Rational Number Project

Initial Fraction Ideas

Abridged edition for use with third graders

Original Authors: Kathleen Cramer, Merlyn Behr, Thomas Post, Richard Lesh

Revision Authors: Kathleen Cramer, Terry Wyberg, Susan Ahrendt, Debbie Monson, Christina Miller
The original authors of the RNP fractions lessons are, Kathleen Cramer, Merlyn Behr, Thomas Post, and Richard Lesh. Contributors to this revised module for use with third graders include, Dr. Terry Wyberg of the University of Minnesota, Dr. Susan Ahrendt, University of Wisconsin, River Falls, Dr. Debra Monson, St. Thomas University and Christina Miller, Phd student at the University of Minnesota. We are also grateful to the third grade teachers from Minneapolis Public Schools for their ideas for revising the original RNP lessons to better meet the needs of third graders.

ABOUT THE AUTHORS for the Original RNP Fraction Lessons

Kathleen Cramer is an Associate Professor in the College of Education and Human Development at the University of Minnesota. She teaches graduate and undergraduate mathematics education classes for students majoring in elementary education. Her research interests continue to focus on better understanding how to help elementary aged students build meaning for rational numbers and proportionality. Kathleen has a Ph.D. from the University of Minnesota in mathematics education. She has published articles and book chapters dealing with the teaching and learning of fractions and proportional reasoning. She has done numerous workshops for teachers dealing with fraction instruction.

Professor Cramer has been involved with the Rational Number Project (RNP) since 1980. She participated in the initial teaching experiments with fourth and fifth graders. She has taken the primary responsibility for revising the lessons developed from the research to form the two sets of RNP Fraction Curriculum Modules.

Merlyn Behr was for over 25 years a professor of mathematics education at Northern Illinois University in DeKalb, Illinois. He was also a faculty member at Florida State University where he received his Ph.D., and at Louisiana State University at Baton Rouge. Merlyn’s primary interest was in children’s learning of elementary- and middle-grades mathematical concepts. He contributed a great deal to our understanding of children’s cognitive processes in these areas. He was very active in the research community and served on the editorial board of the Journal for Research in Mathematics Education (JRME) and as chair of the North American chapter of the research group of the Psychology of Mathematics Education. As a co-founder of the RNP, Merlyn was instrumental in charting its course and providing much valued intellectual leadership in many aspects of RNP activity.

Merlyn died in February 1995. His wit and professional contributions are sorely missed.

Thomas Post, former high school mathematics teacher in New York State, joined the faculty of the College of Education at the University of Minnesota in 1967 after receiving his Ed.D. from Indiana University. Professor Post’s interest is closely allied with other RNP members, as he is especially interested in children’s and teachers’ perceptions of middle-school mathematics. He also has an interest in interdisciplinary approaches to curriculum. He was a co-founder of the RNP and has been active in the mathematics education research community. Along with Kathleen Cramer, Merlyn Behr and Richard Lesh, he has been one of the co-authors of some 70 papers, book chapters and technical reports produced by the RNP since it’s inception in 1979. Tom has also served on the editorial board of the JRME and has been chair of the North American chapter of the research group Psychology of Mathematics Education.
Richard Lesh, former professor and dean at Northwestern University, received his Ph.D. from Indiana University. He spent 5 years overseeing computer software development in mathematics and science at WICAT systems in Provo, Utah. He then served as senior research scientist at ETS in Princeton, NJ where he developed innovative strategies and materials for assessing outcomes in mathematics classrooms. Professor Lesh has served as project manager of the program unit - Research on Teaching and Learning - at the National Science foundation. Currently, he is a professor of mathematics at the University of Massachusetts-Dartmouth helping to further advance our thinking about authentic assessment, principles and strategies. Dick is on of the original co-founders of the RNP and has worked on each of its six grants since 1979. He currently leads the Massachusetts site of the RNP’s Middle-Grades Teacher Enhancement Project, which is the latest of the projects funded by NSF.
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Preface

The Rational Number Project (RNP) is a cooperative research and development project funded by the National Science Foundation. Project personnel have been investigating children’s learning of fractions, ratios, decimals and proportionality since 1979. The original book of fraction lessons was the product of several years of working with children in classrooms as we tried to understand how to organize instruction so students develop a deep, conceptual understanding of fractions.

The lessons were originally published in 1997 under the title: RNP: Fraction Lessons for the Middle Grades Level 1. They have been revised (August, 2009) and renamed to better reflect its content. This module called, Initial Fraction Ideas, includes lessons to develop meaning for fractions using a part-whole model, to support students’ construction of informal ordering strategies based on mental representations for fractions, to build meaning for equivalence concretely. The lessons also develop meaning for adding and subtracting fractions using concrete models. Initial Fraction Ideas module does not include formal algorithms, and instruction with formal algorithms was not part of this original RNP curriculum module.

This edition includes a subset of the 23 lessons found in the RNP: Initial Fraction Ideas curriculum. The lessons selected and subsequently revised for third graders reflects the third grade Common Core Standards for fractions as well as the Minnesota State Standards for fractions at this grade level. The revisions are based on input from third grade teachers who used the original RNP lessons in their classrooms. Four additional lessons dealing with the number line as a
model for fractions were created and field tested with third graders and revised based on that experience. The number line is a challenging model for young learners but lessons were created as the Common Core Standards and MN State Standards both include the number line model in their third grade standards. The third grade module consists of 20 lessons in all.

The complete set of 23 fraction lessons from the module, Initial Fraction Ideas, can be found at:
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html

A companion module to this module has been developed with NSF support. This module, Fraction Operations and Initial Decimal Ideas, extends students’ fraction ideas to develop fraction operations of addition, subtraction, multiplication and division with symbols. That module also introduces students to decimal ideas – naming decimals, order, equivalence, addition and subtraction. This module can be found on the RNP website at this address:
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp2.html

- **This revised set of lessons** provide teachers with an alternative to the textbook scope and sequence for fraction instruction and are appropriate for students in grade 3 but will be effective in remedial settings with older students.

- **This revised set of lessons** help students develop number sense for fractions because they invest time in the development of concepts, order and equivalence ideas.
• This revised set of lessons provides students with daily “hands-on” experiences. Fraction circles, chips and paper folding are the manipulative models used in these lessons to develop initial fraction ideas.

• This revised set of lessons provides teachers with daily activities that involve children in large group and small group settings. All the lessons involve students using manipulative materials. Our work with children has shown that students need extended periods of time with manipulatives to develop meaning for these numbers.

• This revised set of lessons offers teachers insight into student thinking as captured from the RNP research with children. The “Notes to the Teacher” section shares examples of students’ misunderstandings, provides anecdotes of student thinking, and contains information on using manipulative materials.

• These lessons will help teachers and students attain the goals set forth in the Common Core Standards for grade 3.

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>CCSS Objective</th>
<th>RNP 1 Lesson #</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Develop understanding of Fractions as numbers</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Understand a fraction 1/b as a quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size b</td>
<td>1-5; 12, 13</td>
</tr>
<tr>
<td></td>
<td>2. Understand a fraction as a number on a number line; represent fractions on a number line diagram.</td>
<td>15-18</td>
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<td></td>
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<tr>
<td>a. Represent a fraction $1/b$ on a number line diagram by defining the interval between 0 and 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has the size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</td>
<td>15-18</td>
<td></td>
</tr>
<tr>
<td>b. Represent a fraction $a/b$ on a number line diagram by marking off a length of $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line.</td>
<td>15-18</td>
<td></td>
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<tr>
<td>3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</td>
<td>8 – 10</td>
<td></td>
</tr>
<tr>
<td>a. Understand two fractions are equivalent (equal) if they are the same size or the same point on the number line</td>
<td>8 – 10 (Not with # line model)</td>
<td></td>
</tr>
<tr>
<td>b. Recognize and generate simple equivalent fractions. Explain why the fraction are equivalents by using a visual model</td>
<td>8 – 10</td>
<td></td>
</tr>
<tr>
<td>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with symbols $&gt;$, $&lt;$, $=$ and justify conclusions by using a visual fraction model.</td>
<td>6, 6.5, 7, 11</td>
<td></td>
</tr>
</tbody>
</table>
• **These lessons** will help teachers and students attain the goals set forth in the Minnesota Mathematics Standards for grade 3 and expose students to fraction equivalence ideas found in the grade 4 standards.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Benchmark (3rd Grade)</th>
<th>RNP lesson(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand meanings and uses of fractions in real-world and mathematical situations</td>
<td>Read and write fractions with words and symbols. Recognize that fractions can be used to represent parts of a whole, parts of a set, points on a number line, or distances on a number line.</td>
<td>1-5; 12-14; 15-18</td>
</tr>
<tr>
<td></td>
<td>Understand that the size of a fractional part is relative to the size of the whole.</td>
<td>1 - 5</td>
</tr>
<tr>
<td></td>
<td>Order and compare unit fractions and fractions with like denominators by using models and an understanding of the concept of numerator and denominator.*</td>
<td>6, 6.5, 7, 11</td>
</tr>
<tr>
<td>Standard</td>
<td>Benchmark (4th grade)</td>
<td></td>
</tr>
<tr>
<td>Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities</td>
<td>Represent equivalent fractions using fraction models such as parts of a set, fraction circles, fraction strips, number lines and other manipulatives. Use models to determine equivalent fractions.</td>
<td>8-11</td>
</tr>
</tbody>
</table>

• Order lessons also include comparing non-unit fractions with same numerator and exploration of ½ as a benchmark.
Teacher’s Guide
RNP: Initial Fraction Ideas

Abridged edition for use with third graders
Background of the Original RNP lessons

The original RNP lessons reflect research on children’s fraction learning conducted by the National Science Foundation-sponsored Rational Number Project (RNP). Since 1978, Merlyn Behr (Northern Illinois University), Kathleen Cramer (University of Minnesota), Thomas Post (University of Minnesota) and Richard Lesh (Indiana University) have studied how elementary-aged children learn to extend their understanding of numbers to include fraction ideas.

The RNP staff conducted teaching experiments with fourth- and fifth-grade children. In a teaching experiment, researchers entered the classroom as teachers and worked with a curriculum created from a well thought-out theoretical framework. During the teaching and learning process, researchers study children’s learning as they progress through the lessons. This is done through classroom observations, student interviews and written assessments.

The curriculum created for the teaching experiments that evolved into these lessons reflected the following beliefs: (a) Children learn best through active involvement with multiple concrete models, (b) physical aids are just one component in the acquisition of concepts: verbal, pictorial, symbolic and real-world representations also are important, (c) children should have opportunities to talk together and with their teacher about mathematical ideas, and (d) curriculum must focus on the development of conceptual knowledge prior to formal work with symbols and algorithms.

The teaching experiments were conducted in two parts. The first phase of the project was ten weeks long with small groups of fourth and fifth graders at two different sites (Minnesota and Illinois). The second phase was conducted with a classroom of 30 students. A class of fourth graders participated from January of their fourth grade, through January of their fifth grade. Instruction was four days per week and covered the following topics: part-whole model for fractions, ratio and quotient models for fractions, decimals, and number lines.
Throughout both teaching experiments a subset of children were interviewed every two weeks. The interviews provided information on children’s thinking about fraction ideas. We were interested in what role manipulative materials played in their thinking as well as what understandings and misunderstandings children have with fractions.

Our work with children helped explain why children have so much difficulty with fractions. It also informed us as to the type of experiences children need to develop a deep, conceptual understanding of fractions. Consider a few of the insights garnered from these teaching experiments:

1. Children have difficulty internalizing that the symbol for a fraction represents a single entity. When asked if $\frac{2}{3}$ was one or two numbers, many children would say that the symbol represented two numbers. When students consider $\frac{2}{3}$ as two numbers then it makes sense to treat them like whole numbers. For example, when students add two fractions by adding the numerators and then denominators, they are interpreting the symbols as four numbers, not two. Many errors with fractions can be traced to students' lack of mental images for the quantity the symbol represents.

2. Ordering fractions is more complex than ordering whole numbers. Comparing $\frac{1}{4}$ and $\frac{1}{6}$ conflicts with children’s whole number ideas. Six is greater than four, but $\frac{1}{4}$ is greater than $\frac{1}{6}$. With fractions, **more** can mean **less**. The **more** equal parts you partition a unit into, the **smaller** each part becomes. In contrast, $\frac{3}{5}$ is greater than $\frac{2}{5}$ because 3 of the same-size parts are greater than 2 of the same-size parts. In this case, **more** implies **more**. Being able to order plays an important part in estimating fraction addition and subtraction. Ideally when a student adds, for example, $\frac{1}{4} + \frac{1}{3}$, she should be able to reason from her mental images of the symbols that (a) the answer is greater than $\frac{1}{2}$, but less than one and (b) $\frac{2}{7}$ is an unreasonable answer because it is less than $\frac{1}{2}$. 
3. Understanding fraction equivalence is not as simple as it may seem. Some children have difficulty noting equivalence from pictures. Imagine a circle partitioned into fourths with one of those fourths partitioned into three equal parts. Some children we worked with were unable to agree that \( \frac{3}{12} \) equals \( \frac{1}{4} \) even thought they agreed that physically the two sections were the same size. Children said that once the lines were drawn in, you could not remove them. [Therefore \( \frac{3}{12} \neq \frac{1}{4} \)]. In reality, that is just what must be done to understand fraction equivalence from a picture.

4. Difficulties children have with fraction addition and subtraction come from asking them to operate on fractions before they have a strong conceptual understanding for these new numbers. They have difficulty understanding why common denominators are needed so they revert to whole number thinking and add numerators and denominators.

**The RNP Curriculum**

The RNP curriculum offers an alternative scope and sequence to one suggested in fourth- or fifth-grade textbooks. The RNP philosophy is that extended periods of time invested with manipulative materials developing concepts, order, and equivalence ideas are needed before students can operate on fractions in a meaningful way. We call these skills, initial fraction ideas. These goals are consistent with the instructional goals set forth in the National Council of Teachers of Mathematics in their *Principles and Standards for School Mathematics*. The RNP curriculum provides teachers with carefully researched lessons to meet these goals.

The RNP Level 1 materials develop the following topics: (a) part-whole model for fractions, (b) concept of unit, (c) concepts of order and equivalence and (d) addition and subtraction of fractions at the concrete level. The concrete models used are fraction circles, paper folding and chips. It de-emphasizes written procedures for ordering fractions, finding fraction equivalences, and
symbolic procedures for operating on fractions. Instead it emphasizes the development of a quantitative sense of fraction.

To think quantitatively about fractions, students should know something about the relative size of fractions and be able to estimate a reasonable answer when fractions are operated on. Below, find an example of a fourth-grade student’s thought process for estimating a fraction addition problem. This student used the RNP curriculum; her thinking reflects a quantitative sense of fraction. Students using the RNP lessons develop this type of understanding for fractions.

**Problem:** John calculated the problem as follows: \( \frac{2}{3} + \frac{1}{4} = \frac{3}{7} \).

**Do you agree?**

*Student:* I don’t agree. He did it weird. You don’t add the top numbers and bottom numbers.

*Teacher:* What would be an estimate?

*Student:* It would be…greater than 1/2 because \( \frac{2}{3} \) is greater than 1/2.

*Teacher:* Would it be greater or less than one?

*Student:* Less than one. You’d need 1/3 and 1/4 is less than 1/3.

*Teacher:* What about 3/7?

*Student:* 3/7 is less than 1/2.

*Teacher:* How do you know?

*Student:* Because 3/7 isn’t 1/2. I just know.

State and National mathematics Standards have moved fraction content that in the past was fourth grade content to third grade. This abridged version of the RNP level 1 materials was developed based on third grade teachers’ feedback who used a subset of the original RNP level 1 materials in their classrooms and covers content for naming fractions, ordering fractions and fraction equivalence. In addition 3 new lessons were developed to introduce students to the number line model for fractions.
Theoretical Framework

Children using these lessons will be using several manipulative models and will consider how these models are alike and different. They will work in small groups talking about fraction ideas as well as interacting with the teacher in large group settings. They will be drawing pictures to record their actions with fraction models. They will be solving story problems using manipulatives to model actions in the stories.

This model for teaching and learning reflects the theoretical framework suggested by Jean Piaget, Jerome Bruner, and Zoltan Dienes. Richard Lesh, a long time RNP member, suggested an instructional model that clearly shows how to organize instruction so children are actively involved in their learning. Consider this picture.

Lesh suggests that mathematical ideas can be represented in the five ways shown here. Children learn by having opportunities to explore ideas in these different ways and by making connections between different representations. This model guided the development of the RNP curriculum.

Lesson Format

The lessons reflect a classroom organization that values the important role a teacher plays in student learning as well as the need for students to work cooperatively, talking about ideas, and using manipulative models to represent

![Diagram of five ways to represent mathematical ideas](image-url)
rational number concepts. Each lesson includes an overview of the mathematical idea developed. Materials needed by teachers and students are noted. The lesson begins with a class Warm Up. Warm Ups are used to review ideas developed in previous lessons and should take only 5-10 minutes of class time. There is a Large Group Introduction section in each lesson. The teacher’s lesson plans provide problems and questions to generate discussion and target the exploration. Small Group/Partner Work is included in each lesson where students together continue the exploration of ideas introduced in the large group. The class ends with a Wrap Up. A final activity is presented to bring closure to the lesson. At times this will be a presentation by students of select problems from the group work. We found that students like to share their thinking. At other times the Wrap Up will be another problem to solve as a group. The amount of time needed for each lesson will vary from classroom to classroom. A single lesson does not necessarily reflect one day’s work, though teachers often will find that one day is sufficient to cover the material.

An important part of each lesson is the “Comments” section. Here insights into student thinking captured from the initial RNP teaching experiments are communicated to teachers. These notes clarify a wide variety of issues, such as why mastery at the symbolic level is not the primary objective for many of the earlier lessons. The notes also share examples of students’ misunderstandings for teacher’s reflection and anecdotes of student thinking from earlier RNP projects. These notes to the teachers also clarify methods for using manipulative materials to model fraction ideas.

**Manipulative Materials**

Fraction circles, two-sided colored counters and paper folding are the manipulative models used. Our research has shown that the fraction circles are the most important manipulative models for developing mental images of fraction symbols.
**Fraction Circles**

The master for the fraction circles are in the appendix with a page showing the different partitions and colors used for the fraction circles. The circles should be duplicated on index using colors noted on each master. Teachers who have used the fraction circles have relied on their students, parents or teacher-aids to cut out the circles and to organize them in two-pocket folders. If you choose to send home the fraction circles to be cut out with the parent’s help, you will find in the appendix a parent and child activity sheet for them to do together once the circles are cut out.

**Counters**

Two-sided colored counters are available from most publishers of mathematics manipulative materials. A less expensive way is to purchase from a tile store, one square inch tiles (white on one side, tan on other). These cost less than 1.5 cents per tile. Thirty per student should be enough.

