decimals than with fractions. We build on students' understanding of the number line model for decimals to support their work with the number line for fraction addition, subtraction, multiplication, and division. There are 28 lessons in all.

Our Philosophy for Teaching the Algorithms

Algorithms are tools for understanding and doing mathematics. In order to effectively use these tools students need to have meaning for the numbers they are operating upon and have a sense of what happens to the numbers when a particular operation is used. In our lessons students spend a time building meaning for fractions and decimals. We rely on different models to do that: fraction circles, paper folding, 10 x 10 grids, and number lines. We rely on different models and contexts to help students understand what happens to numbers when added, subtracted, multiplied and divided. We pay careful attention to finding out which models were most effective for helping students understand the impact different operations have on the numbers involved.

To teach the common denominator algorithm for addition and subtraction we carefully help students build an understanding as to why common denominators are needed. We used fraction circles to do this. Students' understanding of equivalence (concretely and symbolically) also supports the steps to adding and subtracting fractions using common denominators. Students use their algorithm to solve problems in context. But the algorithm is not the end. In our lessons students use their common denominator algorithm to make sense of the number line model for adding and subtracting fractions. This step builds meaning for the algorithm and reinforces the procedure students have learned.

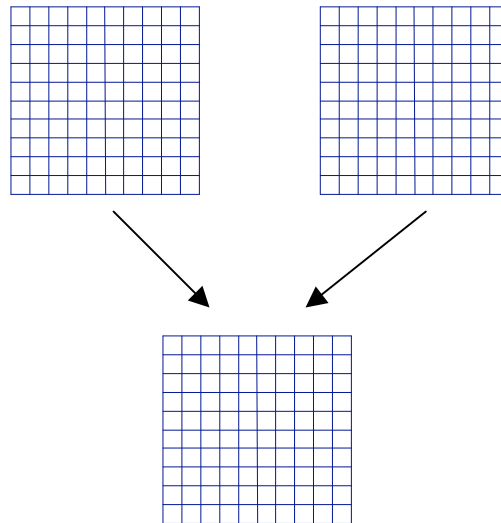
At the end of our lessons for fraction multiplication we believe students will have an extended view of what multiplication is. Students build on their understanding of multiplication of whole numbers and extend their understanding to fractions. We carefully sequenced the numbers involved and use of models so students move from multiplying whole number by a fraction to a fraction times a fraction. The models change as the numbers change. As with fraction addition and subtraction once students construct the multiplication algorithm they apply this procedure to a number line model. The procedure helps students make sense of this new model.

Overcoming whole number thinking students often apply to adding and subtracting decimals was a major goal of these lessons. Again, students first spend time building meaning for decimals before they operate on them. We constructed a new model to help students understanding how to add and subtract decimals. The model is a decimal +/- board and is shown on the next page. We found that using this model helped students see the need for “lining up the decimal points”. While we never used that language with students, they constructed this idea for themselves partly because of the decimal +/- board used in instruction. As with fraction operations, students applied their newly learned addition and subtraction algorithms to adding and subtracting on a number line. The connections students make among pictorial representations, the symbolic algorithm, and the number line only deepens students’ facility with decimal procedures.

Helping students develop algorithms for operating on fractions and decimals requires time. But we strongly believe the time invested in building meaning for

fraction and decimal operations is worthwhile if our goal is for students to use these tools to do mathematics.

Decimal +/- Board



Special Notes on English Language Learners

Martha Bigelow, Associate Professor of Second Languages and Cultures at the University of Minnesota, has read the curriculum and has shared some ideas of how to use the RNP lessons with English Language Learners (ELLs). She feels that the strength of the curriculum for ELLs is that it allows students to discuss their interpretation of a problem, formulate hypotheses, identify steps to finding a solution and give a rationale for their answer/ideas. These experiences will help them acquire the academic language of mathematics. The scaffolding of learning that takes place with the use of models and manipulatives are particularly beneficial to the students. The curriculum uses many of the things that literature suggests are good for ELLs when learning mathematics and other subjects (e.g., visualizing, using manipulatives).

