Rational Number Project

Initial Fraction Ideas
Lesson 16: Overview
In order to make sense of the number line model for fractions students will make connections between the paper folding model for fractions and the number line model for fractions.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 8.5” by 11” paper strips for students and teacher</td>
</tr>
<tr>
<td>• Student Pages A - C</td>
</tr>
</tbody>
</table>

Teaching Actions

Warm Up
Order these fractions from smallest to largest:

\[
\frac{3}{4} \quad \frac{1}{2} \quad \frac{3}{3} \quad \frac{4}{8} \quad \frac{1}{10}
\]

Large Group Introduction

1. Present this story: The distance between your home and the Dairy Queen is only 1 mile. Using one paper strip to represent 1 mile, represent 1/3 of a mile. Then use another paper strip to model ¾ of a mile.

2. Ask students to justify why each fraction strip they folded represents the fraction of the mile. Encourage students’ language that communicates they understand that the strip is the whole unit and represents 1 mile. Students should be able to explain that each unit or mile was partitioned into a certain number of equal parts based on the denominator; amount shaded is based on the numerator.

3. Use Student Page A for students to follow along as you complete steps 4 – 6 of the lesson.

Comments

Note: This is a 2-day lesson.

Please see teacher notes at the end of the lesson for a more detailed discussion of strategies for building meaning for the number line model.

There is a lot of detail in this lesson. Please read through carefully to make sure you understand the process for developing meaning for the unit and how to interpret partitioning on a number line.

Lesson 16 emphasizes identifying the unit and how to count partitions within the unit. Instead of going directly to the number line, students construct a “number line” first with multiple paper strips lined up next to each other.
Teaching Actions

4. Take out a paper strip and show fourths. (Or develop pictures of paper strips to use with your Smart Board). State again the story context: distance between their home and Dairy Queen is 1 mile. State that the one strip is your unit or whole and stands for one mile. Tape the strip on the board for all to see (You may want to use larger paper strips like 2” by 14”). A picture of this is in the first box in Student Page A.

5. Label the start of the paper strip zero miles. Ask students what 0 mile represents (home: you might draw a picture of a house on top of the strip at 0). Ask: Where should you put the number 1 to show the distance of 1 mile modeled with the paper strip? (You might want to write DQ on top of the strip at 1 mile). How many equal parts is the unit partitioned into? What is another name for 1 mile based on those partitions? (4/4) What is another name for 0 miles based on the partitions? (0/4)

6. Ask: You walked ¼ of a mile from your home to the Dairy Queen. (You may want to act this out by actually walking in front of the picture stopping at the ¼ fold.) Ask students to use their pencil point to walk that ¼ mile. Where are you on the paper strip? Point to the spot on the paper strip showing where you are. What fraction of a mile is it from 0 to that
**Teaching Actions**

**first partition?** If I shaded 1 of the four equal parts on the paper strip, what fraction of the whole strip is shaded? Label the **fold line** at the end of the shaded ¼ part the number ¼. Repeat for 2/4, ¾ and 4/4. (Students may label the boxes instead of the fold lines. As we are focusing on the attribute of length and the value of the point at the end of the length, encourage students to label the fold lines and not the boxes.)

![Diagram showing 1 mile divided into quarters](image)

7. Ask: What is another name for 4/4 miles? How many miles is that? Label that amount under 1 mile.

8. Explain that sometimes students have trouble counting up the number of equal parts. One way to count up the total number of equal parts is to count the number of equal parts using loops. Tell the students that another third grader showed the class how to do this as shown below. Ask: How many jumps to the end starting at zero?

![Diagram showing loops](image)

9. Repeat for paper strips folded into halves and thirds (Problems 2 and 3). **Do not do Problem 4 at this point.** Continue to use the context of 1 mile as you

**Comments**

Students are encouraged to count up the partitions by using loops if they miscount the number of equal parts by counting all the tick mark instead of the number of lengths.
## Teaching Actions

partition a fraction strip to show ½ mile or thirds of a mile.