**Paper Folding**

Use 8.5” by 11” sheets of paper cut into strips 1” by 8.5”. Have lots on hand for students to use for lessons 7 and 10.

**Number line for fractions**

The number line is considered to be an important model for fractions even though it is a more abstract one than other concrete and pictorial models used in fraction instruction. But this model is unique as compared to other models used to teach fractions and is more challenging for students. The unit on the number line is a length and totally continuous with no separation between units. This makes if difficult for students to identify the unit on the number line. The number line uses symbols and visual cues to convey part of its meaning to
students. Students have to coordinate symbolic and visual cues to bring meaning to the model. This coordination is not needed with other more concrete models. Students also struggle interpreting the tick marks (partitions) on the number line often counting the tick marks and not the distances to identify fractions. The number line lessons in the third grade edition of the RNP lessons tries to address the challenges students have with this model for fractions.

Context and translations from paper folding to the number line are two strategies used to build meaning for the number line as a model for fractions. We are continuing to study how to effectively build this model into the RNP lessons. The 3 lessons currently available are working drafts. These will be updated periodically as we implement them ourselves with students to study student’s thinking related to the number line and evaluate the effectiveness of the lessons with 3rd graders.

**Special Notes on Students’ Thinking**

From our interviews with children we noted that they constructed what we now refer to as informal strategies for ordering fractions. These strategies reflect students’ use of mental images of fractions to judge the fraction’s relative size. These informal strategies do not rely on procedures usually taught: least common denominators and cross-products. We have named the four strategies noted in students’ thinking as: same numerator, same denominator, transitive and residual strategies.

When comparing $\frac{2}{3}$ and $\frac{2}{6}$ (fractions with the same numerator) students can conclude that $\frac{2}{3}$ is the larger fraction because thirds are larger than sixths and two of the larger pieces must be more than two of the smaller pieces. This
strategy involves understanding that an inverse relationship exists between the number of parts a unit is partitioned into and the size of the parts.

The same denominator strategy refers to fractions like $\frac{3}{8}$ and $\frac{2}{8}$. In this case, the same denominator implies that one is comparing parts of the unit that are the same size. Three of the same-size parts are greater than two of the same-size parts.

The student strategy that has been termed the *transitive* strategy can be modeled by comparing $\frac{3}{7}$ and $\frac{5}{9}$. When making this comparison, a student can conclude that $\frac{3}{7}$ is less than $\frac{5}{9}$ because $\frac{3}{7}$ is less than $\frac{1}{2}$, while $\frac{5}{9}$ is greater than $\frac{1}{2}$. This is the transitive strategy because students use a single outside value to compare both fractions.

When comparing $\frac{3}{4}$ and $\frac{5}{6}$, a student can reflect that both fractions are one “piece” away from the whole unit. Because $\frac{1}{6}$ is less than $\frac{1}{4}$, $\frac{5}{6}$ must be closer to the whole and is therefore the bigger fraction. This thinking strategy has been called a *residual* strategy because students focus on the part “leftover” in judging the relative size of the fractions.

These four strategies closely parallel students’ actions with manipulatives. They are in contrast to the paper and pencil procedures, which require changing both fractions to common denominators or calculating cross-products. RNP lessons developed only these student-constructed strategies. The order questions on the interviews will assess whether students construct these strategies. Students who have constructed these strategies have developed are on the way to developing number sense for fractions.

**Final Comments**

You will find at the end of each lesson a form for you to record your adaptations for each lesson. Any curriculum will need to be “personalized” by the teacher who uses it, so it best meets the needs of his/her students. This form
will act as a reminder about changes you feel are important to make the next time you teach the lesson.
The RNP Lessons
Initial Fraction Ideas

Abridged edition for use with third graders

Scope and Sequence
## RATIONAL NUMBER PROJECT
### Initial Fraction Ideas
#### Scope and Sequence

<table>
<thead>
<tr>
<th>LESSON</th>
<th>MANIPULATIVE</th>
<th>TOPIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fraction Circles</td>
<td>Exploration with the circles.</td>
</tr>
<tr>
<td>2</td>
<td>Fraction Circles</td>
<td>Model and verbally name: 1-half, 1-third</td>
</tr>
<tr>
<td>2.5</td>
<td>Fraction Circles</td>
<td>Model and verbally name: 1-half, 1-third, and 1-fourth</td>
</tr>
<tr>
<td>3</td>
<td>Fraction Circles</td>
<td>Model and verbally name unit fractions with denominators greater than 4.</td>
</tr>
<tr>
<td>4</td>
<td>Paper Folding</td>
<td>Compare paper folding to fraction circles. Model and name (verbally and with written words) unit and non-unit fractions.</td>
</tr>
<tr>
<td>5</td>
<td>Fraction Circles</td>
<td>Model fractions and record with symbols a/b.</td>
</tr>
<tr>
<td>6</td>
<td>Fraction Circles</td>
<td>Model the concept that the greater the number of parts a unit is divided into, the smaller each part is.</td>
</tr>
<tr>
<td>6.5</td>
<td>Fraction Circles</td>
<td>Order fractions with like numerators and like denominators embedded in division story problems.</td>
</tr>
<tr>
<td>7</td>
<td>Paper Folding</td>
<td>Reinforce the concept that the greater the number of parts a unit is divided into, the smaller each part is.</td>
</tr>
<tr>
<td>8</td>
<td>Fraction Circles</td>
<td>Fraction Equivalence</td>
</tr>
<tr>
<td>9</td>
<td>Fraction Circles</td>
<td>Fraction Equivalence</td>
</tr>
<tr>
<td>10</td>
<td>Paper Folding</td>
<td>Fraction Equivalence</td>
</tr>
<tr>
<td>11</td>
<td>Fraction Circles</td>
<td>Order fractions by comparing to 1-half.</td>
</tr>
<tr>
<td>12</td>
<td>Chips</td>
<td>Introduce new model for fractions less than one by comparing to a familiar model.</td>
</tr>
<tr>
<td>13</td>
<td>Chips</td>
<td>Model fractions using several units for the same fraction.</td>
</tr>
<tr>
<td>14</td>
<td>Fraction Circles</td>
<td>Model fractions greater than one using mixed and improper fraction notation.</td>
</tr>
<tr>
<td>15</td>
<td>Number Line</td>
<td>Review characteristics of a number line with whole numbers</td>
</tr>
<tr>
<td>16</td>
<td>Context, Paper folding and Number lines</td>
<td>Model fractions using pictures of paper folding strips as a model for number line. Problems embedded in story contexts connected to lengths</td>
</tr>
<tr>
<td>17</td>
<td>Number lines</td>
<td>Make connections between paper folding model and the number line</td>
</tr>
<tr>
<td>18</td>
<td>Number lines</td>
<td>Make connections between fraction circle model and the number line</td>
</tr>
</tbody>
</table>
Post Lesson Reflection

Lesson__________________

1) Number of class periods allocated to this lesson: ______________

2) Student Pages used: ________________

3) Adaptations made to lesson: (For example: added extra examples, eliminated certain problems, changed fractions used)

4) Adaptations made on Student Pages:

5) The next time I teach this lesson I should:
Rational Number Project

Initial Fraction Ideas
Lesson 1: Overview
Lesson provides guided exploration with fractions circles. Students start to become familiar with colors and relationships like 3 browns cover 1 black and 1 brown is bigger than 1 red.

Materials
• Fraction Circles for students and teacher
• Student Page A
• Transparency 1

Teaching Actions

Large Group Introduction
1. Start the Lesson by asking children to sort through their fraction circles to answer these questions:
   (a) How many blues cover the black circle?
   (b) Which is bigger, 1 brown or 1 gray?
   (c) How many pinks cover 1 yellow?
   (d) How many browns cover the black?
   (e) Which is bigger, 1 brown or 2 reds?
   (f) How many purples cover 1 yellow?
   (g) How many dark blues are there? Light blues?

Small Group/Partner Work
2. Explain to the students that they are to continue their exploration by using the circles to complete Student Page A.

Wrap Up
3. End the lesson by working through Transparency 1. The figure on the left represents the circle part you want to cover. To the right are the circle parts. Students are to determine which combination of parts will cover the shape on the left.

Comments
Students need to play with the fraction circles before developing a formal language for describing relationships among the pieces.

There are two different blues: a set of 4 dark blue pieces; a set of 7 light blue pieces. In the lessons the color “blue” refers to the set of 4 dark blue pieces. “Light blue” will refer to the set of 7 blues.

Different ways to approach Student Page A:
Students do page individually and then compare with a partner.
Students do page with a partner.
Do a few problems together and then students finish on their own.
If some students finish Student Page A ahead of others, ask them to create their own problems and record them on the back of the page or put them on the board for others to solve.
### Teaching Actions

All pieces selected do not have to be of the same color.

**Example**

| Black | Blue  | Yellow | Brown | Blue |

4. Encourage students to guess first and then use their fraction circles to find the exact combination. In the above example, 2 blues and 1 yellow would cover the circle.

### Comments

You may want to duplicate Transparency 1 for students.

To encourage students to guess you might want to emphasize making “hypotheses”. Write the word hypothesis on the board. Record students’ guesses, test them out and reach a group consensus.

### Translations

- Verbal to manipulative
- Picture to manipulative to verbal
- Manipulative to written symbols
<table>
<thead>
<tr>
<th>Black</th>
<th>Blue</th>
<th>Brown</th>
<th>Blue</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow</td>
<td>Blue</td>
<td>Blue</td>
<td>Pink</td>
<td>Pink</td>
</tr>
<tr>
<td>Blue</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Black</td>
<td>Yellow</td>
<td>Blue</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Brown</td>
<td>Blue</td>
<td>Pink</td>
<td>R</td>
<td>Pink</td>
</tr>
<tr>
<td>Yellow</td>
<td>Brown</td>
<td>Brown</td>
<td>Pink</td>
<td>Pink</td>
</tr>
</tbody>
</table>
Exploring with the Fraction Circles

1. ____________ browns equal 1 whole circle.

2. 1 whole circle equals ____________ pinks.

3. ____________ reds equal 1 whole circle.

4. ____________ pinks equal 1 brown.

5. 1 brown equals ____________ reds.

6. 1 brown is (less than, equal to, greater than) 1 pink.

7. 1 red is (less than, equal to, greater than) 1 brown.

8. 1 yellow is (less than, equal to, greater than) 1 brown.

9. 1 yellow and 1 brown and 1 ____________ equals 1 whole circle.

10. 1 yellow equals 1 brown and 2 ____________.

11. 3 pinks and 1 ____________ equal 1 whole circle.

12. ____________ grays and 1 blue and 1 yellow equals 1 whole circle.

13. 2 grays and ____________ blue equals 1 yellow.

14. 1 pink equals ____________ reds.

15. 4 ____________ equal 1 yellow.
Initial Fraction Ideas
Lesson 2: Overview
Students explore relationships among circle pieces, modeling and orally naming fraction amounts for: 1-half and 1-third.

Materials
• Fraction Circles for students and teacher
• Student Page A

Teaching Actions

Warm Up
Find two different ways to cover 1 yellow piece. Find two different ways to cover 1 brown piece.

Large Group Introduction

1. Ask students to take out a black circle. Model how to divide the black circle into 2 equal parts by showing that 2 yellow parts cover the whole circle.

2. Ask: How many yellows equal the black circle? Are the 2 parts covering the black circle equal?

3. Explain: the black circle is the unit or the whole you partitioned into two equal parts. When 2 equal parts equal one unit or whole, each part (pick up 1 yellow) is called one-half. This yellow piece is one-half of the black circle.

4. Show one-half by placing 1 yellow on the black circle. “1 yellow covers half of the black circle.”

Comments
Most lessons will have a “Warm Up” problem that reviews ideas from previous lessons. These warm ups are not meant to take more than 5 minutes of class time.

Flexibility of unit is stressed right from the beginning by having students find multiple representations for 1/2, 1/3, and 1/4.

The critical variable with fractions is that a unit or a whole is divided into equal parts. A single part can be given a fraction name that depends on what it is being compared to.

Example:

2 blues equal 1 yellow so 1 blue is one-half of the yellow. [Here 1 yellow is the unit]

Example:

4 blues equal 1 black. 1 blue is one-fourth of the black circle. [Here black is the unit]

Students confuse the terms unit and whole. Sometimes students will use the word whole for the object partitioned and use the term units to
5. Continue by looking for other examples for one-half. Show one blue piece and ask: This one blue is my unit or whole that is to be divided. How can we divide this piece into 2 equal parts? What color pieces will do this?

6. Use these questions:  Are the 2 parts equal? 1 gray is 1 of 2 equal parts; what fraction of the blue piece is 1 gray? [1 half]

7. Show 1 yellow and ask students to consider the yellow as the unit or the whole to be divided. Which piece divides the yellow into two equal parts? What name can we give to each piece? What fraction of the yellow is each piece?

8. Model, using the black circle as the unit or the whole, representations for thirds.

9. 3 browns cover 1 black; 1 brown is 1 of 3 equal parts; 1 brown is one-third of the black. Show as:

- Students are naming fractions in the verbal mode only. In the lesson 3 students will record as: 1-fourth.
- You may want to show how other units can be partitioned into halves:
  - 1 pink → 2 reds
  - 1 brown → 2 pinks
  - 1 orange → 2 purples
## Teaching Actions

| 10. Find other examples for 1-third using 1 yellow and 1 brown and then 1 blue as the unit. |

## Small Group/Partner Work

| 11. Student page A presents problems similar to ones presented in large group. Assign to students in pairs, as they are to name the fractions verbally. |

## Wrap Up

| 12. To assess the “big idea” in this lesson present the following scenario display one of each of these colors: yellow, blue, pink, and red on the and ask: “You have called all of these 1-half, yet they are different sizes. How is that possible?” |

## Translations

- Manipulative to verbal
- Pictures to verbal
Find two different ways to cover 1 yellow piece.

Find two different ways to cover 1 brown piece.
Work with a partner to complete this activity. Use your fraction circles. Say the fraction name to each other when asked.

1. Take out one yellow piece. This is the unit or the whole you will divide into equal parts.
   How many blues cover one yellow piece? ______
   1 blue is _______ of one yellow.
   (Say the fraction name)

2. Take out one brown piece. This is the unit or the whole you will divide into equal parts.
   How many pinks cover one brown piece? ______
   1 pink is _______ of one brown.
   (Say the fraction name)

3. One yellow piece is the unit or whole you will divide into equal parts.
   How many pinks cover one yellow piece? ______
   1 pink is _______ of one yellow.
   (Say the fraction name)

4. What color piece is 1-half of one blue? ______

5. What color piece is 1-third of one yellow? ______

6. What color piece is 1-half of one black circle? ______

7. What color piece is 1-third of one black circle? ______

8. What color piece is 1-third of one orange piece? ______

9. What color piece is 1-half of one pink piece? ______
**Initial Fraction Ideas**

**Lesson 2.5: Overview**

Students explore relationships among circle pieces, modeling and orally naming fraction amounts for: 1-half, 1-third, and 1-fourth.

---

**Materials**

- Fraction Circles for students and teacher
- Student Page A, B

---

**Teaching Actions**

**Warm Up**

Using two different units, show the fraction 1-third. How are the two examples alike? How are the two examples different?

**Large Group Introduction**

1. Model fourths using 1 black, 1 yellow, and 1 brown as units.

2. Ask students to use the black circle as their unit and to find the color that partitions the black circle into equal parts. Ask: What fraction of the black circle is one blue piece?

3. Say: I want to show $\frac{1}{4}$ in another way. If one yellow piece is my unit or whole to be divided, what piece is $\frac{1}{4}$ of the yellow?

4. Say: Let’s try to show $\frac{1}{4}$ in another way. If one brown is my unit, what piece is $\frac{1}{4}$ of the brown?

5. Ask: Can anyone explain why one blue, one gray, and one red can all be called 1-fourth?

6. End the development part of the lesson with a non-example. Show how 2 blues and 1 yellow cover the black circle. Pick up 1 blue and say that this piece is 1 of 3 parts of the circle so it is one-third of the circle. Ask: Is this true? If I wanted to know what part of the black circle 1 blue is, what must I do?

7. [Repeat showing 2 browns and 2 pinks covering

**Comments**

Students continue to explore unit fractions using the fraction circles extending to fourths.

While students are using the fraction circles, they are developing mental images for fractions that will later support their ability to order fractions.
the black circle. 1 pink does not equal 1-fourth.

Small Group/Partner Work

8. Student pages A & B present problems similar to ones presented in large group as well as problems within realistic contexts. Assign to students in pairs, as they are to answer the questions orally.

Wrap Up

9. To assess the “big idea” in this lesson present the following scenario:

Lianna said that 1 red piece is one-third; Rodrigo said 1 red is one-fourth. Who is correct?

[Note that 1 red is one-third of the blue; 1 red is also one-fourth of the brown. Both Lianna and Rodrigo are correct once you know what unit they are comparing the red to].

Translations

- Manipulative to verbal
- Pictures to verbal
Using two different units, show the fraction 1-third.

How are the two examples alike?

How are the two examples different?
The class will work together in groups or in pairs on these problems. Answers are to be given orally or by drawing a picture. On some of the problems children may want to use the fraction circles to help solve the problem.

1. The yellow piece is the unit.  
   How many grays cover the yellow piece? ____________  
   1 gray is ___________ of the yellow.  
   (Say the word)

2. The blue piece is the unit.  
   How many reds cover the blue piece? ____________  
   1 red is ___________ of the blue.  
   (Say the word)

3. The brown piece is the unit.  
   How many reds cover the brown piece? ____________  
   1 red is ___________ of the brown.  
   (Say the word)

4. What color is 1-fourth of one black circle? ____________

5. What color is 1-third of one black circle? ____________

6. Draw a picture of a round pizza. Show on your drawing the pizza cut into 2 fair shares.

   Each fair share is ___________ of the whole pizza.  
   (Say the fraction name)
7. Here is a picture of a candy bar that someone has started to cut into pieces. Draw lines in to finish cutting the candy bar into equal parts.

The small piece of candy is ______________ of the whole candy bar.
(Say the fraction name)

8. Karla has a large chocolate cookie. Draw a picture of Karla’s cookie. Show on your drawing how she could divide the cookie into 4 fair shares.

Each part is ______________ of the whole cookie.
(Say the fraction name)

9. William has a square pan of brownies. Draw a picture of William’s pan of brownies. Show on your drawing how William could divide the pan of brownies into 3 fair shares.

Each part is ______________ of the whole pan of brownies.
(Say the fraction name)
Rational Number Project

Initial Fraction Ideas Lesson 3: Overview
Students model and name (orally and in written words) unit fractions with denominators greater than 4.

Materials
• Fraction Circles for students and teacher
• Student Page A

Teaching Actions

Warm Up
Find the piece that is 1-half of each of these colors: yellow, blue, brown, orange.