With the above in mind, here are some ideas based on Dr. Bigelow's review of these lessons for RNP teachers to keep in mind when using this curriculum with ELLs:

- Encourage students to talk with peers of the same native language to process concepts.
- Identify ways that students who display understanding, but cannot verbally articulate understanding, can participate in group work (e.g., draw pictures, put things in order, categorize things that are similar). Through the use of fraction circles, paper folding, number lines and pictures ELLs have many opportunities to show their thinking in ways other than words and symbols.
- The RNP lessons include many opportunities for class presentations. It is important that ELLs who seem to be speaking English and understanding concepts of the lesson are invited to give answers in whole-class discussion.
- Some ELLs have not had a great deal of experience with cooperative learning and need help with how to make this learning format work for them. Assigning roles and reminding them to use their peers to help answer questions, rather than always the teacher, is important.
- Though calculators are only used briefly in only one of the lessons, do not assume that all ELLs have experience using a calculator.
- It is important to invite ELLs to participate and give them the floor in math classes. You can let them know ahead of time that you are going to ask them to contribute at a certain point, if you think they need to prepare a little more.

- The reading that students need to do in mathematics classes has to be very close and careful. This is different than in other classes where they often read for the gist. ELLs often need to be reminded to look at every word and make sure they really understand every word in the word problems. When working on story problems there should be an emphasis on the action in the story to uncover the operation.

Assessment issues:

While the language in the curriculum is very important, it might present a barrier to assessing students because it is sometimes difficult to figure out if the answer is wrong because the word problem was in English or because the student does not understand the operation/concept. One way to be sure is to have a couple word problems offered to the student in the student's native language, if he/she reads in that language.

Building confidence:

For students with limited or interrupted formal schooling, it may be necessary to do a bit more to remind them of what they already learned to give them more confidence that they know what is going on in class. You can ask them things like this: Have we had similar problems before? How did we solve them? Were any strategies useful? Remind them to use positive self-talk (e.g., Yes, I can do this. I did it before and I can do it again. I know when I'm stuck. I know how to ask for help.) The RNP group has found that ELLs do particularly well with the decimal models; teachers can build on these student successes in more challenging work.

Language in the curriculum

Some words in the curriculum have a specific meaning in Mathematics and also have an everyday word: *table, round, times*. This is tricky for ELLs because the word may seem familiar, but then in math class there is a new meaning. It is best to make the new meaning explicit.

There are a number of skills that are important across the language modalities. Items that the ESL teacher may wish to work on with students, in addition to the math teacher. It is important to be aware that these are the ways language is being used in the class:

Listening:

- Understand explanations with and without concrete referents.
- Understand oral numbers.
- Understand oral word problems.
- *Reading:*
- Understand specialized vocabulary
- Understand word problems
- *Speaking:*
- Answer questions and explain your answer
- Ask questions for clarification
- Explain problem-solving procedures (e.g., "...because")
- Describe applications of concepts (e.g., "if...then")
- Compare and contrast (e.g., uses "greater/less than", "as...as")

- Estimate and answer verbally
- *Writing:*
- Write verbal input numerically
- Write word problems
- Estimate and answer in writing

Again, ELLs will benefit from the use of concrete models and the opportunities to interpret problems, develop hypotheses, work towards a solution, and justify their conclusions. Though it is important that the teacher invite ELLs to contribute to class discussions and use their successes as a way to connect future ideas.

Final Comments

At the end of each lesson you will find a form to record your adaptations for each lesson. Any curriculum will need to be “personalized” by the teacher who uses it so it best meets the needs of his/her students. This form will act as a reminder about changes you feel are important to make the next time you teach the lesson.

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RNP: Fraction Operations and Initial Decimals Ideas Scope and Sequence

| | Lesson | Topic Overview | Materials | Homework |
|--------------------------|--------|--|----------------------|----------|
| Review fraction circles | 1 | Students review how to order fractions using informal strategies. Students compare unit fractions, use $\frac{1}{2}$ as a benchmark and compare fractions close to one. | Fraction Circles | * |
| | 2 | Students review equivalence ideas with paper folding. Students develop “multiplication” rule from paper folding. | Paper folding strips | |
| Addition and Subtraction | 3 | Students use order and equivalence to estimate sums and differences. | Fraction Circles | * |
| | 4 | Students use fraction circles to construct their own plan for adding two fractions. Students explain their plan and show a way to record the steps of their plan symbolically. | Fraction Circles | |
| | 5 | Students transition from using fraction circles and symbols to adding fractions using only symbols. Connections between concrete model and symbols are emphasized. | Fraction Circles | |
| | 6 | Students build on their understanding of adding fractions with symbols to subtract fractions using fraction circles and symbols. | Fraction Circles | * |
| | 7 | Students solve story problems involving addition and subtraction. The subtraction stories go beyond the take away model used in lesson 7. How much more and compare problems are introduced. | Fraction Circles | * |
| | 8 | Students have developed a strategy for adding and subtracting fractions using their equivalence ideas in previous lessons. This lesson extends their work to special cases: fractions > 1 ; sums > 1 ; differences with fractions > 1 ; sums of more than two fractions. | | * |