## Comments

Observe how students solve the problems, forming side groups for students who may need support. Encourage students to talk aloud to think how to count up the number of equal parts. Do students focus on counting the equal lengths between the fold lines?

This is a good place to end the lesson; pick up with the Large Group section the next class period. You may want to have a quick closure by sharing answers to Problems 1 and 3 on Student Page B.

## Small Group/Partner Work

10. Assign Student Page B to students to practice the ideas so far developed. Problems 2 & 4 ask students to consider fraction greater than one. Observe how students process these tasks and use that information to start the next large group presentation.

## Large Group

11. Go back to Student Page A, problem 4. Present this context: The distance from your house to Cub foods is 2 miles. Take another strip of paper divided into fourths. Tape this one next to the 4th strip already on the board (or set this up on your Smart Board). Ask: How should we label this strip? What does the beginning of the strip represent? (Your house) What number should we use to label that spot? (0) Why? If each paper strip represents 1 mile and we stop at this point (1 mile marker), what number should we put here? Why? Let’s keep walking to the end of the strip where Cub is located. What number should I put here?

![Number Line](image)

0 miles | 1 mile | 2 miles

12. Ask: How can we label the partitions between 0
Teaching Actions

miles and 1 mile? What about 1 mile and 2 miles? (Encourage language like: “1 mile and ¾ more”. Students might want to label using improper fractions; don’t rush this idea as just labeling as mixed numbers is challenging enough at this point).

![Diagram of a strip divided into segments with labels: 0, ¼, ½, ¾, 1, 1¼, 1½, 1¾, 2 miles]

13. Now look at problems 2 and 4 from Student Page B. Ask students to check their work and make changes as needed. Then have students explain how they counted the number of equal partitions and why the fraction is located between 1 and 2 miles on the strip.

Small Group/ Partner Work

14. Assign Student Page C

Wrap Up

15. Share students’ solutions to problems from Student Pages C. Look for errors mentioned in the teacher notes section of this plan. Make the errors public so students can adjust how they count the equal parts of the paper strip shown as a picture. Ask student to explain how they determined the number of partitions? Ask: Did anyone find an effective strategy that didn’t use the loops?

Translations

- Context to Manipulative to picture to verbal
- Context to Manipulative to picture to symbol
Teacher Notes for Number Line Lessons 16-18

Students have difficulty making sense of a number line as a model for fractions. Because this model is not as concrete as the other models students use to develop meaning for a fraction as a number, the model is not introduced until students have a good grounding in the part-whole model using fraction circles, paper folding and chips. We use contexts related to length to help students make sense of the unit on the number line.

As you teach the number line lessons, you should consider what is distinctive about this model. To make sense of the number line as a model for fractions students have to coordinate visual and symbolic information. This coordination involves three major ideas: (a) making sense of the unit; (b) partitioning a length; and (c) identifying the fraction as a number located as a point on the number line.

The visual information includes numerical symbols and tick marks or points that provide the students with clues to identify the unit on the number line, the number of equal partitions, and the location of the fraction as a number on the line in relation to zero.

For example, to locate the number $\frac{3}{4}$ on a blank number line a student should construct a unit on the line as a distance from 0 to 1 marked out with points or tick marks on the line. To locate fractions $> 1$ additional units can be added on the number line by iterating that length from 1 to 2 and so on with points or tick marks to designate these additional units. Multiple units are represented on a number line, and the units are continuous with no separation.

Partitioning the unit into equal parts is the next step. One way to partition the unit is to first divide the length between 0 and 1 into equal lengths using tick marks or points on the line. To show $\frac{3}{4}$, a student would mentally separate the unit length into 4 equal parts, indicating this partitioning using 3 marks (not 4) between 0 and 1. Then a student would count over 3 equal lengths (or the value in the numerator) to interpret the
fraction \( \frac{3}{4} \). Students are interpreting \( \frac{a}{b} \) using a part-whole construct onto a length; it involves partitioning into equal parts first, and then iterating or counting over a certain number of equal parts.