Large Group Introduction

1. Show one yellow piece. State that this piece is the unit or whole that they are to divide into equal parts. Ask students to divide it into six equal parts.

2. Explain that because 6 reds cover 1 yellow, it is 1 out of 6 equal parts or 1-sixth of the yellow.

3. Ask students, what fraction piece is 1-sixth of the black? How do you know?

4. Make this chart to show the relationship between the number of equal parts a unit is divided into and the word name for that number of divisions.

<table>
<thead>
<tr>
<th>Number of Equal Parts Unit is divided into</th>
<th>Word Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>half</td>
</tr>
<tr>
<td>3</td>
<td>third</td>
</tr>
<tr>
<td>4</td>
<td>fourth</td>
</tr>
<tr>
<td>5</td>
<td>fifth</td>
</tr>
<tr>
<td>6</td>
<td>sixth</td>
</tr>
<tr>
<td>7</td>
<td>seventh</td>
</tr>
</tbody>
</table>

[Continue to include 8, 9, 10, 12, 15]

Comments

Make a large classroom chart for students to use as a reference for the rest of the fraction unit. You might include a third column showing a picture of a unit (not always a whole circle) divided into the appropriate number of equal parts.

You may want to have students make their own personal chart to keep in their fraction circle folder.
Teaching Actions

For each item in the chart show at least 2 physical models. For example:

- Halves

- Thirds

Students should model each example with their fraction circles.

5. Students should help you find these different representations.
   - You may suggest the unit and ask them to divide it into a certain number of equal parts.
   - You might ask students to suggest the unit. For example, say: “The next value in the chart is to model sixths. What unit can we use?”

6. Once the chart is completed, work through these problems:
   - Using the black circle as the unit, ask students to find the color that divides the unit into 4 equal parts. Hold up 1 of 4 equal parts, call it “one-fourth”, and record the written name as 1/4.
   - Using the yellow circle as the unit, ask students to find the color that divides the unit into 4 equal parts. Hold up all for parts; call it “one-fourth”; record 1/4.
   - Ask: “How are the two models for 1/4 alike? Different?”

7. Repeat for sixths and twelfths using two different units.

Comments

Students initially record fractions in words like: 1/4; 1/6. Research suggests that students make fewer reversals with the symbols (for example, writing 3/2 for 2/3) when they first write fractions in words.
Teaching Actions

8. To prepare students for Student Page A ask the following questions. Have students record answers using word names.
   
   o The blue piece is the unit. What fraction name can you give 1 gray piece? 1 red piece?
   
   o The brown piece is the unit. What fraction name can you give 1 pink? 1 white? 1 gray?

Small Group/Partner Work

9. Assign Student Page A.

Wrap Up

10. End the class with this game: Teacher says: “Two of the colors I am thinking of equal one yellow. What color is it? What fractional name can I give each piece?

   Extra challenges:
   
   If the yellow piece is the unit, what value does the black circle have?
   
   If the blue piece is the unit, what value does the yellow piece have? The black one?
   
   These questions may lead to a nice discussion. Students may question how to express the answer. If the yellow piece is the unit (or one whole) then the black circle is 2 units, 2 wholes or just 2.

Translations

- Manipulative to verbal to written symbols

Comments

You might consider assigning students Page A without this introduction. This will make the activity more of a problem solving activity.

If students struggle to name the fraction, go back and ask them to describe what they see in their display. Try to get them to verbalize that, “There are 4 equal parts. 1 blue is 1 of 4 equal parts”. Often once students can describe the display in this way, they can then jump to naming the amount as 1-fourth.
Find the piece that is 1-half of each of these pieces:
• 1- yellow
• 1- blue
• 1- brown
• 1- orange
Use fraction circles to find the names of the different fraction pieces.

I. The black circle is the unit. What fraction name can you give these pieces?
   1 yellow ___________ 1-half ___________ 1 brown ___________
   1 blue _____________ 1 gray _____________
   1 white _____________ 1 green _____________
   1 red _____________ 1 pink _____________

II. Now make 1 yellow unit. What fraction name can you give these pieces?
   1 blue _____________ 1 gray _____________
   1 pink ______________ 1 red ______________

III. Change the unit to 1 blue. What fraction name can you give these pieces?
   1 gray ______________ 1 red ______________

IV. Change the unit to 1 orange. What fraction name can you give these pieces?
   1 purple _____________ 1 green ______________
Rational Number Project

Initial Fraction Ideas
Lesson 4: Overview

Students use paper folding to model and name unit and non-unit fractions. Students compare the paper-folding model to fraction circles. Students record fractions in words: one-fourth, two-thirds.

Materials

- Paper strips for folding for students
- Fraction Circles for teacher
- Student Pages A-L

Teaching Actions

Warm Up

Name the red piece in three different ways by changing the unit. What different units did you use?

Large Group Introduction

1. Prior to using paper strips to model fractions it is necessary to practice folding strips into 2, 3, 4, 6, and 8 equal parts.

   Ask students to follow along with you as you model how to fold paper strips. Fold paper strip into two equal parts:

   ![Folded strip]

   2. Keep it folded. Now fold it again into two equal parts. Ask: how many equal parts do you think we have? Unfold:

   ![Unfolded strip]

   3. Ask students to verbalize how to fold paper strips to form four equal parts.

   4. Model folding into three equal parts. Form the letter "S" with a paper strip to get close to 3 equal parts. Press down on paper.

   5. Model sixths. Fold paper strip into thirds and then fold into two equal parts. Have students do this and

Comments

Cut paper strips from 8.5” by 11” sheets of paper about 1 inch wide and 8.5” long.

This lesson should take two class periods. Students are still recording fractional amounts using word names; symbols are introduced in lesson 5.

One way to break down the lesson is to complete steps 1 to 6 where students learn to fold paper into 2, 3, 4, 6, and 8 equal parts; then have students practice on their own using Student Page A. Wrap up the lesson with students sharing how they folded paper strips. Select students you have seen who are successful so they can model for others how to fold paper strips into equal parts.

Then pick up the lesson at Step 7 starting with the second warm up and continue on in lesson where students shade equal parts of the paper strips to show fraction amounts. Here students compare the paper strip model with fraction circles. Students can do selected student pages for practice and homework.

Folding into thirds is tricky. For some students you may want to use precut strips with the partitioning lines drawn in. (Student Page L)
**Teaching Actions**

6. Ask students if they could have obtained sixths by folding first in halves and then in thirds? Try it.

7. Now that students have had some practice folding paper strips, they can shade equal parts of paper strips to show fractions. Using fraction circles, show one-fourth using a black circle as the unit.

![Fraction circle diagram]

Say: To show one-fourth of a black circle I divided it into four equal parts. Pick up one of the parts to show one-fourth.

9. Ask: How can you show me one-fourth with a paper strip? Have students fold into 4 equal parts and shade in one of the 4 equal parts. Record fraction name as 1-fourth.

10. Discuss how the two displays for one-fourth are alike and different.

11. Repeat for 1-third and 1-eighth

12. Look at two displays for one-third:

   ![Fraction strip diagram]

**Comments**

Students often will expect 5 equal parts (3+2). They are more apt to think additively than multiplicatively.

The similarity between the two displays is what’s important. A unit or whole is divided into equal parts and one or more equal parts are highlighted in some way. This is a *manipulative to manipulative* translation.
### Teaching Actions

13. Shade in another third on the paper strip.

![Paper strip with shaded third]

Ask: how many thirds are shown now? How can I show two-thirds with circles? (Pick up two browns and say these are two-thirds of black.) State that 2-thirds is 1-third and 1-third more:

![Fraction circle]

14. Now draw a picture of a square. Divide it into 4 equal parts and shade 3 of 4 parts. Ask students to fold paper to show the same fraction that you drew. Record fractions as 3-fourths: 1-fourth + 1-fourth + 1-fourth.

15. Return to fraction circles. Model problems as in lesson 3, this time with non-unit fractions.

**Examples:**
- The black circle = 1. What is the value of 1 blue; 3 blues; 1 brown; 2 browns; 3 reds.
- The yellow piece = 1. What is the value of 1 blue; 2 reds; 3 grays; 2 pinks.

### Comments

Non-unit fractions are introduced as sums of unit fractions: 2-fourths is 1-fourth and 1-fourth.

Students now have seen two models for fractions. Practice pages that follow this lesson give students a chance to apply their new learning to pictures of units in different shapes.

### Teacher Notes for Student Pages:

**E:** Clarify with students that a picture may need to be modified to determine if 2-fourths are shaded in. For example:

- Is 2-fourths shaded?

2-fourths can easily be seen once the picture is completed by drawing in the needed lines.

### Small Group/Partner Work

16. There are several student pages in this lesson. **Select the most appropriate ones for your students.** Students may need some assistance to do some of the pages. See **Comments** for clarification.

### Wrap Up

17. Ask students to identify problems on Student Pages, H, I & J, that equal zero. Ask: How can you show 0 using paper strips? Ask students to do this in more than one way. (For example: 0/2; 0/4; 0/3)
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. Ask: What are the different ways to model one and zero with paper folding strips?</td>
<td></td>
</tr>
</tbody>
</table>

**Translations**

- Manipulative to verbal
- Manipulative to manipulative to verbal
- Manipulative to verbal to written symbols (word names)
Name the red piece in three different ways by changing the unit. What different units did you use?
Fold a paper strip into 6 equal parts. Explain to a friend how you did this.

Did you both do it in the same way?
Practice Folding Paper

You and your teacher folded paper strips into different numbers of equal parts. Now it is your turn to practice paper folding on your own. You will need at least 4 paper strips.

1. Imagine that one of these paper strips is a candy bar. You want to share the candy bar evenly so you and three friends get the same amount. Show how to do that with the paper strip and then draw a picture of it below.

2. Take out one of the paper strips and try to fold it into 3 equal parts. Describe below how you did this.

3. How can you partition the paper strip into 6 equal parts? Before you do this, write up the steps below. Then act out those steps to fold the paper strip into 6 equal parts.

4. Fold a paper strip into 8 equal parts. Then draw a picture of the strip below.
For each problem, look at the figure and then answer the question and write the word name for each fraction. Word names may include: half, halves, third, fourth, fifth, sixth, eighth or tenth.

1. _______ equal-sized parts.
   Each part is _____ __________ of the whole.

2. _______ equal-sized parts.
   Each part is _____ __________ of the whole.

3. _______ equal-sized parts.
   Each part is _____ __________ of the whole.

4. _______ equal-sized parts.
   Each part is _____ __________ of the whole.

5. _______ equal-sized parts.
   Each part is _____ __________ of the whole.
6. _______ equal-sized parts.
Each part is _____ _________ of the whole.

7. _______ equal-sized parts.
Each part is _____ _________ of the whole.

8. _______ equal-sized parts.
Each part is _____ _________ of the whole.

9. _______ equal-sized parts.
Each part is _____ _________ of the whole.

10. _______ equal-sized parts.
Each part is _____ _________ of the whole.
Directions:
You'll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction you have shown (use words, not symbols).

1.  

5.  

2.  

6.  

3.  

7.  

4.  

8.
Directions:

You’ll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction you have shown (use words, not symbols).

1.  
   
2.  
   
3.  
   
4.  
   
5.  
   
6.  
   
7.  
   
8.  
   

fractions
Look at each picture carefully. Place an “X” beside each picture that shows 2-fourths shaded in. You may need to draw in lines to determine if 2-fourths are shaded.
For each diagram, fill in the blanks to tell about the diagram.

a. Number of equal parts ________
   Number of equal parts shaded ______
   The fraction shaded is ______ -sixth

b. Number of equal parts ________
   Number of equal parts shaded ______
   The fraction shaded is 1- ________

c. Number of equal parts ________
   Number of equal parts shaded ______
   The fraction shaded is ____________

d. Number of equal parts ________
   Number of equal parts shaded ______
   The fraction shaded is ____________
e. Number of equal parts _____________
   Number of equal parts shaded _____
   The fraction shaded is _____________

f. Number of equal parts _____________
   Number of equal parts shaded _____
   The fraction shaded is _____________

Write words like 2-fourths, 3-fifths, and so on for the fraction shaded by each diagram.

Write _________________

Write _________________

Write _________________
Write the fraction that is shown in words:

a. ____________________

b. ____________________

c. ____________________

d. ____________________

e. ____________________

f. ____________________
Circle the figures that have equal-sized parts.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9.
Problem Solving

Directions:

For each of the drawings write the color corresponding to the part marked a, b, c, and so on. Then write the word name for the fraction that the color represents. You can use fraction circles if you need them. Your teacher will help you with exercise 1.

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractions in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. yellow</td>
<td>1-half</td>
</tr>
<tr>
<td>b. blue</td>
<td>1-fourth</td>
</tr>
<tr>
<td>c.</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractions in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
</tr>
<tr>
<td>d.</td>
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</tr>
<tr>
<td>e.</td>
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</table>

<table>
<thead>
<tr>
<th>Color</th>
<th>Fractions in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
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<tr>
<td>b.</td>
<td></td>
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<tr>
<td>c.</td>
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<tr>
<td>d.</td>
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</table>
Rational Number Project

Initial Fraction Ideas
Lesson 5: Overview

<table>
<thead>
<tr>
<th>Materials</th>
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</thead>
<tbody>
<tr>
<td>• Fraction Circles for students</td>
</tr>
<tr>
<td>and teacher</td>
</tr>
<tr>
<td>• Student Pages A – E</td>
</tr>
</tbody>
</table>

Students are introduced to fraction symbols by translating from manipulatives to verbal to symbols.

Teaching Actions

Warm Up

Use paper strips to show 1-third and 1-fourth. Which is the largest fraction?

Large Group Introduction

1. Ask students to use fraction circles to show 3-fourths. They are to show two models. For example:

   ![Fraction Circles]

   - 3 blues are 3-fourths of 1 black.
   - 3 grays are 3-fourths of 1 yellow.

2. Ask how the two models are alike.

3. Record in words fraction name: 3-fourths. Explain that there is also a symbol name for 3-fourths and it is $\frac{3}{4}$.

4. Discuss the meaning of $\frac{3}{4}$. Ask how many equal parts each unit is divided into? Point to the bottom of the fraction symbol and explain that this 4 tells us that. The 3 tells us that we are interested in 3 of these 4 equal parts. The fraction means $\frac{1}{4}$ and $\frac{1}{4}$.

Comments

It’s not important for students to memorize the words: numerator and denominator.

It’s very important to help children verbalize the meaning of fraction symbols.

Have them talk through what they are doing with the fraction circles.

The action on the manipulative reinforces the meaning of the symbol.

You can also return to previous student pages and have students record answers in symbol form.
Teaching Actions

and $\frac{1}{4}$.

5. Write $\frac{2}{3}$ on the board and ask students to show that fraction with the fraction circles. Have them verbalize why their model does indeed represent $\frac{2}{3}$.

First divide the whole circle into 3 equal parts … then explain

$I$ divided the circle into 3 equal parts to find what color is thirds. Then I only want two of them so

shows 2 of 3 equal parts. It is $\frac{1}{3}$ and $\frac{1}{3}$ more.”

6. Repeat for $\frac{5}{6}$, $\frac{4}{8}$, $\frac{3}{3}$.

Embed examples in context:

A spinner for a game was divided into 5 equal parts. 3/5 of the spinner was blue. Show that amount with the fraction circles.

A pizza was cut into 6 equal parts. You ate 2/6 of the pizza. Show that amount with the fraction circles.

Small Group/Partner Work

7. Student pages that follow reinforce the meaning of the symbol. Select the most appropriate (and amount of) practice that your students need. Students are not expected to do all the pages.

Comments

You might want to create a list of number words to display that students can reference when doing the small group work. Example: half, thirds, fourths, etc.
Teaching Actions

Wrap Up

8. Ask students to describe 2-3 instances that fractions are used in everyday life or in science class.

9. Record situations from these examples that lead to recording a fraction with symbols. For example, to make chocolate chip cookies, you need to use \( \frac{3}{4} \) of a cup of brown sugar. Draw a picture of a measuring cup, partition it into 4 equal parts and show \( \frac{3}{4} \).

Translations

- Manipulative to verbal to written symbols
- Written symbols to manipulative to verbal
- Real life to manipulative to written symbols
- Written symbols to written symbols
- Written symbols to pictures
- Pictures to written symbols
Use paper strips to show $\frac{1}{3}$ and $\frac{1}{4}$. Which is the largest fraction?
1. Write each fraction in words.

   a. \( \frac{2}{4} \) 2-fourths  
   b. \( \frac{3}{7} \) ____________________  
   c. \( \frac{6}{8} \) ____________________  
   d. \( \frac{3}{11} \) ____________________  
   e. \( \frac{7}{10} \) ____________________  
   f. \( \frac{7}{15} \) ____________________  
   g. \( \frac{3}{12} \) ____________________  
   h. \( \frac{7}{9} \) ____________________  

2. Write the word name and the symbol name for each fraction described.

   a. 3 of 5 equal-size parts are shaded. 3-fifths \( \frac{3}{5} \)  
   b. 5 of 7 equal-size parts are shaded. ____________________   ______  
   c. 3 of 13 equal-size parts are shaded. ____________________   ______  
   d. 12 of 17 equal-size parts are shaded. ____________________   ______  
   e. 0 of 3 equal-size parts are shaded. ____________________   ______  

3. Write the fraction symbol for each fraction word.

   a. 9-tenths \( \frac{9}{10} \)  
   b. 7-eighths ____________________  
   c. 6-sixths ____________________  
   d. 15-nineteenths ____________________  
   e. 13-twenty-firsts ____________________  
   f. 17-eighteenths ____________________  
   g. 0-fourths ____________________  

4. Imagine a circle divided into 4 equal parts.
   
   Three $\frac{3}{4}$ parts are shaded!
   
   What fraction tells how much is shaded in all? _______________
   
   Draw a picture.

5. Imagine a rectangle divided into 5 equal parts.
   
   Four $\frac{4}{5}$ parts are shaded!
   
   What fraction tells how much is shaded in all? _______________
   
   Draw a picture.

6. Write the word name and the symbol name each fraction describes.
   
   a. A rectangle is folded into 7 equal-size parts.  
      5 parts are shaded.

   b. A circle is folded into 8 equal-size parts.  
      4 parts are shaded.
Directions:
Match each picture with its symbol or word name by writing the letter of the picture next to its symbol. The first one is done for you.

