Lesson 16: Overview
Students extend their fraction concepts by reconstructing the unit when given the fraction part.

Materials
- Fraction Circles for students and teacher
- Student Page A

Teaching Actions

Warm Up
Order fractions from smallest to largest. Explain your thinking.

\[
\begin{align*}
\frac{6}{7} & \quad 2 & \quad \frac{99}{100} & \quad \frac{9}{10} & \quad \frac{3}{4}
\end{align*}
\]

Large Group Introduction

1. Lead a discussion around the concept of unit. Possible questions include:

   - To show \(\frac{1}{3}\), what possible units could I use if I used fraction circles?
   - If I used chips, what units could I use?
   - If I used paper folding, how would I show \(\frac{1}{3}\)?
      [Use paper as a unit; partition into equal-sized parts; highlight a certain number of parts]

2. Explain that so far we have done a lot of problems in which we started with a unit and divided it into equal sized parts. Now we will reverse the process. You will know one or more of equal-sized parts and have to find the unit.

3. Model the idea of reconstruction the unit. Show 1 pink piece and say that this is 1 of 3 equal parts – it is \(\frac{1}{3}\) of some amount, some unit.

4. Show \(\text{pink piece}\) and ask: because this is 1 of 3 equal sized parts, how many more parts do I need to build a whole unit? What size parts do I need? (All must...
Teaching Actions

be pink).

5. Place and count

\[ \begin{array}{ccc}
\text{pink} & \text{pink} & \text{pink} \\
1 \text{ part} & 2 \text{ parts} & 3 \text{ parts}
\end{array} \]

The whole unit is 3 pinks or 1 yellow.

We know that 1 pink is \( \frac{1}{3} \) of 1 yellow. We found the unit starting with \( \frac{1}{3} \) of it.

6. Ask students to take out 1 blue piece. State that this blue is \( \frac{1}{4} \) of some whole unit.

7. Ask: Will the unit be bigger or smaller? How many fourths make a whole unit? Use your circles to find the unit.

8. Repeat for these pieces and values:

\[
\begin{align*}
\text{gray} &= \frac{1}{4} \quad \text{[unit is yellow]} \\
\text{red} &= \frac{1}{3} \quad \text{[unit is blue]} \\
\text{pink} &= \frac{1}{2} \quad \text{[unit is brown]} \\
\text{red} &= \frac{1}{4} \quad \text{[unit is brown]} \\
\text{gray} &= \frac{1}{2} \quad \text{[unit is blue]}
\end{align*}
\]

9. Present this example and explain that it is tricky:

1 gray = \( \frac{1}{3} \). Find the unit.

Ask the student to explain how to construct the unit. Then ask how we can describe the unit. Is there 1 piece to cover this amount? Since there isn’t, students can name the unit as 3 grays. You can trace the 3 grays and say this amount is the unit.

Comments

Have students do these independently at their desks and then have them verbalize the process in a large group.

In each of the examples so far, the answer could be expressed as a single piece.

Ex: 1 gray = \( \frac{1}{4} \), so the unit equals 4 grays or 1 yellow.

This won’t always be the case. If 1 blue = \( \frac{1}{3} \), then the unit equals 3 blues.
### Teaching Actions

10. Repeat for this example:
   \[
   1 \text{ blue} = \frac{1}{3}. \quad \text{Unit} = ?
   \]

### Small Group/Partner Work

11. Student Page A provides practice.

### Wrap Up

12. End the class with this problem:
   
   I know that this blue piece is half of something. How can I find the value of these pieces?
   
   1 red 2 pinks
   1 gray 2 yellows

13. Accept suggestions. Then model a solution by reconstructing the unit (using their new skill). Show
   
   \[
   \[
   \text{Explain that if } \frac{1}{2} \text{, then } \frac{1}{6} \text{ and make the unit. The unit is 1 yellow:}
   \]
   
   Show 6 reds covering 1 yellow so 1 red is \(\frac{1}{6}\).
   
   Now ask students to do the other 3.
   
   \[1 \text{ gray } = \frac{1}{4}; \quad 2 \text{ pinks } = \frac{2}{3}; \quad 2 \text{ yellows } = 2\]

14. Repeat for this problem: \(\text{blue } = \frac{1}{3}\). Find the value of these pieces.
   
   1 red 1 yellow
   1 gray 9 reds
   
   \[1 \text{ red } = \frac{1}{9}; \quad 1 \text{ yellow } = \frac{2}{3}; \quad 1 \text{ gray } = \frac{1}{6}; \quad 9 \text{ reds } = 1\]

### Translations

- Written symbols to manipulative to verbal
- Written symbols to manipulative to written symbols

### Comments

No single piece covers 3 blues. Some children may express the unit as 3 blues; others as 1 yellow and 1 blue. Others will think their reasoning is wrong because there isn’t one piece to cover 3 blues.

To name the other pieces, student should find the unit and then compare the other pieces to that unit. Don’t try to solve this abstractly:

blue = \(\frac{1}{2}\)
red is \(\frac{1}{3}\) of blue, so \(\frac{1}{2}\) of \(\frac{1}{3}\) = \(\frac{1}{6}\).

This problem is challenging. Step back and let students problem solve. Ask them to explain their strategies and reasoning.
Order fractions from smallest to largest. Explain your thinking.

\[
\frac{6}{7} \quad \frac{2}{3} \quad \frac{99}{100} \quad \frac{9}{10} \quad \frac{3}{4}
\]
Problem Solving and Fraction Circles

I. Find the unit given the following information. Explain how you solved the problem. [You may want to draw pictures]

a) The red piece is \( \frac{1}{4} \) of some amount. Find that amount. ______

b) The gray piece is \( \frac{1}{6} \) of some amount. Find that amount. ______

c) The green piece is \( \frac{1}{5} \) of some amount. Find that amount. ______

II. If the pink piece is \( \frac{1}{4} \) what value do these have? Explain your reasoning.

   a) 1 brown     b) 1 red     c) 1 white

Challenge: If the yellow piece is \( \frac{2}{3} \) what value does one gray piece have?