To show \( \frac{3}{4} \) in another way a student could iterate a unit length (1/4) across the length of 0 to 1, using tick marks or points to mark off the lengths. The student would have to estimate the size of the unit length so, for example, four, 1-fourths are equally spaced across the unit length with \( 4 \times \frac{1}{4} = 1 \). This strategy involves knowing that the iteration of four, \( \frac{1}{4} \)'s must exhaust the unit length. Here students are interpreting the fraction \( \frac{a}{b} \) as \( a \times \frac{1}{b} \).

Finally a student must understand that the point or tick mark at the end of the 3 equal lengths counted from 0 is the number \( \frac{3}{4} \). The number line shows that \( \frac{3}{4} \) is a number between 0 and 1.

**Misconceptions: Understanding the Unit**

It is helpful for a teacher to be aware of common errors students make while learning to model fractions on the number line. Error students make include, misunderstanding what the unit is on the number line. For example, this student circled 3 when asked to locate \( \frac{3}{4} \) on this number line. This error shows that the student thought that the unit is the whole number line shown and not the length between 0 and 1. When using the number line students need to bring to the task their understanding of the relative size of a fraction. In this case \( \frac{3}{4} \) is less than one but greater than zero. Therefore is a number between 0 and 1.
Here is another example of student’s misunderstanding of the unit. When asked to label the tick marks this student ignored the meaning behind the numbers 1 and 2 on the number line and considered the whole line as the unit.

Student’s issue with the unit becomes evident when you ask them to create their own number line. Initially students might identify a unit with symbols for 0 and 1 when placing a fraction on the number line. For example, this student located $\frac{3}{4}$ on the blank line as shown below. She explained as follows: “I drew four things because there are four loops. I did $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$”. When asked what the whole unit she said, “the whole is like 4”. She iterated $\frac{1}{4}$ from the start of the number line, and proceeded to count on.

While she focused on lengths of her partitions (rather than actual number of tick marks), her fractions were not presented in relationship to the numbers 0 and 1.

In this example, the student located thirds without designating the unit using symbols 0 and 1.
Another misunderstanding is to locate 0 and 1 on the number line at the arrows. This showed students did not see 0 and 1 as numbers on the line.

Notice that this student did not “use up” the whole unit, but just counted by fourths with equal spacing.

With a number line students have to coordinate symbolic and visual information in ways not required with more concrete models. Zero and one in particular are necessary symbols to identify the unit length on the number line. With other models students can be flexible in selecting a unit to partition to model a fraction. A student can use 8 chips or 12 chips to build a model for \(\frac{3}{4}\); a student can use a complete circle or half a circle to partition to show \(\frac{3}{4}\) as 3 of 4 equal groups. With these models the choice for the unit is implicitly connected to the number one; often words like “whole” or “unit” are used instead of saying “one”. With a number line, the unit is explicitly a length between two points labeled 0 and 1. While the length between 0 and 1 may vary, the unit on the number line is not the entire length of the line or a length between 0 and 4 for example. Part of making meaning of the number line is reinterpreting what is meant by the “whole”, the “unit” and the locations of zero and one.

Obviously making sense of the unit is an important concept for students to master if they are to reinterpret fractions on a number line. Emphasizing what the unit is becomes an important part of instruction. In the RNP number line lessons we rely on story contexts as a way to make sense of the unit on the number line. We build on students’ understanding of the relative size of fractions so they consider the size of a fraction like \(\frac{1}{4}\) as a number less than one before they locate the fraction on a number line.

**Misconceptions: Partitioning**

Partitioning errors are common. Students often count the tick marks incorrectly. For example consider this student’s error:
In this response the student appeared to count the number of tick marks between 0 and 1 and 1 and 2 rather than the number of equal intervals between 0 and 1 and 1 and 2. In addition the students did not interpret 2/2 as 1 or 1 2/2 as 2. This error is similarly seen in this student’s response. Here the student counted 3 tick marks instead of counting the lengths between the tick marks.