A. \[ \begin{array}{c|c|c} & & \ \end{array} \] \[ \begin{array}{c} 1/6 \end{array} \] \[ \begin{array}{c} \text{F} \end{array} \]

B. \[ \begin{array}{c|c|c|c} & & & \ \end{array} \] \[ \begin{array}{c} \text{2-halves} \end{array} \] 

C. \[ \begin{array}{c|c|c|c|c} & & & & \ \end{array} \] \[ \begin{array}{c} 3/4 \end{array} \]

D. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} \text{2-thirds} \end{array} \]

E. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} 3/3 \end{array} \]

F. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} \text{1-fourth} \end{array} \]

G. \[ \begin{array}{c|c|c|c|c|c|c} & & & & & & \ \end{array} \] \[ \begin{array}{c} 6/6 \end{array} \]

H. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} 1/3 \end{array} \]

I. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} \text{3-sixths} \end{array} \]

J. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} 4/6 \end{array} \]

K. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} \text{2-fourths} \end{array} \]

L. \[ \begin{array}{c|c|c|c|c|c} & & & & & \ \end{array} \] \[ \begin{array}{c} 1/2 \end{array} \]
Shade each circle to show the fractional amount.

\[
\begin{array}{ccc}
\frac{1}{4} & \frac{2}{2} & \frac{1}{6} \\
\frac{5}{12} & 0 & \frac{5}{6} \\
\frac{1}{3} & \frac{11}{12} & \frac{4}{4} \\
\frac{2}{12} & \frac{1}{2} & \frac{6}{8} \\
\frac{1}{6} & 0 & \frac{6}{12} \\
\frac{6}{6} & \frac{8}{8} & \frac{1}{2} \\
\end{array}
\]
Write the name for the shaded part of each rectangle in words and then in symbols.

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1. | 1 - half | $\frac{1}{2}$ |   |   |   |   |   |   | 9. |   |   |   |   |   |   |   |   |   |   |   |   |
|  2. |   |   |   |   |   |   |   |   | 10. |   |   |   |   |   |   |   |   |   |   |   |   |
|  3. |   |   |   |   |   |   |   |   | 11. |   |   |   |   |   |   |   |   |   |   |   |   |
|  4. |   |   |   |   |   |   |   |   | 12. |   |   |   |   |   |   |   |   |   |   |   |   |
|  5. |   |   |   |   |   |   |   |   | 13. |   |   |   |   |   |   |   |   |   |   |   |   |
|  7. |   |   |   |   |   |   |   |   | 15. |   |   |   |   |   |   |   |   |   |   |   |   |
|  8. |   |   |   |   |   |   |   |   | 16. |   |   |   |   |   |   |   |   |   |   |   |   |
Initial Fraction Ideas
Lesson 6: Overview

Students observe with circles that as the unit is divided into more and more equal parts, the unit parts become smaller.

Materials
- Fraction Circles for students and teacher
- Student Pages A and B

Teaching Actions

Warm Up
Show this fraction with your fraction circles using two different units or wholes. Then draw pictures for your display: $\frac{3}{4}$

Large Group Introduction

1. Start the lesson by reviewing ordering of whole numbers. For example, ask a student to select the greater of these 2 numbers, 720 or 702, and to explain his/her strategy for doing so.

2. Give another example using a context. José earns $42,175 a year. Mara earns $51,275 a year. Who earns more?

3. Introduce the idea of ordering fractions with this example. Draw two whole circles. On one circle draw in fourths. In the other draw in eighths.

4. Which circle has the larger parts? Why is this so? So which is bigger one of the four equal parts or one of the eight equal parts?

5. Explain: Let’s develop this idea between number of parts and size of parts using Student Page A.

6. Ask students to use their fraction circles as you work together; name the black circle as the unit.

7. Ask: How many brown pieces cover the whole circle? How many orange? Which color takes more pieces to cover the whole unit? Which

Comments
To think quantitatively about fractions, students should know something about the relative size of fractions. Lesson 6 is the first of several lessons to help students construct informal strategies for ordering fractions. At Level 1, we want to provide the concrete experiences that students need if they are ever to reason intuitively about fraction symbols.

Activities in this lesson will lead students to reason, for example, that $1/4 > 1/8$ because if you divide a circle into 8 equal parts, the parts will be smaller than if you divide the same unit into 4 equal parts.

This notion of more and greater can lead to misunderstandings. Some students may want to say that $1/8 > 1/4$ because $8 > 4$ or because with eighths, you have more pieces than you do with fourths. Whole-number reasoning has a strong influence on how children think about fractions.

Children need to be reminded that to compare fractions, we look at the “size of piece,” not the “number of pieces”.

Lesson 6 ©RNP 2013
Teaching Actions

8. Record that information in a chart.

<table>
<thead>
<tr>
<th>Color</th>
<th>How many cover 1 circle</th>
<th>Which color takes more...</th>
<th>Which color has smaller...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>5</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

12. When completed, ask students if they see any patterns between the number of pieces to fill the whole unit and the size of the pieces.

13. As a group, write a rule similar to either of these:
   • As the number of pieces needed to fill the whole decreases, the size of each piece gets larger.
   • As the number of pieces needed to fill the whole increases, the size of each piece gets smaller.

14. Once the rule is generated use it in examples without the circular pieces.

   Examples:
   20 purples = 1 whole
   80 greens = 1 whole
   Which is larger, 1 purple or 1 green?

Small Group/Partner Work

15. Assign Practice Pages B to reinforce the day’s lesson.

Wrap Up

18. Ask students to picture the whole circle in their mind. What color pieces would divide the circle into twelfths? (red). Now picture the color pieces that would cover the whole circle into 3 equal parts (brown). Which piece is bigger red or brown? Why does that make sense?

19. Repeat for blue vs yellow; orange vs red. Then ask:
which is bigger and why: ½ or ¼?

16. Conclude by asking: Does more always mean less with fractions? Give this example: Imagine that it takes 10 maroon pieces to cover the whole circle. Which is smaller, 2 maroon pieces or 3 maroon pieces? How do you know?

17. Ask: How is this example different from all the rest we’ve talked about today?

20. Ask which is bigger: ¾ or ¼? What do you picture in your mind to answer this question?

This lesson helps students to compare unit fractions. These are fractions with one in the numerator. The term “unit” is used differently here than when you call the whole circle the unit. This can be confusing for students so there is not need to label the fractions as unit fractions.

In the next lesson students compare fractions with the same numerators other than one.

Translations

- Real life to picture to verbal
- Manipulative to written symbols to verbal
- Written symbols to manipulative
- Real life to manipulative to pictures
Show this fraction with your fraction circles using two different units. Then draw pictures for your display:

\[
\frac{3}{4}
\]
Directions: Use fraction circles to fill in the table.

<table>
<thead>
<tr>
<th>Color</th>
<th>How many cover 1 whole circle?</th>
<th>Which color takes MORE pieces to cover 1 whole?</th>
<th>Which color has SMALLER pieces?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Brown</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>5</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>2. Orange</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Purple</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Gray</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. White</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Orange</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purple</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Gray</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>8. Brown</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Green</td>
<td></td>
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</table>
Comparing Fractions

1. Use the whole circle as your unit. Show $\frac{1}{3}$ and $\frac{1}{2}$. Which fraction is the smaller of the two?

2. Use the whole circle as your unit. Show $\frac{1}{5}$ and $\frac{1}{10}$. Which fraction is the larger of the two?

3. Use the whole circle as your unit. Show $\frac{1}{6}$ and $\frac{1}{3}$. Which fraction is the smaller of the two?

4. Use the whole circle as your unit. Show $\frac{1}{12}$ and $\frac{1}{9}$. Which fraction is the larger of the two?

5. Imagine the whole circle is a pizza. You cut the pizza into 4 equal parts. How much of the pizza is one slice? Would you get more or less pizza if you divided it into 8 equal parts? Draw pictures to show how you answered the questions.
**Rational Number Project**

**Initial Fraction Ideas**

**Lesson 6.5: Overview**

<table>
<thead>
<tr>
<th>Materials</th>
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</thead>
<tbody>
<tr>
<td>• Fraction Circles for students and teacher</td>
</tr>
<tr>
<td>• Student Pages A and B</td>
</tr>
</tbody>
</table>

Students order fractions embedded in division story problems. Students order fractions with like numerators and with like denominators.

---

**Teaching Actions**

**Warm Up**

Which is larger: 1-fourth or 3-fourths? Draw a picture to support your answer.

**Large Group Introduction**

1. Introduce this story problem. Ask students to draw a picture to answer the question. Kara entered the Pizza Factory. She saw 2 friends in 1 booth and 3 friends in another booth. Both groups have just been served a large pizza. Which group should she sit with so that she gets the most to eat?

2. Draw this diagram:

3. Ask students to show Kara’s share in booth 1 (with 2 friends) and in booth 2 (with 3 friends).

4. Which group has the most people? In which group does a person have the smallest share of pizza?

5. Conclude that 1/3 of the pizza is more than 1/4 of the pizza. [Repeat with 6 people at a table; 8 people at a table.]

---

**Comments**

In this lesson students build understanding of ordering non-unit fractions with the same numerator and reviews ordering fractions with the same denominators. Students use fair share story problems to reinforce the idea that the more you divide a unit into equal parts, the parts become smaller.

Students also reinforce this idea by using fraction circles to model fractions with same numerator or same denominator.

All of this work is to help students build mental images for fractions so they can judge the relative size of fractions. This is an essential skill to have in order to estimate when adding and subtracting fractions. For example to estimate \( \frac{3}{4} + \frac{1}{8} \) students with strong mental images for fractions will see that \( \frac{3}{4} \) is \( \frac{1}{4} \) away from the whole. \( \frac{1}{8} \) is smaller than \( \frac{1}{4} \) so the sum must be less than one. With strong mental images students will also know that \( \frac{3}{4} > \frac{1}{2} \).
6. Present this story: You have two small square pans of brownies. Both pans are divided into 8 equal parts. Allie ate 2 brownies from the first pan with nuts on top. Hamdi ate 3 brownies from the second pan that didn’t have nuts. How much of each pan did each girl eat? Who ate more? How do you know?

Small Group/Partner Work

7. Assign Practice Pages A and B

Wrap Up

8. Ask students to imagine these two fraction amounts. If the whole circle is the unit, what does \(\frac{3}{4}\) look like? What does \(\frac{3}{12}\) look like? Which is bigger? How do you know? Select students you know can verbalize clearly how to think about these two fractions. You want students to model for others the idea of acknowledging both the numerator and denominator in their thinking.

Fourths are larger than twelfths. So 3 larger pieces are more than 3 smaller pieces. \(\frac{3}{4} > \frac{3}{12}\)

Translations
- Real life to picture to verbal
- Manipulative to written symbols to verbal
- Written symbols to manipulative
- Real life to manipulative to pictures
Which is larger: 1-fourth or 3-fourths? Draw a picture to support your answer.
Draw pictures or use fraction circles to solve each problem.

1. Four children share one large pizza at the blue table in the lunchroom. Three children share one large pizza at the green table. At which table does each child get more pizza? Why?

2. Who gets more candy: a child at a table where 6 children are sharing a candy bar or a child at a table where 3 children are sharing a candy bar?

3. Jessica and Kim shared a large pizza. Jessica ate $\frac{2}{6}$ of a pizza. Kim ate $\frac{3}{6}$ of the pizza. Who ate more? How do you know?

4. Josie eats 2 brownies from a small pan of brownies that was divided into 6 equal parts. You ate 2 brownies from a small pan of brownies (the same size pan as Josie’s) divided into 8 equal parts. How much of each pan did Josie and you eat? Who ate more? How do you know?
**Directions:**
Use fraction circles to compare the two fractions. Circle the **larger** fraction.

<p>| | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>2/3</td>
<td>1/4</td>
<td>3/4</td>
<td></td>
</tr>
<tr>
<td>4/5</td>
<td>3/5</td>
<td>2/7</td>
<td>2/6</td>
<td></td>
</tr>
<tr>
<td>6/7</td>
<td>2/7</td>
<td>4/12</td>
<td>4/15</td>
<td></td>
</tr>
<tr>
<td>8/12</td>
<td>11/12</td>
<td>6/7</td>
<td>3/7</td>
<td></td>
</tr>
<tr>
<td>2/7</td>
<td>2/9</td>
<td>9/10</td>
<td>3/10</td>
<td></td>
</tr>
<tr>
<td>4/8</td>
<td>4/6</td>
<td>Try these without manipulatives.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/3</td>
<td>2/8</td>
<td>13/100</td>
<td>27/100</td>
<td></td>
</tr>
<tr>
<td>7/10</td>
<td>7/9</td>
<td>9/10</td>
<td>9/100</td>
<td></td>
</tr>
</tbody>
</table>
Rational Number Project

Initial Fraction Ideas

Lesson 7: Overview

This lesson reinforces the idea that as the number of parts the unit is divided into increases, the size of the parts decreases.

Materials

- 8.5” X 1” strips of paper for each student and teacher
- Student Pages A, B, C

Teaching Actions

Warm Up

Order these fraction pairs. Write an explanation for each pair (use pictures in your explanation).

\[ \frac{3}{3} \quad \frac{5}{3} \]
\[ \frac{4}{8} \quad \frac{6}{6} \]

Large Group Introduction

1. Ask children to fold a strip of paper into 2 equal parts. Using the same strip of paper ask them how they can increase the number of equal parts to 4. Have them do this, but before they open up the strip of paper to show fourths ask: Before you open up the strip, can you tell me if the size of the equal parts will be larger or smaller than halves? Why?

2. Repeat for:
   - Fourths to eighths
   - Thirds changed to sixths (Use pre-divided strips for thirds if needed)
   - Halves to sixths

3. Now ask students to fold, shade, and label these fractions with paper folding:

\[
\begin{array}{ll}
\frac{1}{3} & \frac{1}{4} \\
\frac{1}{3} & \frac{1}{6} \\
\frac{3}{4} & \frac{1}{4} \\
\end{array}
\]

[Do more if needed]

Comments

Children need opportunities to use new ideas in order to ensure they internalize them.

Many experiences with physical models are needed to overcome the influence of children’s whole number thinking.

In this lesson students use paper folding to reexamine the relationship between size of piece and number of pieces the whole is divided into.

Encourage children to explain their ordering. Don’t let them refer to only one part of the fraction, as for example: 1/3 vs. 1/4 “thirds are bigger”. Thirds may be bigger, but that information is enough to order 2 fractions only if the numerators are the same. “Thirds are bigger so 1 of a larger piece is greater than 1 of a smaller pieces.” By talking like this children are coordinating numerator and denominator to approximate the size of the fraction. You want to build the notion of a fraction as a single entity!

Students may over generalize and think bigger is always more. Check for this.
Teaching Actions

Small Group/Partner Work

4. Put students in pairs and assign Student Page A. Student 1 will make fraction 1 with paper folding; student 2 will make fraction 2. They will then compare and circle the larger fraction.

5. Student Pages B and C offer extra practice.

Wrap Up

6. Conclude the lesson by first asking children to draw a picture to solve each problem below. Have students explain their ordering strategies.

   Mary 1/2 of a large Domino pizza; Joan had 1/4 of a large Domino pizza. Who ate more?

   Lianna ate 4/8 parts of a candy bar; Rodrigo ate 4/6 of a same-sized candy bar. Who ate more?

7. Now ask: Picture ¾ in your mind using the whole circle as the unit. Now picture ½ of the whole circle. Which is bigger ½ or ¾?

8. Look to see if students say ½ is greater because halves are bigger than fourths. Encourage a discussion of the role of the numerator and how 3 smaller parts may be bigger than one bigger piece.

Translations

- Written symbols to manipulative to verbal

Comments

Some children may be able to compare without manipulatives

\[
\frac{1}{3} \text{ vs. } \frac{1}{5}; \quad \frac{2}{10} \text{ vs. } \frac{2}{20}
\]

but there is no need to push abstraction at this level.

Some students may try to compare fractions without the manipulatives and make errors. Encourage them to use paper folding at least to verify their guesses.

Look for students to say something about the numerator and denominator. “Halves are bigger than fourths and you have one of each”.

Order these fraction pairs. Write an explanation for each pair (use pictures in your explanation).

\[
\begin{array}{cc}
\frac{3}{4} & \frac{3}{10} \\
\frac{5}{7} & \frac{3}{7} \\
\frac{1}{9} & \frac{1}{4}
\end{array}
\]
Directions:
Circle the larger fraction. Use your paper strips to determine the answers.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{6}$</td>
<td>$\frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{6}$</td>
<td>$\frac{3}{3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{0}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{6}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Without your paper strips, circle the larger fractions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{100}$</td>
<td>$\frac{1}{99}$</td>
</tr>
<tr>
<td>$\frac{3}{40}$</td>
<td>$\frac{3}{50}$</td>
</tr>
<tr>
<td>$\frac{2}{10}$</td>
<td>$\frac{4}{10}$</td>
</tr>
</tbody>
</table>
Directions:
Partition and shade each picture to show the fraction. Circle the **SMALLER** fraction in each pair.

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{3}{8}$</td>
</tr>
</tbody>
</table>
Directions

A friend has been out of school for two days and missed the math lessons dealing with comparing fractions. Write your friend a letter explaining how to compare fractions like the ones you have been working with. [You may want to draw pictures.]
Rational Number Project

Initial Fraction Ideas
Lesson 8: Overview

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Fraction Circles for students and teacher</td>
</tr>
<tr>
<td>• Student Pages A and B for students</td>
</tr>
</tbody>
</table>

Students explore fraction equivalence by naming fractions equal to 1/2 with fraction circles and by finding other fraction equivalences with fraction circles.

Teaching Actions

Warm Up

Use paper folding strips to show these two fractions: \( \frac{3}{4} \) and \( \frac{1}{2} \). Compare the strips – which fraction is larger? Why?

Large Group Introduction

1. Start the lesson with a quick review of naming fractions using the fraction circles. Set the stage for this by telling students that the whole circle is our unit, the amount to be divided into equal parts. Ask: If the whole circle is our unit, what fraction name is 1 blue? 2 browns? 6 grays? 4 pinks?

2. If students struggle naming the fraction amount ask: how many blues cover the whole circle? What fraction is one blue?

3. At the overhead, cover the whole circle with 1 yellow and ask students to find different ways to cover the remaining half of the circle. Record answers by color.

   Ex: 1 blue and 2 grays 2 blues

4. Repeat this activity, but specify that they have to use the same color pieces to cover half the circle. Record

Comments

The idea of equivalence is a prerequisite for fraction operations. To add \( \frac{1}{2} + \frac{3}{4} \), you will explain that \( \frac{1}{2} \) can be exchanged for \( \frac{2}{4} \) because \( \frac{1}{2} = \frac{2}{4} \).

Equality should first be developed from concrete models before explaining a rule that generates equal fractions \( \left( \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \right) \).

You are defining equivalent fractions by showing that fractions are equivalent if they cover the same amount of the circle. Partitioning is different so the digits in the fraction symbols are different; but 1 of 2 equal parts covers the same amount as 2 of 4 equal parts.

One-half equivalences are the most important ones for children to learn. Students with good “fraction number sense” use \( \frac{1}{2} \) as a reference point for estimating the size of other fractions.