* Homework available for use after this lesson is completed

| | | | | |
|----------------|--|---|--|-------------|
| Decimals | 9 | Students create a model for decimals using 10 x 10 grids to show tenths and hundredths. They record amounts in words, fractions, symbols, and decimals | 10 x 10 grids | * |
| | 10 | Students develop an understanding of thousandths and begin to look at equivalence among tenths, hundredths, and thousandths. Students develop decimal order strategies by identifying the larger of two decimals, by sorting sets of decimals and by finding a decimal between two decimals | 10 x 10 grids | * |
| | 11 | Students estimate sums and differences using mental images of 10 x 10 grids. Students develop strategies for adding and subtracting decimals using Decimal +/- boards. Students find exact answers to decimal addition and subtraction using mental math. | Decimal boards | * |
| | 12 | Students review ordering and equivalence and practice adding and subtracting decimals in problem solving contexts. | | |
| | 13 | Students use a meter stick as a model for decimals by connecting this new model to the 10 x 10 grid model. | Meter stick | * |
| | 14 | Students model decimal addition and subtraction problems using a number line, 10 x 10 grid and symbols. | Number line and decimal +/- boards | * |
| | Fraction Addition and subtraction with number line | 15 | Students learn to show fractions on a number line. | Number line |
| 16 | | Students make connections between adding and subtracting fractions using a symbolic procedure to adding and fractions using a number line. | Number line | * |
| Multiplication | 17 | Students are able to use fraction circles to find the product of a whole number and a fraction. Students are able to explain that the expression $a \times b$ can be read as "a groups of b". (whole number x fraction) | Fraction circles | * |

| | | | | |
|-------------------------|----|---|-----------------------------|---|
| | 18 | Students are able to multiply a whole number and a fraction using fraction circles, drawing pictures, and using mental images. (whole number x fraction) | Fraction circles | * |
| | 19 | Students will use number lines to multiply a fraction and a whole number. (fraction x whole number) | Number line | * |
| | 20 | Students will use number lines to multiply a whole number by a fraction. (fraction x whole number) Students will be able to explain the differences between multiplication involving a whole number of groups and a fractional number of groups. | Number line | * |
| | 21 | Students will use patty paper (area model) to multiply two fractions. (fraction x fraction) | Patty paper | * |
| | 22 | Students will use patty paper (area model) to multiply two fractions. (fraction x fraction) Students will develop the algorithm for multiplying fractions by noticing patterns. | Patty paper | * |
| | 23 | Students will multiply fractions using a number line. (fraction x unit fraction) | Number line sheets. | * |
| | 24 | Students will use a variety of models and the algorithm to multiply a fraction by another fraction. Students will describe connections among the number line, pictures, and the algorithm. | Patty paper and Number line | * |
| Division with fractions | 25 | Students solve measurement division story problems by using fraction circles and pictures. Students explain their solution strategies. Story problems involve whole numbers divided by a fraction <1 ; mixed numbers divided by a fraction <1 ; fraction <1 divided by another fraction <1 . All answers are whole numbers. | Pictures | |
| | 26 | Students solve measurement division story problems by using pictures. Students explain their solution strategies. Story problems mixed numbers divided by a fraction <1 ; fraction <1 divided by another fraction <1 . All answers include fractions. | Pictures | * |

| | | | |
|----|---|----------|---|
| 27 | Students solve measurement division story problems by using pictures. Students write division sentences with common denominators. Students build on their experiences with different models and contexts to estimate quotients to fraction division problems. | Pictures | |
| 28 | Students find estimates to fraction division problems. | Pictures | * |