Partitioning a number line is complex. To partition a length on a number line to locate a fraction, students have to focus on constructing equal lengths between two whole numbers using points or tick marks to do this. The number of tick marks is one less than the number of partitions needed (as designated by the denominator of the fraction). Students are constructing equal lengths (continuous idea) using tick marks (discrete idea). The equal parts constructed with the tick marks need to exhaust the length from 0 to 1. These tick marks need to be interpreted as tools to partition the unit length but also as designations for actual numbers, whole numbers and fractions.

Partitioning with other concrete models is primitive in comparison. With fraction circles, students find pieces of the same color that partition a unit into equal parts; with chips they sort chips into equal groups; with paper folding they physically fold paper into partitions. These models are more forgiving when errors are made; students just clear the workspace and try again. With experience students remember the colors that partition the circles into equal parts; with chips students realize that multiples of the denominator are sets that can be partitioned; with paper folding, sequences for folding paper become internalized. The number line requires the students to spend more time planning their physical actions before making them.
Importance of the Number line as a Model for Fractions

Despite the challenges of this model, the number line provides students the opportunity to consolidate and extend their rational number knowledge they have garnered with other models. The number line provides students the opportunity to represent their understanding of fraction as a number onto a new model that involves explicitly attending to fraction values in relation to each other, and to whole numbers. While the development of the part-whole interpretation is important, the number line interpretation applies understandings of order, equivalence, and magnitude represented within our entire number system. This raises the question as what prior understandings are needed to make sense of this model.

In previous work with fraction operations, RNP found that 6th graders struggled using a number line to build meaning for the steps involved in adding and subtracting fractions, in particular seeing the rationale for a common denominator. Students who successfully used the number line to operate with fractions were guided by knowing the procedures before instruction. Those who did not know the procedures struggled to make sense of them using the number line.

The number line might not be an embodiment to build initial ideas for operating with fractions, but one to apply conceptual and procedural skills constructed with other models. In other words, the number line is a model that gives students a chance to coordinate the knowledge that they have built with more concrete representations. When sequenced after students used other models, students then have the opportunity to apply and extend their rational number knowledge. It is for this reason the number line lessons end the grade 3 fraction unit.
We see the number line as a model that solidifies students’ rational number knowledge built with more concrete models. In grade 3 students translate their understanding of unit and partitioning using fraction circles, chips and paper folding to a more complex model, the number line. In doing this, students solidify their understanding of fraction as a number in relationship with other numbers on the number line.

**The Number Line Lessons**

The number line lessons reflect our understanding of the complexity inherent in the number line model and common misconceptions students have when first working with this new model. The lesson activities are orchestrated to help students overcome these errors. Building on what we learned from students’ thinking and their struggles related to the unit, partitioning, and interpreting a fraction as a number in relationship to other numbers, the number line lessons have been revised again. The original lessons introduced the number line as a translation from the paper-folding model to the number line. This translation idea is fundamental to this particular curriculum. In the new lessons we emphasize multiple translations.

To address students’ issues with reinterpreting what a unit is on the number line, as well as the locations of 0 and 1 on the line, the new lessons now use a translation from context to paper folding to the number line to introduce the number line. The contexts involving length give meaning for 0 and 1 as positions that represent a named unit such as miles as well as help students focus on lengths between tick marks as the salient feature of the partitioning. The contexts also emphasize the naming of the tick marks themselves as locations that are the labeled distances from zero. The lessons leading up to the number line lessons were revised so students encounter more explicitly fractions
equal to 0 and 1 with the other models. More time is spent with fractions greater than one with the other models so students experience the partitioning action with multiple instances of a given unit while naming these representations in terms of a given amount as one, as they need to do with the number line.