A student will use his/her concept of \( \frac{1}{2} \) to estimate, for example,
Teaching Actions

results by color and fraction name.

| 1 yellow = | 2 blues | 1 = 2 |
| 1 yellow = | 3 pinks | 1 = 3 |
|           |         | 2 = 6 |

5. Ask what each display has in common. They all cover 1-half of the black circle therefore they are all equivalent. Introduce the term, equivalence.

Small Group/Partner work

6. Students continue to explore fraction equivalences for fractions other than \( \frac{1}{2} \). Student Page A is a review for naming fractions using fraction circles; Student Page B, asks students to concretely find fraction equivalences and to record them with words and then symbols.

Wrap Up

7. At this point it the lessons you should assess which students have strong mental images for fractions based on the fraction circles. Ask the following questions. Have students describe their mental representations:
   a. If the whole circle is our unit, what color represents thirds?
   b. What color represents sixths?
   c. Which is bigger one brown or one pink?
   d. Which is bigger \( \frac{1}{3} \) or \( \frac{1}{6} \)?
   e. What color is fourths?
   f. Which is bigger \( \frac{1}{4} \) or \( \frac{3}{4} \)?

8. Ask students to picture the fraction circles in their mind: Which fractions do you see that are equal to \( \frac{1}{2} \)? What colors do you see? How many of each color covers \( \frac{1}{2} \) exactly?

Translations

- Written symbols to manipulative to written symbols
- Manipulatives to verbal
Use paper folding strips to show these two fractions:

\[
\frac{3}{4} \quad \text{and} \quad \frac{1}{2}
\]

Compare the strips – which fraction is larger? Why?
# Naming Fractions

**Directions:** Use the **black circle** as the unit. Name the fraction amount for the number of pieces shown below.

<table>
<thead>
<tr>
<th>Color</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 yellow</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2 browns</td>
<td></td>
</tr>
<tr>
<td>1 blue</td>
<td></td>
</tr>
<tr>
<td>2 oranges</td>
<td></td>
</tr>
<tr>
<td>2 reds</td>
<td></td>
</tr>
<tr>
<td>2 yellos</td>
<td></td>
</tr>
<tr>
<td>6 grays</td>
<td></td>
</tr>
<tr>
<td>0 blues</td>
<td></td>
</tr>
<tr>
<td>4 pinks</td>
<td></td>
</tr>
<tr>
<td>3 yellows</td>
<td></td>
</tr>
</tbody>
</table>
### Practicing Fraction Equivalence with Our Fraction Circles

<table>
<thead>
<tr>
<th>Show the Equal Fractions with your Fraction Circles</th>
<th>Record the Equivalent Fractions with Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 yellow = _____________ blues</td>
<td>$\frac{1}{2} = -$</td>
</tr>
<tr>
<td>1 yellow = _____________ pinks</td>
<td>$\frac{1}{2} = -$</td>
</tr>
<tr>
<td>1 yellow = _____________ grays</td>
<td>$\frac{1}{2} = -$</td>
</tr>
<tr>
<td>1 blue = _____________ grays</td>
<td>$\frac{1}{4} = -$</td>
</tr>
<tr>
<td>1 pink = _____________ grays</td>
<td>$\frac{1}{6} = -$</td>
</tr>
<tr>
<td>1 brown = _____________ pinks</td>
<td>$\frac{1}{3} = -$</td>
</tr>
<tr>
<td>3 blues = _____________ grays</td>
<td>$\frac{3}{4} = -$</td>
</tr>
</tbody>
</table>
### Rational Number Project

#### Initial Fraction Ideas

**Lesson 9: Overview**

Students continue to explore fraction equivalence using fraction circles to find missing parts of equivalences presented using symbols.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Fraction Circles for students and teacher</td>
</tr>
<tr>
<td>• Student Pages A and B for students</td>
</tr>
<tr>
<td>• Fraction numeral cards for the teacher (cut out)</td>
</tr>
</tbody>
</table>

### Teaching Actions

#### Warm Up

Ty and Shanya both ate part of a Hershey candy bar. Ty ate $\frac{1}{2}$ of his while Shanya ate $\frac{4}{8}$ of hers. Who ate more? Draw a picture to justify your answer.

#### Large Group Introduction

1. Start the lesson by asking students to take out their fraction circles and find two fractions equal to $\frac{1}{3}$. Ask: What does it mean for two fractions to be equal? From your model, why does $\frac{1}{3} = \frac{2}{6}$?

2. Pose this question: Does $\frac{1}{4} = \frac{4}{8}$?

3. Encourage students to answer this question by either showing with the fraction circles that 1 blue = 2 grays or by mentally picturing the fraction circles to conclude that $4/8 = \frac{1}{2}$ and not $\frac{1}{4}$.

4. Pose this question: $\frac{3}{4} = $ how many eighths? Ask how can you record that equivalence with symbols? ($\frac{3}{4} = \frac{6}{8}$)

#### Small Group/Partner work

5. Students continue to explore fraction equivalences using their fraction circles translating from symbols to manipulatives to symbols. Assign Student Pages A

### Comments

1. We want students to think about equivalence as covering up the same amount of the fraction circle. In the next lesson the idea of equivalence will extend to paper folding where students will see that folding the paper into more parts does not change the total amount shaded. Folding only increases the number of parts.
5. Play the Fraction Fill game.

**MATERIALS:** Fraction numeral cards
Fraction Fill Board

**DIRECTIONS:** Teacher randomly selects a numeral card and shows it to students. Students choose to shade that amount on one of the circles. They can only shade 1 representation for that fraction amount.

Ex: \( \frac{1}{4} \)

Student can shade

\[ \text{ or } \]

Student cannot shade the circle divided into two equal parts by adding lines to the circle to divide it into fourths.

Ask students from time to time, what equivalences they used. Record them on the board. Continue showing numeral cards. Students refer to equivalence chart to make selections. The first to shade two complete circles says “Fraction Fill.”

**Translations**
- Written symbols to manipulative to written symbols
Ty and Shanya both ate part of a Hershey candy bar.

Ty ate \( \frac{1}{2} \) of his while Shanya ate \( \frac{4}{8} \) of hers.

Who ate more? Draw a picture to justify your answer.
## Practicing Fraction Equivalence with Our Fraction Circles

<table>
<thead>
<tr>
<th>Show the Equal Fractions with your Fraction Circles</th>
<th>Record the Equivalent Fractions with Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 yellow = _________ blues</td>
<td>$\frac{1}{2} = \frac{2}{4}$</td>
</tr>
<tr>
<td>2 blues = _________ reds</td>
<td></td>
</tr>
<tr>
<td>1 brown = _________ whites</td>
<td></td>
</tr>
<tr>
<td>4 blues = _________ grays</td>
<td></td>
</tr>
<tr>
<td>2 pinks = _________ browns</td>
<td></td>
</tr>
<tr>
<td>1 brown = _________ pinks</td>
<td></td>
</tr>
<tr>
<td>1 blues = _________ reds</td>
<td></td>
</tr>
</tbody>
</table>
The black circle is the unit or the whole. Use your fraction circles to determine if the following number sentences are true or false. Circle TRUE if you think the two fractions are equivalent. Circle FALSE if you think the two fractions are not equivalent.

<table>
<thead>
<tr>
<th>Fraction Sentences</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} = \frac{4}{6} )</td>
<td>TRUE</td>
</tr>
<tr>
<td>( \frac{1}{3} = \frac{2}{6} )</td>
<td>FALSE</td>
</tr>
<tr>
<td>( \frac{1}{4} = \frac{4}{6} )</td>
<td>TRUE</td>
</tr>
<tr>
<td>( \frac{3}{12} = \frac{1}{4} )</td>
<td>TRUE</td>
</tr>
<tr>
<td>( \frac{1}{3} = \frac{4}{12} )</td>
<td>FALSE</td>
</tr>
<tr>
<td>( \frac{1}{2} = \frac{5}{8} )</td>
<td>FALSE</td>
</tr>
</tbody>
</table>

Use your fraction circles to find the missing numerator in each pair to make the number sentences equal.

<table>
<thead>
<tr>
<th>Fraction Sentences</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} = \frac{?}{6} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{3} = \frac{9}{\phantom{9}} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{2} = \frac{\phantom{2}}{6} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{4} = \frac{12}{\phantom{12}} )</td>
<td></td>
</tr>
</tbody>
</table>
Fraction Fill
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
Rational Number Project

<table>
<thead>
<tr>
<th>Initial Fraction Ideas</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 10: Overview</td>
<td>• Scissors and glue</td>
</tr>
<tr>
<td>Students explore equivalence ideas with</td>
<td>• Strips of paper (8.5” x 1”) for</td>
</tr>
<tr>
<td>paper folding.</td>
<td>folding for each student.</td>
</tr>
<tr>
<td></td>
<td>• Student Pages A-C</td>
</tr>
</tbody>
</table>

### Teaching Actions

#### Warm Up

Using your fraction circles find two fractions equal to $\frac{1}{4}$. Draw a picture for each equivalent fraction. Record the equivalent fraction under each picture.

#### Large Group Introduction

1. Throughout this activity, teacher and students do the examples. Teacher may choose to use larger strips of paper for demonstration purposes.

2. Ask students to fold strips of paper into halves and shade $\frac{1}{2}$ of the paper. Write the symbol for amount shaded on that strip.

3. Now have students fold the same strip to show 4 equal parts. Before they actually open up the folded paper, ask them to guess the number of shaded parts.

4. Open up the amount and record that amount on the paper strip.

5. Ask: Do you have more than 1 fraction written on your paper? Explain why.

6. Ask: How did the paper strips change after the second folding? Did the amount shaded change? Did the total number of parts changed? What else change? Does $\frac{1}{2} = 2/4$? Why is this true?

### Comments

Students will benefit from seeing equivalent fractions with more than one manipulative.

Remember all this work with manipulatives is an investment that will pay off later as children learn to operate with fractions. The manipulative experiences will give them the mental images they need to operate (+, -, x, ÷) on fractions in a meaningful way.
### Teaching Actions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>Repeat for ( \frac{1}{4} ). Fold to show that ( \frac{1}{4} = \frac{2}{8} )</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Ask students to use their paper folding strips to show that ( \frac{3}{4} \neq \frac{7}{8} )</td>
<td></td>
</tr>
</tbody>
</table>

### Small Group/Partner Work

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>9.</td>
<td>Practice is provided in Student Pages A – C.</td>
<td></td>
</tr>
</tbody>
</table>

### Wrap Up

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>Ask: What does it mean for two fractions to be equivalent? What are some fractions equal to ( \frac{1}{2} )?</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Record on the board this set of equivalences for 1-half: ( \frac{1}{2} = \frac{2}{4} ), ( \frac{1}{2} = \frac{3}{6} ), ( \frac{1}{2} = \frac{5}{10} ). Ask students if they see any interesting number patterns in fraction pairs equal to ( \frac{1}{2} ).</td>
<td></td>
</tr>
</tbody>
</table>

### Comments

While students are working on the student pages, model as needed how to solve the problems with paper folding strips. Students could answer questions on Student Page A by drawing in the folding lines on strips of Page B.

Students may notice that for 1-half, the denominator is twice the size of the numerator; they may notice that if the numerator is doubled the denominator is doubled or if the numerator is tripled then the denominator is tripled. You are not formalizing the symbolic rule for equivalence, but just helping students start to notice the multiplicative nature of fractions starting with \( \frac{1}{2} \).

### Translations

- Written symbols to manipulative to written symbols
- Pictures to written symbols
Using your fraction circles find two fractions equal to \( \frac{1}{4} \).

Draw a picture for each equivalent fraction.

Record the equivalent fraction under each picture.
PAPER FOLDING AND EQUIVALENT FRACTIONS

Cut out the strips on Student page B so you can fold them to solve these problems.

1. Write the symbol for the fraction shaded on strip A: ______
   Fold to make 8 equal sized parts.
   Write the equivalent fraction that is shown: ______

2. Write the symbol for the fraction shaded on strip B: ______
   Fold to make 9 equal sized parts.
   Write the equivalent fraction that is shown: ______

3. Write the fraction for the amount shaded on strip C: ______
   Fold to make 6 equal sized parts.
   Write the equivalent fraction that is shown: ______

4. Write the fraction for the amount shaded on strip D: ______
   Fold to make 12 equal sized parts.
   Write the equivalent fraction that is shown: ______

5. Write the fraction for the amount shaded on strip E: ______
   Fold to make 8 equal parts.
   Write the equivalent fraction that is shown: ______
Paper Folding and Equivalent Fractions

1) Show 3 fractions equal to 1/2. (Hint: you will need to start with 3 sheets of paper folded into 2 equal parts with one part shaded. Draw pictures to show your answers.

2) Use paper folding to find out which of these are true statements. Circle the number sentences that are true.

\[
\frac{1}{3} = \frac{2}{6} \quad \quad \quad \quad \frac{2}{4} = \frac{4}{8}
\]

\[
\frac{1}{4} = \frac{3}{8} \quad \quad \quad \quad \frac{1}{2} = \frac{6}{8}
\]

3) Use paper folding to find these equivalences.

\[
\frac{1}{2} = \frac{8}{8} \quad \quad \quad \quad \frac{1}{3} = \frac{6}{6}
\]

\[
\frac{1}{4} = \frac{8}{8} \quad \quad \quad \quad \frac{1}{2} = \frac{4}{4}
\]
Lesson 11

Rational Number Project

Initial Fraction Ideas
Lesson 11: Overview
Students use fraction circles to order 2 fractions by comparing them to one-half.

Materials
- Fraction Circles for students and teacher
- Student Pages A, B

Teaching Actions

Warm Up
Draw a picture of paper folding strips to show the fraction \(\frac{1}{4}\). Now partition your picture to show how many eighths equal \(\frac{1}{4}\).

Large Group Introduction

1. Ask students to take out the black circle and to cover one-half of the circle with 1 yellow piece.

2. Show on the overhead that 3 blues, which is \(\frac{3}{4}\) of the black, is greater than 1 yellow \((\frac{1}{2}\) of the black).

   Record: 3 blues > 1 yellow so \(\frac{3}{4} > \frac{1}{2}\)

3. Ask students to find 4 other fractions greater than \(\frac{1}{2}\). Model and record their responses on the overhead.

4. Now ask them to imagine fraction pieces greater than 1 yellow or \(\frac{1}{2}\) of the circle. Have them write down at least 3 estimates for amounts greater than \(\frac{1}{2}\). Encourage students to share their estimates and explain what they thought of or pictured.

Ex: A child may say, “I can see that 3 pinks are the same as 1 yellow, so 5 pinks must be greater than \(\frac{1}{2}\).”

Comments

Students need many experiences with concrete materials to develop mental images of fractions so they can develop a quantitative notion of fraction.

Comparing to \(\frac{1}{2}\) is a powerful strategy for judging the relative size of fractions and is a characteristic of having a quantitative notion of fraction.

Looking at specific numerical relationships between numerator and denominator to determine if fractions are greater or less than \(\frac{1}{2}\) is not the goal for all students. Some students may show that they see number patterns for \(\frac{1}{2}\).

We encourage students to rely on their mental images related to the fraction circles or paper folding to guide their ordering strategies.
### Teaching Actions

5. Have students verify each guess with their circles and record results with fraction notation.

### Small Group /Partner Work

6. Student Page A provides independent practice with circles comparing fractions to $\frac{1}{2}$.

7. Student Page B provides more practice with ordering and equivalence ideas developed so far.

### Wrap Up

8. End class by presenting these problems for discussion. Emphasize student verbalization of their thinking as they order these fractions. They may or may not use the circles.

<table>
<thead>
<tr>
<th>Examples:</th>
<th>$\frac{1}{3}$</th>
<th>$\frac{3}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{4}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{4}{100}$</td>
<td>$\frac{4}{70}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{10}$</td>
<td>$\frac{1}{100}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{4}{12}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{3}{6}$</td>
<td></td>
</tr>
</tbody>
</table>

Which is bigger or are they equal?

Ordering fractions using common denominator rule is not part of these lessons. Many students should be able to order these fraction pairs using mental images for fractions.

You can use the problems in the wrap up to evaluate which students can order fractions using their mental images of fraction circles. Keep returning to order tasks like these to informally assess students’ number sense.

A common error students make is to look only at the denominator to make an order decision. For example, when comparing $\frac{1}{3}$ vs. $\frac{3}{4}$, a student may say $\frac{1}{3}$ is the bigger fraction since thirds are larger than fourths.

Encourage students to reflect on the numerator and denominator to determine the fraction’s relative size.

### Translations

- Written symbols to manipulative
- Manipulative to verbal to written symbols
- Written symbols to verbal
Draw a picture of paper folding strips to show the fraction $\frac{1}{4}$.

Now partition your picture to show how many ninths equal $\frac{1}{4}$. 
Exploring $\frac{1}{2}$ With Fraction Circles

Use the whole circle as your unit. Make the fraction $\frac{2}{5}$ with the fraction circles.

Decide if $\frac{2}{5}$ is greater or less than $\frac{1}{2}$.

Record your response in the box:

\[
\frac{2}{5} \quad \text{is less than} \quad \frac{1}{2}
\]

Complete the problems below. Use these choices:

- is less than
- is greater than
- or
- is equal to

\[
\begin{array}{ccc}
\frac{2}{3} & & \frac{1}{2} \\
\frac{1}{4} & & \frac{1}{2} \\
\frac{5}{8} & & \frac{1}{2} \\
\frac{8}{10} & & \frac{1}{2} \\
\frac{3}{4} & & \frac{1}{2} \\
\frac{1}{5} & & \frac{1}{2} \\
\frac{2}{8} & & \frac{1}{2} \\
\frac{4}{6} & & \frac{1}{2} \\
\frac{7}{12} & & \frac{1}{2} \\
\frac{3}{4} & & \frac{1}{2}
\end{array}
\]
Using Fraction Circles to Order Fractions

Use fraction circles to show each fraction. Compare the fractions. Circle the largest fraction. If the fractions are equivalent, circle both.

<p>| | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>(2)</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{12}$</td>
<td>(3)</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{4}$</td>
<td>(5)</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{6}$</td>
<td>(6)</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{4}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>$\frac{4}{8}$</td>
<td>$\frac{1}{2}$</td>
<td>(8)</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{6}{8}$</td>
<td>(9)</td>
<td>$\frac{3}{12}$</td>
<td>$\frac{1}{12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td>$\frac{4}{8}$</td>
<td>$\frac{5}{8}$</td>
<td>(11)</td>
<td>$\frac{2}{4}$</td>
<td>$\frac{3}{6}$</td>
<td>(12)</td>
<td>$\frac{2}{6}$</td>
<td>$\frac{2}{12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(13)</td>
<td>$\frac{3}{6}$</td>
<td>$\frac{5}{6}$</td>
<td>(14)</td>
<td>$\frac{6}{8}$</td>
<td>$\frac{8}{8}$</td>
<td>(15)</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{6}{12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Rational Number Project

#### Initial Fraction Ideas

**Lesson 12: Overview**

Students are introduced to chips as a fraction model. They learn to represent a given fraction using different sets of chips as a unit.

#### Materials

- Chips or tiles with a different color on each side
- Paper strips for folding
- Student Page A

#### Teaching Actions

**Warm Up**

Which fraction is larger? Share your answer and rationale to a partner. What mental images for fractions did you picture to find the larger fraction?

\[
\frac{3}{4} \quad \frac{1}{2}
\]

**Large Group Introduction**

1. You use chips while students use paper folding to show the same fraction.

2. Model the fraction \(\frac{2}{3}\) with the display chips in a way for all students to see.

   Say: I have 6 chips (show 6 chips with the white side up). This is my unit or whole that I am going to divided into equal groups. I’m going to partition them into 3 equal groups. Can you partition your paper strip into 3 equal parts?

   Ask: How are the displays alike and different?

3. Turn over 2 of the 3 equal groups of chips to show the tan side of the chips. Ask children to model this action on their paper strips by shading 2 of 3 equal parts.

   Say: I made 2 of 3 equal-sized groups tan.

#### Comments

This will be a challenging lesson for students. You may want to use two class periods to cover this material.

To reinforce important fraction concepts students are introduced to a new model for fractions (chips) by relating this model to a previous one (paper folding). This is a translation from one physical model to another. Seeing similarities between models helps students abstract important concepts.

Common error: Students model \(\frac{2}{3}\) by making *groups of three* instead of *three equal groups*.

We use white square tiles found at any tile store as they are an inexpensive way to provide all students with their own chips.
Ask: What fraction of the chips is tan?

Ask: How are the chips and paper folding models alike? Different?

4. Summarize what you did by writing on the board.

   I started with a unit of ____ white chips.
   
   I divided the unit into ____ equal groups.
   
   I made ____ groups tan.
   
   ____ of ____ equal-sized groups are tan.
   
   What fraction of the groups is tan? (\(\frac{2}{3}\))

5. Ask students to verbalize what they did with paper in a similar way. Conclude that there are many different models to show fractions.

6. Model other fractions using chips. Students should have their own chips. For the sake of consistency, use the white side to show the unit and use the tan side to show “amount shaded.”

7. SAY: I have 12 chips. I want to show \(\frac{1}{4}\) using these chips as my unit.

   ASK: How many equal-sized groups will I need? (four). Now divide the 12 chips into 4 equal groups.
Teaching Actions

ASK: To show \( \frac{1}{4} \), how many equal groups must I make tan? (one)

8. Repeat for several more fractions.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} )</td>
<td>6</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>8</td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td>12</td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td>8</td>
</tr>
</tbody>
</table>

Small Group/Partner Work


Wrap Up

10. Present these two stories to students. Ask them to decide which model, paper folding or chips, would be best to show the fraction the story.

Story 1: Devan had 15 m & m’s. She shared them equally between herself and 2 friends. What fraction of candy did she get?

Story 2: Lianna has a Nestle Crunch bar. She plans to share it between herself and 2 friends. What fraction of the candy bar will each get?

Translations:

- Manipulative to manipulative to verbal; symbols to manipulative to verbal

Students will struggle a bit. As you move from group to group you may want to model the fraction using paper folding, going through each step as the students use the chips to copy your steps.

Students confuse the number of groups with the number of chips in each group so emphasize that \( \frac{3}{4} \) means 3 out of 4 equal-sized groups. It does not mean to put 4 chips into a group.
Which fraction is larger? Share your answer and rationale to a partner.

What mental images for fractions did you picture to find the larger fraction?

\[
\frac{3}{4} \quad \frac{1}{2}
\]
Modeling Fractions with Chips

1. Show 3-fourths with chips. Use 8 chips in all. Draw a picture of your display.

2. Show 1-fourth with chips. Use 12 chips in all. Draw a picture of your display.

3. Show 2-sixths with chips. Use 12 chips in all. Draw a picture of your display.

4. Show 2-sixths with chips. Use 6 chips in all. Draw a picture of your display.

5. Show 1-fifth with chips. Use 15 chips in all. Draw a picture of your display.

6. On the back of this page, describe steps you would take to show $\frac{2}{3}$ using 12 chips.
Rational Number Project

Initial Fraction Ideas Lesson 13: Overview

Students continue practicing showing fractions with chips. They determine several units that can be used to model a fraction and what units can’t be used to model fractions.

Materials

- Chips for students and teacher
- Student Page A - C

Teaching Actions

Warm Up

Order these fractions from smallest to largest. Be ready to explain your thinking.

\[
\frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{10}
\]

Large Group Introduction

1. Present this picture:

Say: I want to model this fraction using chips as my unit or whole instead of paper. What fraction is shown? If I use 12 chips as my unit, tell me the steps to show \(\frac{3}{4}\).

2. Vary the unit by asking students what they’d do if you used 4 chips as a unit and then 20 chips as the unit. Ask how these chip models are alike and how they are different.

3. Summarize by showing that to show \(\frac{3}{4}\) you used 4, 12, and 20 chips. Ask if you could have used other sets of chips as your unit.

4. Show \(\frac{2}{3}\) with the fraction circles. Ask students to model this fraction with chips. Allow them to choose the unit. Ask students to tell you the number

Comments

Flexibility of unit is emphasized with chips, as was done with the fraction circles. Students should know that to show \(\frac{2}{3}\), a number of sets can be used - 3 chips, 6 chips, 9 chips...

Regardless of the number of chips, the same action to model the fraction is used. (Partition into 3 equal groups and show, 2 of the 3 groups tan.

Students at times confuse the term “whole” and “unit”, thinking whole is the amount partitioned into equal groups while “units” are the equal parts.

Ask questions as you do this lesson: What is the unit you are using to show \(\frac{3}{4}\)? See if the student identifies all the chips as the unit or the number of chips in each equal part as the unit.
Teaching Actions

of tiles they use as the whole or unit.

5. Present this chart to students. Ask if the showed each fraction with paper folding strips, how many equal parts would they need.

6. Then ask them to imagine using chips to show each fraction. Ask: What are some possible sets of chips you can use as the whole (unit) to model each fraction? List 3 possible units that they could use as the unit for each fraction. Ask: How did you figure this out?

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Units you could use</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{5}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclude the introduction by asking students to show the fraction $\frac{1}{4}$ with 14 chips. Discuss why this cannot be done. Ask for units that cannot be used to show the fractions in the above chart. [Make another column in the table].

Small Group/Partner Work

7. Assign Student Pages A - C as a way to practice showing fractions with chips.

Wrap Up

8. End this lesson with some problem solving using chips. Present these two problems and ask students to solve them using chips. Then have a few students share how they solved the problems.

Joe ate 4 jellybeans. This was $\frac{1}{5}$ of all the jellybeans in the bag. How many jellybeans were in the bag?

Comments

The possible units are multiples of the denominator.

$\frac{4}{5}$ 5, 10, 15, 20… are all possible units.

To solve the challenges students have to reconstruct the unit. If 4 jellybeans equals 1-fifth, the there must be 20 jellybeans in the bag as the whole unit is made up of 5-fifths.

If 10 equals 1/4, the 4 groups of 10 would equal the whole.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marta ate 10 jellybeans. This was $\frac{1}{4}$ of all the jellybeans in the bag. How many jellybeans in the bag?</td>
<td></td>
</tr>
</tbody>
</table>

**Translations:**
- Picture to manipulative to verbal
- Written symbols to manipulative
Order these fractions from smallest to largest. Be ready to explain your thinking.

\[
\frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{10}
\]
### Showing Fractions Using Chips

<table>
<thead>
<tr>
<th>What fraction is shaded?</th>
<th>Show that fraction using 6 chips as your unit. Draw a picture in the column to the right.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Shaded fraction]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>What fraction is shaded?</td>
<td>Show that fraction using 12 chips as your unit. Draw a picture in the column to the right.</td>
</tr>
<tr>
<td>[Shaded fraction]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>What fraction is shaded?</td>
<td>Show that fraction using 12 chips as your unit. Draw a picture in the column to the right.</td>
</tr>
<tr>
<td>[Shaded fraction]</td>
<td></td>
</tr>
<tr>
<td>Picture of chips</td>
<td>Fraction shaded</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------------------------</td>
</tr>
</tbody>
</table>
| ![chip image 1](image1) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
| ![chip image 2](image2) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
| ![chip image 3](image3) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
| ![chip image 4](image4) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
| ![chip image 5](image5) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
| ![chip image 6](image6) | How many groups? _____  
How many groups are tan? ____  
What fraction of the chips are tan? ____ |
Show each fraction with chips in two ways. You decide on the unit. Draw a picture or your models.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>![Model 1 for 1/3]</td>
<td>![Model 2 for 1/3]</td>
</tr>
<tr>
<td>1/4</td>
<td>![Model 1 for 1/4]</td>
<td>![Model 2 for 1/4]</td>
</tr>
<tr>
<td>1/6</td>
<td>![Model 1 for 1/6]</td>
<td>![Model 2 for 1/6]</td>
</tr>
<tr>
<td>1/2</td>
<td>![Model 1 for 1/2]</td>
<td>![Model 2 for 1/2]</td>
</tr>
<tr>
<td>2/3</td>
<td>![Model 1 for 2/3]</td>
<td>![Model 2 for 2/3]</td>
</tr>
</tbody>
</table>
Rational Number Project

Initial Fraction Ideas
Lesson 14: Overview

Students name fractions greater than 1 but less than 2 with fraction circles. Students name fractions using both mixed numbers and improper fractions.

Materials
- Fraction Circles for students and teacher
- Transparency 1
- Student Pages A and B

Teaching Actions

Warm Up

You and your friends ate one whole pizza and 1-third of another one. Draw a picture of this amount of pizza. How much pizza is this?

Large Group Introduction

1. Ask students to use their fraction circles with the black circle as the unit to show \( \frac{2}{3}, \frac{4}{4}, \frac{5}{5} \) and \( \frac{12}{12} \). In each example, ask for another name for the amount shown (1 whole or just 1).

2. Have students show \( \frac{6}{8} \) using the whole circle as the unit. Ask if \( \frac{6}{8} \) is greater or less than 1 whole or 1?

3. Present this story and ask students to model it with their circles. Again, use whole circles as the unit.

   Last night Margo ate \( \frac{3}{4} \) of a large pizza. (Show that with circles). In the morning she ate some leftover pizza that equaled \( \frac{2}{4} \) of a pizza.

4. Ask students to try and show the extra \( \frac{2}{4} \). They realize that they do not have enough pieces. Have them work with a partner and use 2 sets of fractions circles to model the story.

5. Continue with the story: How much pizza did

Comments

Modeling fractions greater than one using fraction circles, is easier than with chips, so we concentrate on developing the concept of changing improper fractions and vice versa with fraction circles.

The warm up sets the stage for the lesson dealing with fractions greater than one. Discuss together how students named the fraction amount. Explain that in this lesson they will find ways to name fractions > 1.

Accept both names: 1 and \( \frac{1}{4} \) or \( \frac{5}{4} \). Do not rush any rules about changing improper fractions to mixed fractions. Our goal is for students to change from one notation to another using circles and then just with mental images of circles. No paper/pencil rules.
Teaching Actions

Margo eat altogether?

Questions to lead discussion for naming amount of pizza:

- Did Margo eat more than 1 whole pizza? How do you know?
- Let’s count how many fourths she ate: \(\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}\).
- From the picture I see that \(\frac{5}{4}\) equals \(\frac{4}{4}\) and \(\ldots\).  
  - What’s another name for \(\frac{4}{4}\)?
  - What’s another way of describing the amount of pizza Margo ate?

6. Draw a picture of what you did and restate how \(\frac{5}{4} = 1\frac{1}{4}\).

This shows \(\frac{5}{4}\);
point to \(\frac{4}{4}\) and \(\frac{1}{4}\) more.
This shows \(\frac{5}{4}\);
1 whole and \(\frac{1}{4}\) of another circle.

6. Ask students to use their fraction circles to show these amounts. If the fraction >1, have the students name the amount in two ways.

\[
\frac{4}{3}, \frac{3}{4}
\]
Teaching Actions

7. As students explain their models, ask if the amount is greater or less than one. Try to get them to verbalize concrete actions that show when a fraction is greater than one.

8. Select students to draw pictures for a few examples showing two ways to name the fraction.

Ex: \( \frac{8}{6} \)

Small Group/Partner Work

10. Assign Student Pages A, B, C.

Wrap Up

11. Ask students to imagine each fraction noted below and from their mental image name the amount in another way.

\( \frac{4}{3}, \frac{3}{2}, \frac{3}{3} \)

Translations

- Real world to manipulative to verbal
- Written symbols to manipulative to written symbols
- Written symbols to picture to verbal to written symbols

Comments

We want students to verbalize that, for example, \( \frac{8}{6} \) is greater than 1 because they need more than 1 whole circle to model it. \( \frac{4}{5} \) is less than 1 because they needed only 1 unit to model it.
You and your friends ate one whole pizza and \(\frac{1}{3}\) of another one.

Draw a picture of this amount of pizza.

How much pizza is this?
Write two fraction names for each picture.

<table>
<thead>
<tr>
<th>Improper Fraction</th>
<th>Mixed Number</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Fraction 1" /></td>
<td><img src="image2" alt="Mixed Number 1" /></td>
</tr>
<tr>
<td><img src="image3" alt="Fraction 2" /></td>
<td><img src="image4" alt="Mixed Number 2" /></td>
</tr>
<tr>
<td><img src="image5" alt="Fraction 3" /></td>
<td><img src="image6" alt="Mixed Number 3" /></td>
</tr>
<tr>
<td><img src="image7" alt="Fraction 4" /></td>
<td><img src="image8" alt="Mixed Number 4" /></td>
</tr>
<tr>
<td><img src="image9" alt="Fraction 5" /></td>
<td><img src="image10" alt="Mixed Number 5" /></td>
</tr>
<tr>
<td><img src="image11" alt="Fraction 6" /></td>
<td><img src="image12" alt="Mixed Number 6" /></td>
</tr>
</tbody>
</table>
Shade in the pictures to show each fraction. Write another name for each amount.

<table>
<thead>
<tr>
<th>Mixed Number</th>
<th>Shade In</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \frac{1}{2}$</td>
<td><img src="image1" alt="Shade In" /></td>
<td>$\frac{3}{2}$</td>
</tr>
<tr>
<td>$1 \frac{3}{4}$</td>
<td><img src="image2" alt="Shade In" /></td>
<td>$\frac{7}{4}$</td>
</tr>
<tr>
<td>$1 \frac{1}{4}$</td>
<td><img src="image3" alt="Shade In" /></td>
<td>$\frac{5}{3}$</td>
</tr>
<tr>
<td>$1 \frac{4}{6}$</td>
<td><img src="image4" alt="Shade In" /></td>
<td>$\frac{4}{3}$</td>
</tr>
</tbody>
</table>
Rational Number Project

**Initial Fraction Ideas**

**Lesson 15: Overview**

<table>
<thead>
<tr>
<th>Students reflect on the characteristics of number lines showing integers to highlight meaning for the unit on the number line, points as numbers on the number line and the arrows at the end of the number line to show that our numbers continue infinitely.</th>
</tr>
</thead>
</table>

**Materials**

- Number line Pictures A-D
- Student Pages A-C

---

**Teaching Actions**

**Warm Up**

The thermometer shows the temperature. What do we call the numbers above the 0 point? What do we call the numbers below the 0 point? What is the value of the point at letter A? How do you know this?

**Large Group Introduction**

1. Explain: the thermometer from the warm up is like a number line. Number lines are pictures that show the numbers we work with.

2. Look at this partial number line (A). Let’s imagine that the number line is a picture showing the distance from Janel’s house to her grandparent’s home. Her grandparents live 5 miles from her house. Where is Janel’s house on the number line? (Point 0; label it “Janel’s house” or draw a picture of a house.)

**Comments**

Before locating fractions on a number line students should review the characteristics of a number line from their experiences with whole numbers on the number line. Big ideas in this lesson include the following:

- Unit length
- Point at the end of a unit length is a number
- Arrows at the end of the line communicate that our numbers are infinite
- Fractions are numbers located between two integers
Teaching Actions

above 0) Where would her grandparent’s house be on the number line? (Point 5; label “grandparent’s house or draw a picture of a house above point 5). Explain that the unit on the number line is 1 mile; the distance from point 0 to the first tick mark represents one mile. Where would 7 miles be? (7th tick mark from point 0)

3. Look at this picture of a number line (B). How are the two lines different? Which one do you think is a correct way to represent 1, 2, and 3 miles from Janel’s house or from point 0? Why do you think so?

4. Explain: The distance between whole numbers like 1, 2, 3, 4 should be the same on the number line. The distance from 0 to 1 shows the “unit” length. When we build a number line we can keep on adding unit lengths to show the other numbers. If we have 2 unit lengths then the point at the end of 2 unit lengths is the number 2. In this case it represents 2 miles.

5. Invite a student up to show how to place the numbers 3, 4 and 5 on the number line C to represent 3, 4 and 5 miles from Janel’s house (point 0). Ask student to explain his/her thinking. (Does the student make sure the distances between the tick
Teaching Actions

marks are equal?)

6. Ask: The line in this number line ends. Does that mean the numbers end? What can we do to show that our numbers keep on growing? (Include an arrow at the end of the line on the right)

7. Show number line D. There are tick marks/points to the left of zero. What kind of numbers are those? Imagine a winter in Minnesota. The temperature is 5 below zero. Where is “5 below zero” on the number line? What does each tick mark represent in this scenario? How many unit lengths to the left of zero did you count off? What number is at that point?

8. What can we do to the number line to show that our negative numbers keep on going? (Include an arrow at the end of the line to the left)

9. Look at number line D. Ask: Are these all the numbers we have in the world? What are some other numbers we can show on the number line? About where would 1 \(\frac{1}{2}\) be on the number line D to show 1 \(\frac{1}{2}\) miles from Janel’s house? Where would \(\frac{1}{4}\) of a mile be?

Comments

It is important to only estimate where the fractional amounts will be. Just see if the students can see that 1 \(\frac{1}{2}\) is between the two integers 1 and 2; if \(\frac{1}{4}\) is understood to be between 0 and 1.
Teaching Actions

10. Explain that you will end the fraction unit, by finding out different ways to locate fractions on the number line.

Group/Partner work

11. Assign Student Pages A-B to students in pairs. Explain that you will be asking students to share their thinking during the wrap up of the lesson.

Wrap Up

12. Share responses to problems 1-3 on student pages. Have more than one student share his/her thinking. Focus the discussion on the unit and numbers as points on the number line.

13. For problems 4 and 5 act out the problem with a student to emphasize distances from zero and how a number names the distance from zero. For example, ask a student to stand near zero (Project problem 4 on the Smart board). Explain that zero is the starting place; where you live. Now walk one mile, ask: Where are we on the number line? Keep walking stopping at mile marker 2 and 3. Now ask: Where is $3 \frac{1}{4}$ miles?