The lessons were revised to address students’ use of jumps along the number line to partition the unit. A common error students made was not to exhaust the unit length. This may be a carry over from students work with the number line in younger grades with whole numbers. Young children are often encouraged to jump along the number line making loops as they leap from number to number. This method works for skip counting, adding numbers by counting on, or subtracting whole numbers. But with partitioning the unit on a number line to locate fractions, students must look at the entire interval, and determine the size of the partitions. In essence, students need to “look before they leap” or plan ahead before making equal jumps. In the revised lessons, different ways of partitioning the unit are explicitly addressed. In particular we added opportunities for students to reinterpret partitioning by asking students to translate not only from paper folding to the number line, but from partitioning fraction circles to partitioning the number line. We found to meet the needs of more students we needed to provide multiple opportunities to make sense of how to partition the number line.

Please note that understanding how to model fractions on a number line is more complex than naming ½ on the number line that has been partitioned for the students. Students will be able to identify ½ on the number line but not be able to identify fractions in fourths or thirds. Students will be able to identify halves, thirds and fourths on a pre constructed number line before they can consistently create their own number
line for a given fraction. Below find different number line tasks you can use to informally students’ understanding of the number line.

<table>
<thead>
<tr>
<th>Task with premade number line</th>
<th>Task with blank number line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label the tick marks on the number line. Explain how you figured this out.</td>
<td>Now show me 1/3, 2/3 and 3/3 on this number line. Explain your strategy for constructing the number line to show these three fractions.</td>
</tr>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
</tr>
<tr>
<td>Look at this number line. Where is ¾ on the number line? How do you know? Tell me what you are thinking.</td>
<td>Explain your strategy for constructing the number line to show 3/4.</td>
</tr>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
</tr>
<tr>
<td>This is a number line. What can you tell me about the tick marks?</td>
<td>Construct the fraction 1 ¼ on the number line. Explain your strategy for showing 1 ¼ on the number line? (If the student shows 0 and 1 ask, why she/he did this.)</td>
</tr>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
<td><img src="https://via.placeholder.com/150" alt="Number Line" /></td>
</tr>
<tr>
<td>What number names points A and B. How do you know? Tell me what you are thinking.</td>
<td></td>
</tr>
</tbody>
</table>
Order these fractions from smallest to largest.

Be ready to explain your thinking.

\[
\frac{3}{4}, \quad \frac{11}{2}, \quad \frac{3}{3}, \quad \frac{4}{8}, \quad \frac{1}{10}
\]
Problem 1

Problem 2

Problem 3

Problem 4
<table>
<thead>
<tr>
<th></th>
<th>This picture represents the length of a ribbon that has been folded into equal parts. If you made a cut at the 1-fourth partition, what fraction of the yard is that? Label the picture with that fraction.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1.png" alt="Diagram of a yard into fourths" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>This picture represents 2 miles. If you ran 1 mile and 1/3 more where would that fraction be on this representation?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><img src="image2.png" alt="Diagram of miles" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>You have a licorice strip that is 1-foot long. You are sharing it fairly between yourself and two friends. Label the amount that you will receive below. What fraction of the licorice will you get to eat?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image3.png" alt="Diagram of feet" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>You walk 2 miles to school everyday. You walked 1½ miles when your friend joined you for the rest of the way to school. Show on the picture of the paper strip where 1½ miles are.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td><img src="image4.png" alt="Diagram of miles" /></td>
</tr>
</tbody>
</table>
With your partner, draw a picture of paper folding strips to show each fraction.

<table>
<thead>
<tr>
<th>You ran 1½ miles from school to your home. Model that amount using a picture of paper folding strips. If one paper strip represents one mile, how many units will you need to show 1½ miles?</th>
</tr>
</thead>
<tbody>
<tr>
<td>You cut a length of rope that is $1\frac{1}{3}$ feet in length. Model that amount using a picture of paper-folding strips. If one paper strip represents one foot, how many units will you need to show this amount?</td>
</tr>
<tr>
<td>Your long jump was 2 and $\frac{3}{4}$ yards. Model that amount using a picture of paper-folding strips. If one paper strip represents one yard, how many units will you need to show this amount?</td>
</tr>
<tr>
<td>You ate $\frac{3}{4}$ of the subway sandwich. One paper strip represents a whole subway sandwich. Show how much you ate. How many units will you need to show this fraction?</td>
</tr>
</tbody>
</table>