14. Repeat for Problem 5.

15. Look at Problem 6 together. Have students provide rationales for why the number $1/3$ is between 0 and 1. Don't be surprised if some students say that $1/3$ is

Comments

As students do the seat work, observe if:

- Students place whole numbers on the line by iterating equal unit lengths
- Students determine unit length when given lengths >1
- Students can make reasonable estimates for locating fractional amounts on the line

Identify students to share their solutions to make public any errors or misunderstandings as well as solution strategies that would help all students.

Estimate only distances with fractional amounts.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>the number 1 as 1 is 1/3 of 3. If this happens reflect on 1/3 as a number. Is this number greater than zero but less than one?</td>
<td></td>
</tr>
</tbody>
</table>

**Translations**
- Context to picture to verbal
- Context to picture to symbol
- Symbol to picture
What do we call the numbers above 0?
What do we call numbers below 0?
What is the temperature reading at Point A?
How do you know this?
Number line A

Number line B
Number line C

0 1 2 miles

Number line D

0 1 2 3 4 5
Problem 1

Place the numbers 2 and 3 on the number line. Explain how you did this.

Problem 2

Place the numbers 1, 3 and 4 on the number line. Explain how you did this.

Problem 3

Place the numbers 2 and 4 on this number line. Explain how you did this.
Problem 4

Imagine that you lived 4 miles from schools. You rode your bike $3 \frac{1}{4}$ miles before you got a flat time. Estimate where you think $3 \frac{1}{4}$ would be on this number line. **Put an X at your estimate.** Explain your thinking.

![Number line](image)

Problem 5

The distance from school to the library is 3 miles. What does the number 0 on the number line represent if the 3 miles represents the distance from school to the library?

![Number line](image)

You walked $\frac{1}{2}$ mile to the library from school before your Dad picked you up in his car. Estimate where $\frac{1}{2}$ mile is on the number line. Put an X at that spot. Explain your thinking.
Problem 6

Look closely at the number line. Put in the number 0.

Now consider how big is the number $\frac{1}{3}$. Is $\frac{1}{3}$ between 0 and 1 or 1 and 2 or 2 and 3? Put an X about where the number $\frac{1}{3}$ is on the number line.

![Number line with tick marks](image)

Problem 7

Fill in the missing numbers for the tick marks.

![Number line with tick marks](image)
Rational Number Project

Initial Fraction Ideas
Lesson 16: Overview

In order to make sense of the number line model for fractions students will make connections between the paper folding model for fractions and the number line model for fractions.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 8.5” by 11” paper strips for students and teacher</td>
</tr>
<tr>
<td>• Student Pages A - C</td>
</tr>
</tbody>
</table>

Teaching Actions

Warm Up
Order these fractions from smallest to largest:

\[
\frac{3}{4} \quad \frac{1}{2} \quad \frac{3}{3} \quad \frac{4}{8} \quad \frac{1}{10}
\]

Large Group Introduction

1. Present this story: *The distance between your home and the Dairy Queen is only 1 mile.* Using one paper strip to represent 1 mile, represent 1/3 of a mile. Then use another paper strip to model ¾ of a mile.

2. Ask students to justify why each fraction strip they folded represents the fraction of the mile. Encourage students’ language that communicates they understand that the strip is the whole unit and represents 1 mile. Students should be able to explain that each unit or mile was partitioned into a certain number of equal parts based on the denominator; amount shaded is based on the numerator.

3. Use Student Page A for students to follow along as you complete steps 4–6 of the lesson.

Comments

Note: This is a 2-day lesson.

Please see teacher notes at the end of the lesson for a more detailed discussion of strategies for building meaning for the number line model.

There is a lot of detail in this lesson. Please read through carefully to make sure you understand the process for developing meaning for the unit and how to interpret partitioning on a number line.

Lesson 16 emphasizes identifying the unit and how to count partitions within the unit. Instead of going directly to the number line, students construct a “number line” first with multiple paper strips lined up next to each other.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Take out a paper strip and show fourths. (Or develop pictures of paper strips to use with your Smart Board). State again the story context: distance between their home and Dairy Queen is 1 mile. State that the one strip is your unit or whole and stands for one mile. Tape the strip on the board for all to see (You may want to use larger paper strips like 2” by 14”). A picture of this is in the first box in Student Page A.</td>
<td></td>
</tr>
<tr>
<td>5. Label the start of the paper strip zero miles. Ask students what 0 mile represents (home: you might draw a picture of a house on top of the strip at 0). Ask: Where should you put the number 1 to show the distance of 1 mile modeled with the paper strip? (You might want to write DQ on top of the strip at 1 mile). How many equal parts is the unit partitioned into? What is another name for 1 mile based on those partitions? (4/4) What is another name for 0 miles based on the partitions? (0/4)</td>
<td></td>
</tr>
<tr>
<td>6. Ask: You walked ¼ of a mile from your home to the Dairy Queen. (You may want to act this out by actually walking in front of the picture stopping at the ¼ fold.) Ask students to use their pencil point to walk that ¼ mile. Where are you on the paper strip? Point to the spot on the paper strip showing where you are. What fraction of a mile is it from 0 to that</td>
<td></td>
</tr>
</tbody>
</table>
**Teaching Actions**

**first partition?** If I shaded 1 of the four equal parts on the paper strip, what fraction of the whole strip is shaded? Label the **fold line** at the end of the shaded \( \frac{1}{4} \) part the number \( \frac{1}{4} \). Repeat for \( \frac{2}{4}, \frac{3}{4} \) and \( \frac{4}{4} \). (Students may label the boxes instead of the fold lines. As we are focusing on the attribute of length and the value of the point at the end of the length, encourage students to label the fold lines and not the boxes.)

![Diagram of a paper strip divided into four equal parts]

7. Ask: What is another name for \( \frac{4}{4} \) miles? How many miles is that? Label that amount under 1 mile.

8. Explain that sometimes students have trouble counting up the number of equal parts. One way to count up the total number of equal parts is to count the number of equal parts using loops. Tell the students that another third grader showed the class how to do this as shown below. Ask: How many jumps to the end starting at zero?

![Diagram of counting loops]

9. Repeat for paper strips folded into halves and thirds (Problems 2 and 3). **Do not do Problem 4 at this point.** Continue to use the context of 1 mile as you

**Comments**

Students are encouraged to count up the partitions by using loops if they miscount the number of equal parts by counting all the tick mark instead of the number of lengths.
**Teaching Actions**

partition a fraction strip to show ½ mile or thirds of a mile.

**Small Group/Partner Work**

10. Assign Student Page B to students to practice the ideas so far developed. Problems 2 & 4 ask students to consider fraction greater than one. Observe how students process these tasks and use that information to start the next large group presentation.

**Large Group**

11. Go back to Student Page A, problem 4. Present this context: *The distance from your house to Cub foods is 2 miles.* Take another strip of paper divided into fourths. Tape this one next to the 4th strip already on the board (or set this up on your Smart Board). Ask: How should we label this strip? What does the beginning of the strip represent? (Your house) What number should we use to label that spot? (0) Why? If each paper strip represents 1 mile and we stop at this point (1 mile marker), what number should we put here? Why? Let’s keep walking to the end of the strip where Cub is located. What number should I put here?

```
0 miles  1 mile  2 miles
```

12. Ask: How can we label the partitions between 0
### Teaching Actions

miles and 1 mile? What about 1 mile and 2 miles? (Encourage language like: “1 mile and \( \frac{3}{4} \) more”. Students might want to label using improper fractions; don’t rush this idea as just labeling as mixed numbers is challenging enough at this point).

![Diagram](image)

13. Now look at problems 2 and 4 from Student Page B. Ask students to check their work and make changes as needed. Then have students explain how they counted the number of equal partitions and why the fraction is located between 1 and 2 miles on the strip.

### Small Group/ Partner Work

14. Assign Student Page C

### Wrap Up

15. Share students’ solutions to problems from Student Pages C. Look for errors mentioned in the teacher notes section of this plan. Make the errors public so students can adjust how they count the equal parts of the paper strip shown as a picture. Ask student to explain how they determined the number of partitions? Ask: Did anyone find an effective strategy that didn’t use the loops?

### Translations

- Context to Manipulative to picture to verbal
- Context to Manipulative to picture to symbol
Students have difficulty making sense of a number line as a model for fractions. Because this model is not as concrete as the other models students use to develop meaning for a fraction as a number, the model is not introduced until students have a good grounding in the part-whole model using fraction circles, paper folding and chips. We use contexts related to length to help students make sense of the unit on the number line.

As you teach the number line lessons, you should consider what is distinctive about this model. To make sense of the number line as a model for fractions students have to coordinate visual and symbolic information. This coordination involves three major ideas: (a) making sense of the unit; (b) partitioning a length; and (c) identifying the fraction as a number located as a point on the number line.

The visual information includes numerical symbols and tick marks or points that provide the students with clues to identify the unit on the number line, the number of equal partitions, and the location of the fraction as a number on the line in relation to zero.

For example, to locate the number $\frac{3}{4}$ on a blank number line, a student should construct a unit on the line as a distance from 0 to 1 marked out with points or tick marks on the line. To locate fractions $> 1$ additional units can be added on the number line by iterating that length from 1 to 2 and so on with points or tick marks to designate these additional units. Multiple units are represented on a number line, and the units are continuous with no separation.

Partitioning the unit into equal parts is the next step. One way to partition the unit is to first divide the length between 0 and 1 into equal lengths using tick marks or points on the line. To show $\frac{3}{4}$, a student would mentally separate the unit length into 4 equal parts, indicating this partitioning using 3 marks (not 4) between 0 and 1. Then a student would count over 3 equal lengths (or the value in the numerator) to interpret the
fraction \( \frac{3}{4} \). Students are interpreting \( \frac{a}{b} \) using a part-whole construct onto a length; it involves partitioning into equal parts first, and then iterating or counting over a certain number of equal parts.

To show \( \frac{3}{4} \) in another way a student could iterate a unit length (1/4) across the length of 0 to 1, using tick marks or points to mark off the lengths. The student would have to estimate the size of the unit length so, for example, four, 1-fourths are equally spaced across the unit length with \( 4 \times \frac{1}{4} = 1 \). This strategy involves knowing that the iteration of four, \( \frac{1}{4} \)'s must exhaust the unit length. Here students are interpreting the fraction \( \frac{a}{b} \) as a times \( \frac{1}{b} \).

Finally a student must understand that the point or tick mark at the end of the 3 equal lengths counted from 0 is the number \( \frac{3}{4} \). The number line shows that \( \frac{3}{4} \) is a number between 0 and 1.

**Misconceptions: Understanding the Unit**

It is helpful for a teacher to be aware of common errors students make while learning to model fractions on the number line. Error students make include, misunderstanding what the unit is on the number line. For example, this student circled 3 when asked to locate \( \frac{3}{4} \) on this number line. This error shows that the student thought that the unit is the whole number line shown and not the length between 0 and 1. When using the number line students need to bring to the task their understanding of the relative size of a fraction. In this case \( \frac{3}{4} \) is less than one but greater than zero. Therefore is a number between 0 and 1.
Here is another example of student’s misunderstanding of the unit. When asked to label the tick marks this student ignored the meaning behind the numbers 1 and 2 on the number line and considered the whole line as the unit.

Student’s issue with the unit becomes evident when you ask them to create their own number line. Initially students might identify a unit with symbols for 0 and 1 when placing a fraction on the number line. For example, this student located $\frac{3}{4}$ on the blank line as shown below. She explained as follows: “I drew four things because there are four loops. I did $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$. When asked what the whole unit she said, “the whole is like 4”. She iterated $\frac{1}{4}$ from the start of the number line, and proceeded to count on. While she focused on lengths of her partitions (rather than actual number of tick marks), her fractions were not presented in relationship to the numbers 0 and 1.

In this example, the student located thirds without designating the unit using symbols 0 and 1.
Another misunderstanding is to locate 0 and 1 on the number line at the arrows. This showed students did not see 0 and 1 as numbers on the line.

Notice that this student did not “use up” the whole unit, but just counted by fourths with equal spacing.

With a number line students have to coordinate symbolic and visual information in ways not required with more concrete models. Zero and one in particular are necessary symbols to identify the unit length on the number line. With other models students can be flexible in selecting a unit to partition to model a fraction. A student can use 8 chips or 12 chips to build a model for \( \frac{3}{4} \); a student can use a complete circle or half a circle to partition to show \( \frac{3}{4} \) as 3 of 4 equal groups. With these models the choice for the unit is *implicitly* connected to the number one; often words like “whole” or “unit” are used instead of saying “one”. With a number line, the unit is *explicitly* a length between two points labeled 0 and 1. While the length between 0 and 1 may vary, the unit on the number line is not the entire length of the line or a length between 0 and 4 for example. Part of making meaning of the number line is reinterpreting what is meant by the “whole”, the “unit” and the locations of zero and one.

Obviously making sense of the unit is an important concept for students to master if they are to reinterpret fractions on a number line. Emphasizing what the unit is becomes an important part of instruction. In the RNP number line lessons we rely on story contexts as a way to make sense of the unit on the number line. We build on students' understanding of the relative size of fractions so they consider the size of a fraction like \( \frac{1}{4} \) as a number less than one before they locate the fraction on a number line.

**Misconceptions: Partitioning**

Partitioning errors are common. Students often count the tick marks incorrectly. For example consider this student’s error:
In this response the student appeared to count the number of tick marks between 0 and 1 and 1 and 2 rather than the number of equal intervals between 0 and 1 and 1 and 2. In addition the students did not interpret 2/2 as 1 or 1 2/2 as 2. This error is similarly seen in this student’s response. Here the student counted 3 tick marks instead of counting the lengths between the tick marks.

Partitioning a number line is complex. To partition a length on a number line to locate a fraction, students have to focus on constructing equal lengths between two whole numbers using points or tick marks to do this. The number of tick marks is one less than the number of partitions needed (as designated by the denominator of the fraction). Students are constructing equal lengths (continuous idea) using tick marks (discrete idea). The equal parts constructed with the tick marks need to exhaust the length from 0 to 1. These tick marks need to be interpreted as tools to partition the unit length but also as designations for actual numbers, whole numbers and fractions.

Partitioning with other concrete models is primitive in comparison. With fraction circles, students find pieces of the same color that partition a unit into equal parts; with chips they sort chips into equal groups; with paper folding they physically fold paper into partitions. These models are more forgiving when errors are made; students just clear the workspace and try again. With experience students remember the colors that partition the circles into equal parts; with chips students realize that multiples of the denominator are sets that can be partitioned; with paper folding, sequences for folding paper become internalized. The number line requires the students to spend more time planning their physical actions before making them.
Importance of the Number line as a Model for Fractions

Despite the challenges of this model, the number line provides students the opportunity to consolidate and extend their rational number knowledge they have garnered with other models. The number line provides students the opportunity to represent their understanding of fraction as a number onto a new model that involves explicitly attending to fraction values in relation to each other, and to whole numbers. While the development of the part-whole interpretation is important, the number line interpretation applies understandings of order, equivalence, and magnitude represented within our entire number system. This raises the question as what prior understandings are needed to make sense of this model.

In previous work with fraction operations, RNP found that 6th graders struggled using a number line to build meaning for the steps involved in adding and subtracting fractions, in particular seeing the rationale for a common denominator. Students who successfully used the number line to operate with fractions were guided by knowing the procedures before instruction. Those who did not know the procedures struggled to make sense of them using the number line.

The number line might not be an embodiment to build initial ideas for operating with fractions, but one to apply conceptual and procedural skills constructed with other models. In other words, the number line is a model that gives students a chance to coordinate the knowledge that they have built with more concrete representations. When sequenced after students used other models, students then have the opportunity to apply and extend their rational number knowledge. It is for this reason the number line lessons end the grade 3 fraction unit.
We see the number line as a model that solidifies students’ rational number knowledge built with more concrete models. In grade 3 students translate their understanding of unit and partitioning using fraction circles, chips and paper folding to a more complex model, the number line. In doing this, students solidify their understanding of fraction as a number in relationship with other numbers on the number line.

**The Number Line Lessons**

The number line lessons reflect our understanding of the complexity inherent in the number line model and common misconceptions students have when first working with this new model. The lesson activities are orchestrated to help students overcome these errors. Building on what we learned from students’ thinking and their struggles related to the unit, partitioning, and interpreting a fraction as a number in relationship to other numbers, the number line lessons have been revised again. The original lessons introduced the number line as a translation from the paper-folding model to the number line. This translation idea is fundamental to this particular curriculum. In the new lessons we emphasize multiple translations.

To address students’ issues with reinterpreting what a unit is on the number line, as well as the locations of 0 and 1 on the line, the new lessons now use a translation *from context to paper folding to the number line* to introduce the number line. The contexts involving length give meaning for 0 and 1 as positions that represent a named unit such as miles as well as help students focus on lengths between tick marks as the salient feature of the partitioning. The contexts also emphasize the naming of the tick marks themselves as locations that are the labeled distances from zero. The lessons leading up to the number line lessons were revised so students encounter more explicitly fractions
equal to 0 and 1 with the other models. More time is spent with fractions greater than one with the other models so students experience the partitioning action with multiple instances of a given unit while naming these representations in terms of a given amount as one, as they need to do with the number line.

The lessons were revised to address students’ use of jumps along the number line to partition the unit. A common error students made was not to exhaust the unit length. This may be a carry over from students work with the number line in younger grades with whole numbers. Young children are often encouraged to jump along the number line making loops as they leap from number to number. This method works for skip counting, adding numbers by counting on, or subtracting whole numbers. But with partitioning the unit on a number line to locate fractions, students must look at the entire interval, and determine the size of the partitions. In essence, students need to “look before they leap” or plan ahead before making equal jumps. In the revised lessons, different ways of partitioning the unit are explicitly addressed. In particular we added opportunities for students to reinterpret partitioning by asking students to translate not only from paper folding to the number line, but from partitioning fraction circles to partitioning the number line. We found to meet the needs of more students we needed to provide multiple opportunities to make sense of how to partition the number line.

Please note that understanding how to model fractions on a number line is more complex than naming $\frac{1}{2}$ on the number line that has been partitioned for the students. Students will be able to identify $\frac{1}{2}$ on the number line but not be able to identify fractions in fourths or thirds. Students will be able to identify halves, thirds and fourths on a pre constructed number line before they can consistently create their own number
line for a given fraction. Below find different number line tasks you can use to informally students’ understanding of the number line.

<table>
<thead>
<tr>
<th>Task with premade number line</th>
<th>Task with blank number line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label the tick marks on the number line. Explain how you figured this out.</td>
<td>Now show me 1/3, 2/3 and 3/3 on this number line. Explain your strategy for constructing the number line to show these three fractions.</td>
</tr>
<tr>
<td><img src="image1" alt="Number Line" /></td>
<td><img src="image2" alt="Number Line" /></td>
</tr>
<tr>
<td>Look at this number line. Where is ¾ on the number line? How do you know? Tell me what you are thinking.</td>
<td>Explain your strategy for constructing the number line to show 3/4.</td>
</tr>
<tr>
<td><img src="image3" alt="Number Line" /></td>
<td><img src="image4" alt="Number Line" /></td>
</tr>
<tr>
<td>This is a number line. What can you tell me about the tick marks?</td>
<td>Construct the fraction 1 ¼ on the number line. Explain your strategy for showing 1 ¼ on the number line? (If the student shows 0 and 1 ask, why she/he did this.)</td>
</tr>
<tr>
<td><img src="image5" alt="Number Line" /></td>
<td><img src="image6" alt="Number Line" /></td>
</tr>
<tr>
<td>What number names points A and B. How do you know? Tell me what you are thinking.</td>
<td></td>
</tr>
</tbody>
</table>
Order these fractions from smallest to largest.
Be ready to explain your thinking.

\[
\frac{3}{4} \quad 1\frac{1}{2} \quad \frac{3}{3} \quad \frac{4}{8} \quad \frac{1}{10}
\]
<table>
<thead>
<tr>
<th>Problem 1</th>
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<tbody>
<tr>
<td></td>
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<td>Problem 2</td>
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<td></td>
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<td>Problem 3</td>
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<td></td>
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<tr>
<td>Problem 4</td>
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</tbody>
</table>
1. This picture represents the length of a ribbon that has been folded into equal parts. If you made a cut at the 1-fourth partition, what fraction of the yard is that? Label the picture with that fraction.

![Diagram of yard divided into four equal parts, with a cut at the 1-fourth partition marked.]

2. This picture represents 2 miles. If you ran 1 mile and 1/3 more where would that fraction be on this representation?

![Diagram of miles divided into three equal parts, with a cut at the 1 and 1/3 mile partition marked.]

3. You have a licorice strip that is 1-foot long. You are sharing it fairly between yourself and two friends. Label the amount that you will receive below. What fraction of the licorice will you get to eat?

![Diagram of feet divided into three equal parts, with a cut at the 1/3 foot partition marked.]

4. You walk 2 miles to school everyday. You walked 1 ½ miles when your friend joined you for the rest of the way to school. Show on the picture of the paper strip where 1 ½ miles are.

![Diagram of miles divided into three equal parts, with a cut at the 1 ½ mile partition marked.]

With your partner, draw a picture of paper folding strips to show each fraction.

<table>
<thead>
<tr>
<th>You ran 1½ miles from school to your home. Model that amount using a picture of paper folding strips. If one paper strip represents one mile, how many units will you need to show 1½ miles?</th>
</tr>
</thead>
<tbody>
<tr>
<td>You cut a length of rope that is $1\frac{1}{3}$ feet in length. Model that amount using a picture of paper-folding strips. If one paper strip represents one foot, how many units will you need to show this amount?</td>
</tr>
<tr>
<td>Your long jump was 2 and $\frac{3}{4}$ yards. Model that amount using a picture of paper-folding strips. If one paper strip represents one yard, how many units will you need to show this amount?</td>
</tr>
<tr>
<td>You ate $\frac{3}{4}$ of the subway sandwich. One paper strip represents a whole subway sandwich. Show how much you ate. How many units will you need to show this fraction?</td>
</tr>
</tbody>
</table>


**Rational Number Project**

### Initial Fraction Ideas

**Lesson 17: Overview**

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
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<tbody>
<tr>
<td>• Student Pages A - C</td>
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</tbody>
</table>

In order to make sense of the number line model for fractions students will make connections between the paper folding model for fractions and the number line model for fractions.

### Teaching Actions

**Warm Up**

The picture represents a length of 2 feet. Label: 0 feet; 1 foot; 2 feet. Partition each foot into 2 equal parts. What fraction is each part?

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### Large Group Introduction

1. Imagine that you are walking along one of the lakes in Minneapolis. The distance from your car to the ice cream stand is about 2 miles.

2. Explain that if one mile is represented by one paper strip, how many paper strips are needed to show 2 miles?

   How can you label the picture of the paper strips to show 0, 1 and 2 miles?

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</table>

3. You walked \( \frac{1}{4} \) mile from your car to the ice cream stand when you met two of your friends. Ask students where \( \frac{1}{4} \) is on this picture? Encourage different students to explain

   Successful students often partition 4ths by first partitioning the unit length into half and then each half into halves again.

In this lesson, students build on paper folding pictures to draw number lines that are simpler versions of the pictures for paper folding model. **This lesson may take 2 days to complete.**

You may want to act this out in front of the picture projected on the Smart Board.
Teaching Actions

their reasoning for locating \(\frac{1}{4}\) between 0 and 1 and how they partitioned the length between 0 and 1 into 4 equal parts.

4. You continued walking with your friends. When you walked 1 ¼ miles you ran into your teacher. Locate 1 ¼ miles on the picture. (Students now have to partition the length between 1 and 2 into 4 equal parts).
   Encourage different students to explain their reasoning.
   Ask: Why is 1 ¼ between 1 and 2 on the picture?

5. Explain that instead of drawing the whole rectangle for each paper strip we use to model fractions, we can just draw a horizontal line to show each paper strip and use “tick” marks to show each partition.

6. Draw these two pictures or project them on the Smart Board while students follow along with Student Page A.
   Ask students to build the number line below their picture of the paper-folding strip on Student Page A.

   |   |   |   |   |   |   |   |
   0 1/4 1 1 1/4 2
   miles mile miles

   - To emphasize the units, use larger tick marks for 0, 1, 2, etc.

   Students may put 0 and 2 on the arrows. If they do that help them see that 0 is a number and is a point on the number line.
   Remind them that the arrows are just a way to communicate that the numbers keep on going (3, 4, 5... or -1, -2, -3...)

7. Ask: How are the two pictures alike? How many units are shown in the paper-folding picture? How many units do you see on the number line? Where should we put the

Comments

We want to help students to consider \(\frac{1}{4}\) as 1-fourth of the length between 0 and 1 and as a point at the end of that length. The student action is to partition the unit representing 1 mile into 4 equal parts. They might want to shade the first equal part. But help students see that the fraction is a point at the end of the shaded part. At the end of that length you have gone \(\frac{1}{4}\) of a mile.
numbers 0, 1 and 2 on the number line? What do the arrows mean on the number line?

8. Ask: Can anyone show me with loops, how to count the number of partitions between 0 and 1 on the paper folding picture? How can we label the other equal partitions on the paper-folding picture between 0 and 1? How can we label the partitions between 1 and 2?

9. Ask students to remind you how this number line is like the picture of the paper folding. How many equal **lengths** are there between 0 and 1 on the number line? Can you show with loops? What does that tell you about the name for each partition? What fraction should we put under the first tick mark – the end of the first length? The second tick mark? Continue to name all partitions.

10. Suggest that another student thought that the place they labeled ¼ should be 1-eighth because he counted 8 loops from 0 to 2. What do you think about that?

11. Say: I see a difference between the number line picture and the picture for paper folding strips. What do the arrows at the ends of the line mean?

12. Repeat for the other 3 problems on Student Page A as needed. Have students build matching number lines with labels under each picture of paper-folding strips.

Address a common error in the warp up where students ignore the units on the number line and consider the whole line as the unit.

Before going on to the small group work, point out that the length of the unit can vary with a number line. Both examples below are number lines with 2 units shown. But the units are different lengths.

You may want to end the lesson here and use the wrap up section as the next day's lesson.
Teaching Actions

Small Group/ Partner Work

13. Assign Student Pages B and C.

Wrap Up

14. Ask students to explain how they labeled selected answers to problems on Students Pages B and C. For Page C reflect on students’ work by looking to see if
   - Students identified 0, 1 and 2 as needed?
   - Students partitioned 4ths by dividing lengths into half and then half again?
   - Students were able to partition into 3rds accurately using two tick marks between 1 and 2 to equally partition the length into 3 equal parts?
   - Students completed the partitioning first and then located the fraction on the number line?

Show a blank number line. Ask students to draw a blank number line on their paper. Explain that you want to model 2 feet of string. Ask students to show 0 feet, 1 foot and 2 feet on the number line. (Observe if students see the need for equal units.)

Ask students to find these fractions on the number line by partitioning each unit into the needed number of partitions. Where would 1 ½ feet be on the number line? Where would ¾ of a foot be on the number line? Why is 1 ½ between 1 foot and 2 feet while ¾ is between 0 and 1?

Challenge students by asking: Where is 3/8 on the

Comments

This list represents successful strategies for partitioning number line to show fractions.

Observe which students are able to partition and which ones are still struggling. The last number line lesson will try a different translation (Fraction circles to number line) to help students still making errors with the partitioning.
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>number line?</td>
<td></td>
</tr>
</tbody>
</table>

**Translations**

- Context to Picture to picture to verbal
- Picture to picture to symbol
- Picture to symbol
The picture represents a length of 2 feet. Label: 0 feet; 1 foot; 2 feet. Partition each foot into 2 equal parts. What fraction is each part?
Each problem tells the story of a bug walking along a length. A number line is shown to represent the path the bug walks. Your job is to figure out what fraction the bug walked. Be prepared to explain your thinking.

Problem 1: A ladybug is walking along a strip of wood that is 2 yards long starting at 0 yards. It stops at the first dot. What fraction of 1 yard is that? Label that point. The ladybug continues its stroll down the strip of wood. He gets to the 1-yard marker and continues to the next dot. How far did the bug walk? Label that point.

Problem 2: A caterpillar is slowly strolling along a windowpane that is 1 meter in length. He started at the very left edge of the pane. It stops at the dot shown on the number line. What fraction of 1 meter did the caterpillar walk?

Problem 3: The grasshopper stood perched on a branch of a bush. It jumped toward the next bush that was 2 yards away. He didn’t jump very far. The dot on the number line represents how far he jumped. What fraction of a yard did the grasshopper jump? Label that point.

Problem 4: A snail slowly crawled along a log about 1-meter long starting at the very left end of the log. After 5 minutes he didn’t get very far. The dot shows how far he crawled. What fraction of a meter did the snail crawl?
Imagine a bug crawling along a length. With your partner, label the number of units needed then partition the line into equal parts to show how far the bug crawled. Don’t forget to show where the bug started – use the number 0. Clearly label the fraction on the number line to show how far the bug crawled.

Bug started at the end of log. It then crawled $\frac{1}{2}$ yards.

Bug started at the end of log. It then crawled $\frac{1}{3}$ yards.

Bug started at the end of log. It then crawled $\frac{3}{4}$ of a yard.

Bug started at the end of log. It then crawled $2\frac{1}{4}$ yards.
Rational Number Project

**Initial Fraction Ideas**
**Lesson 18: Overview**

Students will continue to build meaning for a fraction as a number on the number line, the role of the unit on the number line, and how to partition a number line by making connections between fraction circles as a model and the number line as a model for fractions.

**Materials**
- Fraction circles
- Student Pages A - C

**Teaching Actions**

**Warm Up**
What fraction is point A?

```
A
0 ———— 1 ———— 2
```

**Large Group Introduction**

1. Ask: What other models have we used to show fractions? (Fraction circles, chips and paper folding).
2. Lead the class through a discussion that translates the actions for modeling fractions using fraction circles to modeling fractions on a number line.
3. To encourage student involvement have each student fold a sheet of paper into 4 equal parts. Each box is for them to record the Fraction circle to number line translation you present. Sample student work is below:
4. Problem 1: Use language as described here to lead the translation from fraction circles to the

**Comments**

In this lesson we are moving away from context to thinking of fractions as numbers on the number line. We build connections between pictures of the familiar fraction circles model for fractions and the number line. This should assist students who are still struggling with how to partition a unit length into equal parts.

As you go over the warm up, see if students correctly labeled the number line without a context. Ask: How many equal parts are there between 0 and 1? What is the value of the tick mark at the end of the length from 0 to the first tick mark? Second tick mark?

Ask students to explain why that tick mark is not 1/8? (Common error is to count the partitions between 0 and 2; there are 8 partitions so it would be 1/8.)
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>number line.</td>
<td>In this lesson students are translating from a familiar model to the number line model. Looking at how the different models are the same, will help students better understand the role of the unit on the number line and how to partition a number line.</td>
</tr>
<tr>
<td>• This whole circle is my unit. ⊙</td>
<td></td>
</tr>
<tr>
<td>I am going to show a fraction with this unit and I want you to show the same fraction on your number line. Draw a blank number line in the upper left corner of your paper. (Do students use arrows at the end of the line?)</td>
<td></td>
</tr>
<tr>
<td>• How can you identify 1 unit on the number line? (Use symbols 0 and 1 as points on the line. Length between the symbols is the unit.)</td>
<td></td>
</tr>
<tr>
<td>• To show the fraction I am thinking of I need to partition my unit circle into 4 equal parts. What fraction is each equal part?</td>
<td></td>
</tr>
<tr>
<td>• How can you partition your unit on the number line into 4 equal lengths? (Ask for volunteers to explain their way to partition into 4ths. Do students partition into ( \frac{1}{2} ) first and then halves again?)</td>
<td></td>
</tr>
<tr>
<td>• Look at your number line. Let’s label each tick mark.</td>
<td></td>
</tr>
<tr>
<td>• Now I am going to show my fraction on the fraction circle. (Shade 3 of the 4 equal parts).</td>
<td></td>
</tr>
</tbody>
</table>
### Teaching Actions

Ask what fraction of the whole circle is shaded?

- Put a point at the number \( \frac{3}{4} \) on your number line. Explain that \( \frac{3}{4} \) is a point at the end of the third length from zero. Because the unit is partitioned into 4 equal parts, the length is \( \frac{3}{4} \) the distance from 0 to 1.

- Ask: how do both models show the number \( \frac{3}{4} \)? What is the same about both models? What is different between the models?

5. Repeat for these fractions: \( 1 \frac{1}{2}; \frac{2}{3}; 1 \frac{1}{4} \)

6. When placing a fraction on a number line students should consider first if the fraction to be shown is greater or less than 1 or greater or less than other whole units. To show a fraction greater than one students will need more than one unit. With fraction circles that is easy to show; just use two or more circles. With the number line students have to consider how to show, for example, two units by using the symbol “2” at the end of a length that is the same distance as from 0 to 1.

### Comments

To make sense of the number line model, students have to bring a good understanding of the relative size of fractions as compared to whole amounts. They need to know ahead of time that, for example, \( 2 \frac{1}{4} \) is between two and three.

For fractions greater than one, students have to interpret the partitions on a number line correctly. With circles students have an easier time looking at this model and counting 3-halves.

With the number line, students often consider the whole line the unit even when 0, 1 and 2 are shown.

Students might label the tick marks fourths ignoring the meaning of the symbols 1 and 2. The dot on this line would then be \( \frac{3}{4} \). That is why it is important to have students ponder the relative size of fractions before locating them on a number line or considering what value the tick marks.

### Small Group/Partner Work

- Assign Student Pages A - C. Student Page B is optional. Students translate from other models to the number line.

### Wrap Up

- Share student responses to selected problems
<table>
<thead>
<tr>
<th>Teaching Actions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>on Student Page A. Go through each example on Student Page C.</td>
<td>have if they are between 1 and 2. As ( \frac{3}{4} &lt; 1 ), the point on the line cannot be ( \frac{3}{4} ).</td>
</tr>
<tr>
<td>• Guiding questions for Student Page C:</td>
<td></td>
</tr>
<tr>
<td>o Where is the unit on the number line? How many units are shown on the number line?</td>
<td></td>
</tr>
<tr>
<td>o How is the length from 0 to 1 partitioned? (Or 1 to 2). How many equal parts is the unit partitioned into? How does that information help you locate the fractions on the number line?</td>
<td></td>
</tr>
<tr>
<td>o If a fraction is greater than one but less than 2 on which unit would you locate the fraction on the number line?</td>
<td></td>
</tr>
</tbody>
</table>

**Translations**

1. Picture to symbol
2. Symbol to manipulative to picture
What fraction is point A?
<table>
<thead>
<tr>
<th></th>
<th>Question</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hamdi drew this model for a fraction. If she used the whole circle as her unit, what fraction did she model? Is that amount greater or less than 1? Show that fraction on the number line.</td>
<td><img src="image1.png" alt="Fraction Model" /></td>
</tr>
<tr>
<td>2</td>
<td>If the whole circle is the unit what fraction of the circle is shaded? Is that amount greater or less than 1? Show that fraction on the number line.</td>
<td><img src="image2.png" alt="Fraction Model" /></td>
</tr>
<tr>
<td>3</td>
<td>If the whole circle is the unit, what fraction is shown? Show that fraction on the number line.</td>
<td><img src="image3.png" alt="Fraction Model" /></td>
</tr>
<tr>
<td>4</td>
<td>If the whole circle is the unit what fraction is shown below? Show that fraction on the number line.</td>
<td><img src="image4.png" alt="Fraction Model" /></td>
</tr>
<tr>
<td></td>
<td>Question</td>
<td>Solution</td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>5</td>
<td>Paula folded her paper strip to show ( \frac{2}{3} ).</td>
<td><img src="image1" alt="Fraction diagram" /></td>
</tr>
<tr>
<td></td>
<td>Show that fraction on the number line.</td>
<td><img src="image2" alt="Number line 0-1" /></td>
</tr>
<tr>
<td>6</td>
<td>Gena showed ( \frac{2}{3} ) using 6 chips as her unit. Is ( \frac{2}{3} ) greater than one or less than one?</td>
<td><img src="image3" alt="Fraction with chips" /></td>
</tr>
<tr>
<td></td>
<td>Show the number ( \frac{2}{3} ) on the number line.</td>
<td><img src="image4" alt="Number line with tick marks" /></td>
</tr>
<tr>
<td>7</td>
<td>Sophia measured her lima bean plant. It measured 1( \frac{1}{2} ) inches high. Show that amount on the number line.</td>
<td><img src="image4" alt="Number line with tick marks" /></td>
</tr>
<tr>
<td>8</td>
<td>Joe drew this picture to model 1( \frac{1}{3} ). Show this fraction on the number line.</td>
<td><img src="image1" alt="Fraction diagram" /></td>
</tr>
</tbody>
</table>
1. Circle the letter that represents the location for the number $\frac{1}{2}$ on the number line. (Is $\frac{1}{2}$ greater or less than 1?) Cross out the letter that represents $1 \frac{1}{2}$ on the number line.

![Number Line](image1)

2. Circle the letter that represents the number $1 \frac{3}{4}$ on the number line. Is $1 \frac{3}{4}$ greater or less than 1?

![Number Line](image2)

3. Place the following numbers on the number line: $\frac{1}{3}$ $\frac{2}{3}$

![Number Line](image3)

4. What fraction names this point on the number line? Is the fraction greater or less than 1?

![Number Line](image4)

5. What fraction names this point on the number line?